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Multi-digit number processing: Cognitive mechanisms and their impairment

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Abstract

The numbers system is one of the widest-used symbolic systems in human culture – almost all of us read, write, and understand numbers. As a formal system, it is very simple: merely 10 digits, few dozens of number words, and a relatively small set of formal rules are sufficient to define how any digit string can be converted to the corresponding sequence of number words and vice versa, and the quantity represented by these sequences. As a cognitive system, however, the number system is not that simple. It involves multiple representations – digits, number words, and quantity – and dedicated conversion pathways between these representations. The complexity of this system is demonstrated by the long period – several years – that it takes children to master it, and by the finding of many different cognitive disorders that disrupt number processing.

One cognitive challenge in handling numbers is converting them from one representation to another. The challenge is especially difficult for multi-digit numbers: they require not only mapping digits and number words between representations, but also taking into account the relations between these elements. Thus, multi-digit numbers boost the number system from a simple symbolic system into a syntactic system, one that represents the number's structure.

This PhD research examined two conversion pathways of multi-digit numbers: how a digit string is converted to quantity when we comprehend it, and how it is converted to a sequence of number words when we read it aloud. For each of these pathways, I focused on the syntactic processes that handle the number' structure.

Converting digit strings to quantity. To investigate this conversion process, we developed a novel research paradigm: participants pointed to the location of multi-digit numbers on a number line, while their finger location was continuously monitored. The location marked on the line reflect the underlying quantity representation, and the intermediate finger movement reflects the process of creating this quantity.

One such question examined using this paradigm was whether the quantity representation relies on a linear or a compressive scale. Previous studies consistently showed that educated adults map numbers to positions linearly. In 8 experiments with 174 participants, they too showed a linear mapping pattern at the trajectory endpoints, but crucially, they showed a transient logarithmic effect in intermediate finger locations. This transient log effect resulted from differential processing speeds of small versus large quantities: small numbers are

processed faster than large numbers, so the finger deviates towards the target position earlier for small numbers than for large numbers. When the finger deviation times were controlled for, the log effect disappeared. This faster processing of small numbers presumably results from nonlinearity in the quantity representation: the encoding of larger quantities is fuzzier, so it takes longer to process.

A second major question was whether the digits of a multi-digit number are processed serially or in parallel. To examine this question, we ran two number-to-position experiments while inducing a lag between the appearance of the decade and unit digits. Inducing a lag in the unit digit delayed the unit effect on finger movement by 35 ms less than the lag duration, indicating an idle time window of 35 ms in the units processing pathway. We propose that this idle time window results from the creation of a syntactic frame, a representation of the multidigit number's structure.

We also examined the decision processes involved in the number-to-position task. Bayesian theory predicts that optimal decisions start from a prior probability distribution of possible responses, acquired from previous trials, and update it according to specific evidence received on the current trial. We examine this prediction with our task by manipulating three different factors: the finger's initial direction, the distribution of prior target numbers, and the current target number. As predicted by the Bayesian model, finger direction was sequentially affected by these three factors, in this order.

The investigation of number comprehension concludes with a detailed cognitive model of the number-to-position mapping task, comprising 3 distinct stages: a quantification stage; a Bayesian accumulation-of-evidence stage, leading to a decision about the target location – first according to prior trials, then according to the current-trial target; and a pointing stage. The model describes several sub-processes in the quantification stage: creating a syntactic frame for the multi-digit number; binding each digit to a decimal role in this frame (units, decades, etc.); quantifying the digit according to this role; and integrating the per-digit quantities into whole-number quantity representations – an exact-linear representation and an approximate-compressive representation.

Converting digit strings to number words. To dissect this conversion pathway, we investigated the number processing abilities of seven individuals with different selective deficits in number reading. Some participants were impaired in visual analysis of digit strings: in encoding the digit order, encoding the number length, or parsing the digit string to triplets.

Others were impaired in verbal production, making errors in the number's structure. Based on the participants' selective impairments and on previous findings, we propose a detailed cognitive model of number reading. The model postulates that within visual analysis, separate sub-processes encode the digit identities and the digit order, and additional sub-processes encode the number's decimal structure: its length, its triplet structure, and the positions of 0. Verbal production consists of a process that generates the verbal structure of the number, in a tree-like structure that is then linearized to a sequence of number-word specifiers; and another process that retrieves the phonological forms of each number word.

The degree of specificity of these number reading processes was investigated in two additional studies. The first of these studies examined whether number reading (converting digit strings to words) uses the same cognitive processes as word reading (converting letter strings to words). We review in detail the various sub-processes involved in reading words and numbers, and ask whether each of them serves only word reading, only number reading, or both. The review of previous studies, together with two new word-number dissociations we found in the present study, led to the conclusion that the reading pathways of words and numbers are almost fully separate. We propose that differences in the morpho-syntactic structure of words and numbers may underlie this separation.

The second study examined whether number reading (converting digit strings to words) uses the same cognitive processes as number comprehension (converting digit strings to quantity). Again, the answer was negative: we investigated ZN, an aphasic patient who could not read two-digit strings but could convert them to whole-number quantities. This dissociation indicates that the syntactic processes that handle the number's structure during number reading are separate from the syntactic processes involved in number comprehension.

Arithmetic facts. The final section of this dissertation investigated a possible source of difficulty in learning the multiplication table: hypersensitivity to interference, a severe difficulty in memorizing similar verbal items. Previous studies showed correlational evidence for relation between hypersensitivity to interference and multiplication deficits; here, we provide causal evidence. In a training study, we manipulated the degree of interference, and we showed that high-interference conditions disrupted the memorization of the multiplication table but low-interference conditions did not. We lay out a possible analogy between this case and other situations of sensitivity to interference, and propose that sensitivity to interference is a property of syntactic systems that represent relations between items. Furthermore, our method indicates that effective teaching of the multiplication table should group together multiplication facts as dissimilar from each other as possible; this may call for reconsideration of the teaching methods in elementary schools.

Overall, this dissertation resulted in two detailed cognitive models of multi-digit number processing: one describing how digit strings are converted to quantity, and the other describing how digit strings are converted to oral number words. As part of this dissertation, we created a novel research paradigm (trajectory-tracked number-to-position task), and a battery for detailed assessment of cognitive disorders in symbolic number processing. Finally, we developed a treatment method for rehabilitation of impaired knowledge of calculation facts.

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Appendices

1. Introduction

Most educated adults can easily read, write, and understand multi-digit numbers, yet this cognitive ability is not at all simple. Culturally, it took millennia to develop the symbolic system of numbers and the place-value decimal system. From a developmental perspective, it takes young children several years until they master the decimal system and the meaning of numbers (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Geary, 1994; Siegler & Booth, 2004; von Aster & Shalev, 2007). And clinically, brain injuries can cause many different numerical deficits (Cipolotti, Butterworth, & Denes, 1991; Dehaene & Cohen, 1991, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Delazer & Bartha, 2001; Delazer, Girelli, Semenza, & Denes, 1999; Deloche & Seron, 1982; Geary, 1993; McCloskey, Sokol, & Goodman, 1986; Willmes, 2008).

The history of multi-digit number systems is long and winding. Most of the early number systems were additive/subtractive systems, where the quantity is obtained by summation or subtraction of the digit values (e.g., in Roman numerals XVII = 10+5+1+1 = 17). Additive systems were abandoned with time, except sporadic use such as the way we use Roman numerals nowadays. They were replaced by positional systems, in which the value of a digit symbol, and often the number word that corresponds with that digit, depends not only on the symbol but also on its position within the string. Even after positional systems were invented, they were still quite different from modern positional systems: in the earliest positional system, the Babylonian system, numbers were written with several symbols per position (e.g., the digits 1 and 4 both occupied one position, in which 1 was represented by a single symbol 𒁹, and 4 was represented by a sequence of symbols: 𒁹𒁹𒁹𒁹). This system did not have a zero digit – a null value in a certain position was indicated by space. For example, the number 7203, which in this base-60 system is broken down into $2 \times 60^2 + 0 \times 60^1 + 3 \times 60^0$, was written as 3 (𒁹𒁹𒁹) in the units (rightmost) position, empty value (space) in the 60^1 position, and 2 (𒁹𒁹) in the 60^2 (third) position: 𒁹𒁹   𒁹𒁹𒁹 . It took about two thousand more years to reach a number system that essentially resembles the modern system, with one character per position and a zero symbol to indicate null-value positions (Chrisomalis, 2010).

The complexity introduced by multi-digit numbers is not only historical but also cognitive, because the leap from single digits to multi-digit numbers involves some additional dedicated cognitive mechanisms. Some of the strongest evidence for the existence of such dedicated mechanisms come from situations in which these mechanisms malfunction. Several case studies reported individuals who had good processing of single digits, but difficulties in processing multi-digit numbers, with error types that suggested a selective deficit in digit-integration mechanisms (Cipolotti, 1995; Cipolotti, Warrington, & Butterworth, 1995; McCloskey, 1992; McCloskey, Sokol, Caramazza, & Goodman-Schulman, 1990). The complexity of multi-digit numbers is experienced not only by individuals with cognitive deficits, but by almost everybody: learning to handle multi-digit numbers is a long and tedious process for children. Once children have learned single digits, it takes them several more years to learn how to read multi-digit numbers (although they do exhibit partial knowledge of the decimal system more quickly, Barrouillet, Thevenot, & Fayol, 2010).

This PhD research investigated the cognitive mechanisms of number processing, focusing on multi-digit numbers – i.e., on the mechanisms that integrate digits into a number. From a theoretical perspective, knowing how our cognitive system handles digit integration would improve our understanding of these numerical cognition mechanisms, and as we shall see below, may also have wider implications regarding the way our cognitive system handles integration of information in general. From a clinical perspective, by improving our knowledge of the cognitive mechanisms of multi-digit number processing, we can help individuals with number processing impairments.

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Historically, the research of numerical cognition has received much less attention than the research of reading or writing. A major advance in the field of numerical cognition was during the mid-1980's and early 1990's, when Michael McCloskey and his colleagues published a cognitive model of number processing (McCloskey, 1992; McCloskey et al., 1986). This model aimed to explain the behavior of healthy individuals as well as that of individuals with number-related cognitive impairments (Mccloskey & Caramazza, 1987), and to allow for assessment of specific cognitive malfunctions in individuals with number-related cognitive impairments (McCloskey, Aliminosa, & Macaruso, 1991; McCloskey, Caramazza, & Basili, 1985; McCloskey et al., 1990, 1986). The general architecture of this model was a central semantic representation of numbers, which serves as the workbench for various operations such as calculation. The model further postulated format-specific peripheral processes that convert sequences of digits and sequences of number words from/to the central semantic representation, and described in detail several of these processes.

McCloskey's assumption of a central semantic representation was challenged by Stanislas Dehaene and Laurent Cohen during the early 1990's: their triple-code model stipulates three distinct representations of numbers – digits, words, and quantity – with no central semantic representation (Dehaene, 1992; Dehaene & Cohen, 1995). According to this model, number-related operations operate on one or more of these representations, and the conversion (transcoding) between each pair of representations is done by a dedicated direct processing route. Several studies supported the notion of three different number representations (Benson & Denckla, 1969; Cohen & Dehaene, 1995; Cohen, Dehaene, Chochon, Lehéricy, & Naccache, 2000; Cohen, Verstichel, & Dehaene, 1997; Dehaene et al., 2003; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Friedsmann, Dotan, & Rahamim, 2010; Marangolo, Nasti, & Zorzi, 2004; Marangolo, Piras, & Fias, 2005; McCloskey et al., 1986; Noël & Seron, 1993) as well as the existence of direct transcoding routes (Cohen & Dehaene, 1991, 2000; Cohen, Dehaene, & Verstichel, 1994). At the onset of the 3rd millennium, the triple-code model is widely accepted.

If numbers are indeed converted between representations by several direct transcoding routes, the structure of each of these transcoding routes becomes a major question. For single-digit numbers, one may perhaps hypothesize that transcoding involves direct mapping from one representation to another. However, the complexity of multi-digit numbers seems unlikely to be solved merely by direct mapping. Consider, for example, the processes that you must perform to read aloud the number 4761 or to understand the quantity it represents. You would have to encode the positional relations between digits, to map each digit to the corresponding word or quantity, and potentially to integrate these words or quantities while respecting the relative order of digits and their decimal positions. The majority of this dissertation concerns two such representation-conversion processes: section A investigates the processes involved in multidigit-to-quantity conversion, namely, how we understand the quantity represented by multi-digit numbers. Section B investigates the processes involved in multidigit-to-verbal conversion, namely, how we read aloud numbers.

Section A: Converting multi-digit numbers to quantity

Within the domain of numerical cognition, research in the recent decades has put much attention to the representation of quantities. Quantities are encoded by the Approximate Number System (ANS), which represents numeric magnitudes in a continuous and approximate manner (Brannon, 2006; Dehaene, 1997; Dehaene & Cohen, 1995; Dehaene, Dehaene-Lambertz, & Cohen, 1998; Dehaene, Izard, Spelke, & Pica, 2008; Piazza, 2010; Xu, Spelke, & Goddard, 2005). With respect to multidigit numbers, existing research raised at least two major questions, both of which are addressed by this dissertation. The first question is whether a multi-digit number is transcoded into single, holistic quantity (Dehaene, Dupoux, & Mehler, 1990; Fitousi & Algom, 2006; Reynvoet & Brysbaert, 1999), or is handled in a decomposed manner, as several single-digit quantities (Meyerhoff, Moeller, Debus, & Nuerk, 2012; Moeller, Fischer, Nuerk, & Willmes, 2009; Nuerk & Willmes, 2005). The second question is whether the digits of a multi-digit number are quantified in parallel or sequentially (Hinrichs, Berie, & Mosell, 1982; Meyerhoff et al., 2012; Moeller, Fischer, et al., 2009).

A third question is the nature of the internal scale used by the ANS to encode quantities. Several studies showed that the ANS exhibits logarithmic rather than linear patterns (Anobile, Cicchini, & Burr, 2012; Berteletti et al., 2010; Booth & Siegler, 2006; Dehaene et al., 2008; Dehaene & Marques, 2002; Lourenco & Longo, 2009; Núñez, Doan, & Nikoulina, 2011; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003; Viarouge, Hubbard, Dehaene, & Sackur, 2010). These findings were explained as indicating some form of nonlinearity in the internal quantity scale – either the scale itself is logarithmic, or the information-to-noise ratio gets worse as the quantity increases (scalar variability, Cicchini, Anobile, & Burr, 2014; Dehaene, 2007). Importantly, in the context of digit-to-quantity transcoding, logarithmic patterns were observed only for young children, suggesting perhaps that the quantity scale changes with aging or following education (Berteletti et al., 2010; Booth & Siegler, 2006; Dehaene et al., 2008; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003). The present research showed such logarithmic patterns even in adults.

Section B: Converting multi-digit numbers to number words

Section B dissects the mechanisms of digit-to-verbal transcoding, i.e., number reading. Several studies showed that number reading is a unique process that is dissociable from the opposite transcoding pathway – verbal-to-digit transcoding, or number writing (Cipolotti, 1995; Cipolotti, Butterworth, & Warrington, 1994; Lochy, Domahs, Bartha, & Delazer, 2004). Digit-to-verbal conversion is also dissociable from digit-to-quantity conversion (Cohen & Dehaene, 2000). Moreover, within number reading, separate sub-processes handle the single-digit identities (“lexical” processes), whereas other sub-processes handle the relations between digits (“syntactic” processes) (Cipolotti, 1995; Cipolotti et al., 1995; Dotan & Friedmann, 2015; McCloskey, 1992; McCloskey et al., 1990). Here, we present a new, detailed cognitive model for the processes involved in number reading. To support this model, we report several neuropsychological case studies, each corroborating certain model parts.

We also examined the relation between number reading (multidigit-to-verbal transcoding) and two related operations: number comprehension (multidigit-to-quantity transcoding) and word reading. The question was whether these two operations use the same cognitive mechanisms as number reading, or different mechanisms. In particular, we examined whether the digit-integration mechanisms that enable reading multi-digit numbers are dedicated to this purpose, or they also serve word reading or number comprehension. In two studies, we showed that multidigit number reading dissociates both from word reading and from multidigit number comprehension.

Section C: Learning the multiplication table

The last section of this dissertation examines a slightly different topic – the learning of the multiplication table. Conceptually, memorizing the multiplication table may be viewed as another form of digit integration – only here, the integration does not take the form of processing several digits within a multidigit string, but of associating a digit pair with the multiplication result. This digits-result association – namely, the knowledge of multiplication facts – is thought to be stored verbally (Dehaene & Cohen, 1997; Dehaene et al., 2003; Lemer, Dehaene, Spelke, & Cohen, 2003).

Learning the multiplication facts is extremely difficult for some individuals (Geary & Hoard, 2001; Gross-Tsur, Manor, & Shalev, 1996; Landerl, Bevan, & Butterworth, 2004; McCloskey, Harley, & Sokol, 1991; van Harskamp & Cipolotti, 2001). Interestingly, a recent line of studies by Alice De Visscher and Marie-Pscale Noël (De Visscher & Noël, 2013, 2014a, 2014b) showed that at least in some cases, this kind of dyscalculia may be explained by a more general verbal difficulty, which they termed "hypersensitivity to interference". Individuals with this disorder have great difficulty in memorizing verbal items that are similar to each other, as is the case in the multiplication table. To make this point, De Visscher and Noël showed that hypersensitivity to interference correlates with impaired knowledge of multiplication facts. Here, we stress this point even further, by showing not only correlational but also causal evidence: we report a woman with hypersensitivity to interference, and we show that when interference is reduced, she could easily learn the multiplication facts. This study also had an important clinical goal: it created an intervention program aimed to teach the multiplication facts to individuals with hypersensitivity to interference, and it showed the effectiveness of this program.

Section A

From digits to quantity

Section A: From digits to quantity

The invention of multi-digit numbers is a major achievement in our culture. It took mankind centuries to develop the idea that large numbers can be represented with merely 10 symbols by relying on their relative positions. During education, the human brain learns the decimal system and, ultimately, it becomes very intuitive that the digit 4 in 41 stands for four decades, while the digit 4 in 14 stands for four units. But what is it exactly that we understand? How does our brain represent multi-digit quantities, and what are the processes that convert a sequence of digit symbols into this quantity representation? In spite of our growing knowledge of the cognitive and neurological brain mechanisms of numerical cognition, the issue of multi-digit quantities was addressed by relatively few studies, and even fewer have investigated the processes that convert digits into these quantities. Section A of this PhD dissertation examines these issues. It investigates the various cognitive representations of numbers in educated adults, dissects the successive stages by which multi-digit Arabic numbers are converted into quantities, and eventually describes a detailed cognitive and mathematical model for these operations.

This section consists of a series of studies that we conducted on these topics. Chapter 2 is the first study in this series, and as such it lays the grounds to the questions we ask and to the methodological paradigm we used throughout this section. The questions concern several aspects of the quantification process of multi-digit numbers and of the resulting representation: holistic quantity representation, linear versus logarithmic quantity scale, and serial versus parallel processing. The paradigm we used was developed as part of this PhD research. It is a version of the number-to-position mapping task, in which participants see numbers and indicate the corresponding location on a number line (Berteletti et al., 2010; Opfer & Siegler, 2007; Siegler & Opfer, 2003). In our version of this paradigm, the participants performed the task on a tablet computer and we tracked their finger trajectories throughout each trial. The position-per-time information reflects the cognitive operations in different stages during the trial (Finkbeiner & Friedman, 2011; Finkbeiner, Song, Nakayama, & Caramazza, 2008; Santens, Goossens, & Verguts, 2011; Song & Nakayama, 2008a, 2008b, 2009), so it allows for a temporal dissection of the cognitive processes involved in the task. The major finding of this study was that the pattern of mapping numbers to positions showed a logarithmic pattern for a short period during the trial, and by the end of the trial, the mapping changed to linear. This study concluded that two-digit numbers have dual quantity representation: a single, holistic quantity that is encoded using a nonlinear compressive quantity scale; and another representation that uses a linear scale.

Chapter 3 described a larger study with 7 experiments, which examined the same issues in more detail. The basic pattern of results in Chapter 2 was replicated – a transient logarithmic pointing pattern. Using new experiments and new analysis methods, we offer a better interpretation of this pattern – that the logarithmic pattern is an artifact of differential processing times for small versus large numbers. This study concludes with a detailed cognitive and mathematical model of converting two-digit numbers to quantity representation, as well as for the processes involved in converting this quantity to a position on the number line where the finger is guided.

The next two chapters further enhance the cognitive model presented in Chapter 3. In Chapter 4, we examine whether the two digits are quantified independently or are dependent on each other. The pattern of dependencies we found led us to offer the existence of a new component in digit-to-quantity conversion: we suggest that to quantify a number, one must first create a representation of its decimal structure – a syntactic representation, in essence.

Chapter 5 examines the number-to-position task from a slightly different aspect – that of decision-making. In each trial in this task, the participants need to determine – sometimes based on partial information – a location to which they should point. We examined the assumption that this decision-making can be modeled as a Bayesian process, where each trial starts with a tentative decision based on a-priori expectation, derived from the participant's previous experience during the experiment, and this expectation is then overridden by the trial's target number.

The summary of this section presents a detailed model of the number-to-position mapping task. The model offers a detailed account to how we convert a two-digit number to quantity and then to a location on the number line.

2. How do we convert a number to a finger trajectory?°

Abstract. How do we understand two-digit numbers such as 42? Models of multi-digit number comprehension differ widely. Some postulate that the decades and units digits are processed separately and possibly serially. Others hypothesize a holistic process that maps the entire 2-digit string onto a magnitude, represented as a position on a number line. In educated adults, the number line is thought to be linear, but the "number sense" hypothesis proposes that a logarithmic scale underlies our intuitions of number size, and that this compressive representation may still be dormant in the adult brain. We investigated these issues by asking adults to point to the location of two-digit numbers on a number line while their finger location was continuously monitored. Finger trajectories revealed a linear scale, yet with a transient logarithmic effect suggesting the activation of a compressive and holistic quantity representation. Units and decades digits were processed in parallel, without any difference in left-to-right versus right-to-left readers. The late part of the trajectory was influenced by spatial reference points placed at the left end, middle, and right end of the line. Altogether, finger trajectory analysis provides a precise cognitive decomposition of the sequence of stages used in converting a number to a quantity and then to a position.

2.1. Introduction

This chapter describes the first in a series of studies in which we explored the conversion of two-digit symbolic numbers, presented as digits, into quantities. The study described here was centered on three major questions: holistic versus decomposed encoding of multi-digit quantities, the use of a logarithmic or a linear quantity scale, and sequential versus parallel processing of the digits in multi-digit numbers.

2.1.1. Holistic vs. decomposed quantity representation

One of the main disputes about two-digit quantity representation is between the holistic and decomposed approaches. The holistic approach claims that two-digit numbers are represented as holistic quantities: similarly to single digits, two-digit numerals are recognized as a whole and mapped onto a memorized quantity (Dehaene et al., 1990; Fitousi & Algom, 2006; Reynvoet & Brysbaert, 1999). The decomposed approach proposes that when a person deals with symbolic multi-digit numerals, only the quantities associated with the individual digits are activated and manipulated (Nuerk & Willmes, 2005). For example, the decomposed approach postulates that

° This chapter is an article published as Dotan, D., & Dehaene, S. (2013). How do we convert a number into a finger trajectory? *Cognition*, *129*(3), 512–529, doi:10.1016/j.cognition.2013.07.007. The text here is identical with the published article, except reformatting and removing some parts that would, if remained, repeat other sections of this dissertation.

comparing two 2-digit numbers is achieved using two separate comparisons – one of the decade digits and another of the unit digits (Meyerhoff et al., 2012; Moeller, Fischer, et al., 2009; Nuerk & Willmes, 2005).

The holistic-decomposed debate often made use of the fact that it takes longer to compare two digits when they are farther apart (Moyer & Landauer, 1967). This distance effect was taken to show that the comparison is performed by converting numbers from the decimal notation to an internal quantity code. The holistic model was supported by the finding of a continuous distance effect even in a comparison task where participants had to compare two-digit number targets to a fixed reference such as 55 (Brysbaert, 1995; Dehaene et al., 1990). Crucially, the unit distance affected the comparison time even when the decade digits were different (e.g., comparing 69 with 55 is faster than comparing 61 with 55), and in certain experimental settings there was no discontinuity at decade boundaries (Dehaene, 1989; Dehaene et al., 1990; Hinrichs, Yurko, & Hu, 1981). To account for this finding, a decomposed model must assume that the unit digits are compared even when they are numerically irrelevant, and that an incompatible unit comparison result interferes with the decade comparison and slows it down. Such an explanation predicts that if the onset of the unit digits is manipulated to be slightly earlier than the decade digit onset, the irrelevant unit comparison should have greater effect and therefore increase its interference in RT. This prediction was refuted, thereby supporting the holistic model (Dehaene et al., 1990).

In a slightly different comparison task, however, in which the subjects have to decide which of two simultaneously presented 2-digit numbers was the larger, the decomposed approach was supported by the discovery that the distance effect is modulated by decade-unit compatibility: for equal overall distance, pairs of two-digit numbers are compared faster when the units comparison result is compatible with the two-digit comparison result (e.g., 32 versus 47, where 2 is smaller than 7) than when the units comparison is incompatible (e.g., 37 versus 52, where 7 is larger than 2). The decomposed model can explain this compatibility effect as an interference from the incompatible unit comparison (Macizo, Herrera, Román, & Martín, 2011; Nuerk, Kaufmann, Zoppoth, & Willmes, 2004; Nuerk, Weger, & Willmes, 2001; Nuerk & Willmes, 2005). The holistic model cannot explain the compatibility effect because such a model considers only the overall distance between the compared numbers. The decomposed model was also supported by a recent study that showed a unit digit quantity effect in two-digit number bisection (Doricchi et al., 2009). However, other studies failed to support the decomposed model because they found no decade-unit compatibility effect, both in number comparison (Ganor-Stern, Pinhas, & Tzelgov, 2009; Zhang & Wang, 2005; Zhou, Chen, Chen, & Dong, 2008) and when using semantic priming paradigms (Reynvoet & Brysbaert, 1999; Reynvoet, Brysbaert, & Fias, 2002).

Holistic and decomposed representations are not necessarily mutually exclusive. Number comparison studies suggest that the decade-unit compatibility effect is found when the compared numbers are presented simultaneously but not when they are presented sequentially, suggesting that subjects can adopt either a holistic or a decomposed strategy according to task demands (Ganor-Stern et al., 2009; Zhang & Wang, 2005; Zhou et al., 2008; but see Moeller, Nuerk, & Willmes, 2009 for an alternative explanation that conforms to a decomposed approach).

2.1.2. Compressive versus linear quantity representation

Much evidence shows that the internal quantity representation is tightly related with space, and that quantities are represented along a mental number line: in left-to-right readers at least, the magnitude of numbers influences manual responses made in the right or left side of space (Dehaene, Bossini, & Giraux, 1993; Shaki, Fischer, & Petrusic, 2009), eye gaze direction (Loetscher, Bockisch, Nicholls, & Brugger, 2010; Ruiz Fernández, Rahona, Hervás, Vázquez, & Ulrich, 2011), and the direction to which spatial attention is shifted (Fischer, Castel, Dodd, & Pratt, 2003). Furthermore, magnitude was shown to be encoded not only categorically as “small” or “large”, but in a continuous manner (Ishihara et al., 2006).

A common paradigm to explore the quantity representation consists in analyzing how individuals map numbers to positions on a number line. How subjects map numbers to space is assumed to reflect, at least in part, the structure of the mental number line, and hence of the quantity representation (Barth & Paladino, 2011; Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Booth & Siegler, 2006; Cappelletti, Kopelman, Morton, & Butterworth, 2005; Siegler & Booth, 2004; Siegler & Opfer, 2003; von Aster, 2000; but see Núñez, Cooperrider, & Wassmann, 2012). Number-to-position studies showed that young children initially map quantities using a compressive scale that resembles a log function, but this changes into a linear encoding during the first years of school (Berteletti et al., 2010; Booth & Siegler, 2006; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003). The log-to-linear shift was hypothesized to result from education, and indeed compressive encoding was found in uneducated non-western adults but linear encoding was found in American adults (Dehaene et al., 2008). Interestingly, a compressive quantity scale can still be found in educated adults in other tasks that tap an implicit level of representation: inattentive mapping of non-symbolic quantities to position along a line (Anobile et al., 2012), quantity estimation with non-spatial responses (Núñez et al., 2011), price estimation (Dehaene & Marques, 2002), number bisection (Lourenco & Longo, 2009), and randomness judgment for sequences of numbers (Banks & Coleman, 1981; Viarouge et al., 2010). Viarouge et al. were even able to define the compressive scale more precisely, because their results fit a log function better than a power function.

The existence of a compressive internal number scale is also supported by neuronal recordings in macaque monkeys: neurons tuned to number in parietal and prefrontal cortex exhibit a Gaussian tuning curve only when plotted on a logarithmic scale (Nieder & Miller, 2003). Functional MRI experiments in human adults strongly suggest that such a representation continues to exist in the adult brain, at least for non-symbolic numerosities presented as concrete sets of objects (Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004).

Another kind of evidence comes from studies showing that reaction time is correlated with the logarithm of the target number (Brysbaert, 1995; Dehaene, 1989; Dehaene et al., 1990). Such findings may imply a logarithmic (or compressive) quantity encoding, but they can also be explained if quantity encoding is linear and is increasingly fuzzy as numbers grow larger (the scalar variability model, Cordes, Gelman, Gallistel, & Whalen, 2001; Gallistel & Gelman, 1992).

In summary, evidence exists for both linear and compressive quantity representations. The two representations were found to co-exist even in the same individuals, when tested in different tasks or in different conditions (Anobile et al., 2012; Dehaene et al., 2008; Lourenco & Longo, 2009; Viarouge et al., 2010).

2.1.3. Parallel versus sequential processing of multi-digit numbers

Another question concerning multi-digit numbers is whether the digits are processed in parallel or sequentially. For words with fewer than 8 letters, expert readers seem to process all of the letters in parallel (Lavidor & Ellis, 2002; Weekes, 1997), and it could be expected that the same would occur with numbers. Indeed, findings from two-digit number comparison suggest that even when effects compatible with a decomposed representation are observed, the quantities of the separate digits are processed in parallel (Moeller, Fischer, et al., 2009). However, longer numbers also involve sequential processing, in number comparison tasks (Hinrichs et al., 1982; Meyerhoff et al., 2012), in recall tasks (Hinrichs & Novick, 1982), and in reading aloud and symbolic comprehension tasks such as same-different judgment and identification of specific sequences of digits (Friedmann, Dotan, & Rahamim, 2010).

2.1.4. The present study

The research described in this chapter seeks to understand the process of encoding two-digit Arabic numbers as quantities. To examine this issue we introduce a novel methodology, which is a variation of the number-to-position task. In the traditional number-to-position task, participants are shown numbers and are required to mark, with a pencil, the corresponding position along a number line. By contrast, our participants performed the number-to-position task on an iPad tablet computer, which allowed continuous measurement of the finger trajectory. On each trial, a two-digit number between 0 and 40 was shown on the iPad screen, and the participants dragged their finger from a fixed starting point at the bottom of the screen to a position along a number line that was at the top of the screen (see Fig. 2.1). The experiment software digitized the entire finger trajectory. Finger trajectories are a powerful measure because the finger position at a certain time during the trial tightly tracks the underlying cognitive operations (Finkbeiner & Friedman, 2011; Finkbeiner et al., 2008; Longtin & Meunier, 2005; Marghetis, Núñez, & Bergen, 2014; Santens et al., 2011, and see Finkbeiner, Coltheart, & Coltheart, 2014; Friedman, Brown, & Finkbeiner, 2013; Song & Nakayama, 2008a, 2008b, 2009, regarding the use of finger trajectories to analyze non-numeric cognitive processes). Thus, analyzing the finger positions at different times in the trial could reveal how the quantity representation of two-digit numbers evolves over time. This is an advantage over using trial-level measures such as errors, reaction times, or the final position along a number line, because such measures can examine only the quantity representation at the end of the trial, whereas the "number-to-position trajectory" paradigm also allows examining the transient quantity representations.

Our study evaluated a broad array of distinct theoretical models that aim to explain how the number-to-position task is performed and what kind of quantity representation is used in this task. The different models assume either holistic or decomposed representations, linear or logarithmic quantity scales, similar or different processing of one-digit and two-digit numbers, and the last model also assumes a spatial strategy to aim the finger to the desired position. Each of these models predicts a certain spatial mapping of numbers to positions as well as a certain temporal pattern of finger trajectories. By analyzing various parts of the finger trajectories, we can reject some models and probe which of the models best fits the observed trajectories.

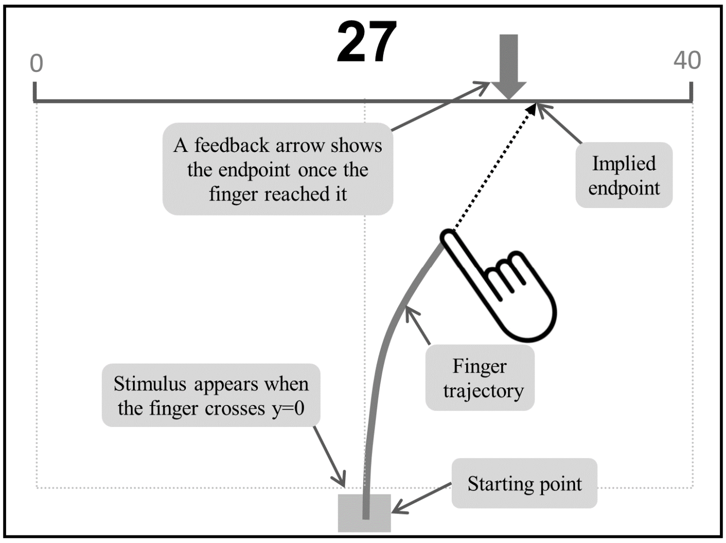

Fig. 2.1. Task and screen layout. Participants were asked to point to the correct location for a 2-digit number on a horizontal line that extended from 0 to 40. On each trial, they first placed their finger on a bottom rectangle. The target appeared when they started moving their finger upward. The entire trajectory was digitized, and the measures were converted into instantaneous estimates of finger coordinates and implied endpoint.

The predictions of the models are illustrated in Fig. 2.2 as simulations of the predicted trajectories. These simulations are admittedly over-simplified and are provided only for visualization purposes. They ignore several parameters such as the fact that the finger changes its direction gradually and not abruptly, the fact that the finger velocity is not constant, and the existence of noise. Their purpose is only to convey graphically the variables that are supposed to influence finger trajectories.

All six simulations in Fig. 2.2 assumed, for illustration only, an overall movement time (from starting point to the number line) of 1300 ms and a constant finger velocity. They also assumed that all trajectories begin with an exact upwards movement of the finger and deviate at a certain time. The models differ from each other with respect to the direction each trajectory takes once they branch apart.

1. The *holistic model* (Fig. 2.2a) assumes a holistic quantity representation mapped to a linearly-organized number line. The simulation assumes that at 400 ms the trajectories branch towards the target position along a linearly-organized number line. Note that although this model is called “holistic”, the same shape of trajectories is also predicted by a decomposed model that assumes that the unit and decade digits are processed in parallel and affect the finger position in exactly 1:10 ratio.

2. The *sequential model* (Fig. 2.2b) assumes that the quantity representation is decomposed, and that the decade digit is processed earlier than the unit digit. The simulation assumes that at t=400 ms the finger starts moving towards the position of the relevant whole decade (because the unit digit information is still unavailable at this time). At t=600 ms the unit digit was processed too and again the trajectories branch apart, this time towards the correct target position along a linearly-organized number line.
3. The next model (Fig. 2.2c) assumes a linear organization of the numbers along the number line, with faster processing of single-digit numbers than of two-digit numbers. The simulation assumed that the single-digit trajectories branch apart at t=400 ms, and at this time the finger starts moving towards the target position along a linearly-organized number line. The two-digit trajectories branch at t=600 ms, and in this case too, the finger starts moving towards a linearly-organized number line.
4. The *transient log* model (Fig. 2.2d) assumes that a holistic, logarithmic quantity representation is first constructed, and this representation is then overridden by a linear representation. The simulation assumed that at t=400 ms the finger starts moving towards the target position along a logarithmically-organized number line, and at t=600 ms the finger direction is re-adjusted to aim towards a linearly-organized number line.
5. The *decomposed digits* model (Fig. 2.2e) assumes that on top of the two-digit quantity, the quantities of each of the digits would also affect the finger position. One possible mechanism which may create such an effect is the existence of an intermediate stage in which the two digits of the target are not yet fully assigned to their respective unit and decades locations (Friedmann, Dotan, & Rahamim, 2010; Greenwald, Abrams, Naccache, & Dehaene, 2003). For a transient period, the two digits would therefore be floating and potentially submitted to illusory conjunctions (Treisman & Schmidt, 1982): the unit digit might be partially bound to the decade location, or vice-versa. The resulting quantity representation will be a linear combination of the two-digit quantity with the single-digit quantities. Our simulation represents the single-digit quantities using a “decomposed-digit factor” which is defined as the average of the two digits, linearly rescaled to the range between -2.5 and 2.5. The simulation assumes that at t=400 ms the finger starts moving towards the position of the target number plus the “decomposed-digit factor”, and at t=1000 ms the finger direction is re-adjusted towards the exact linear position of the target number.

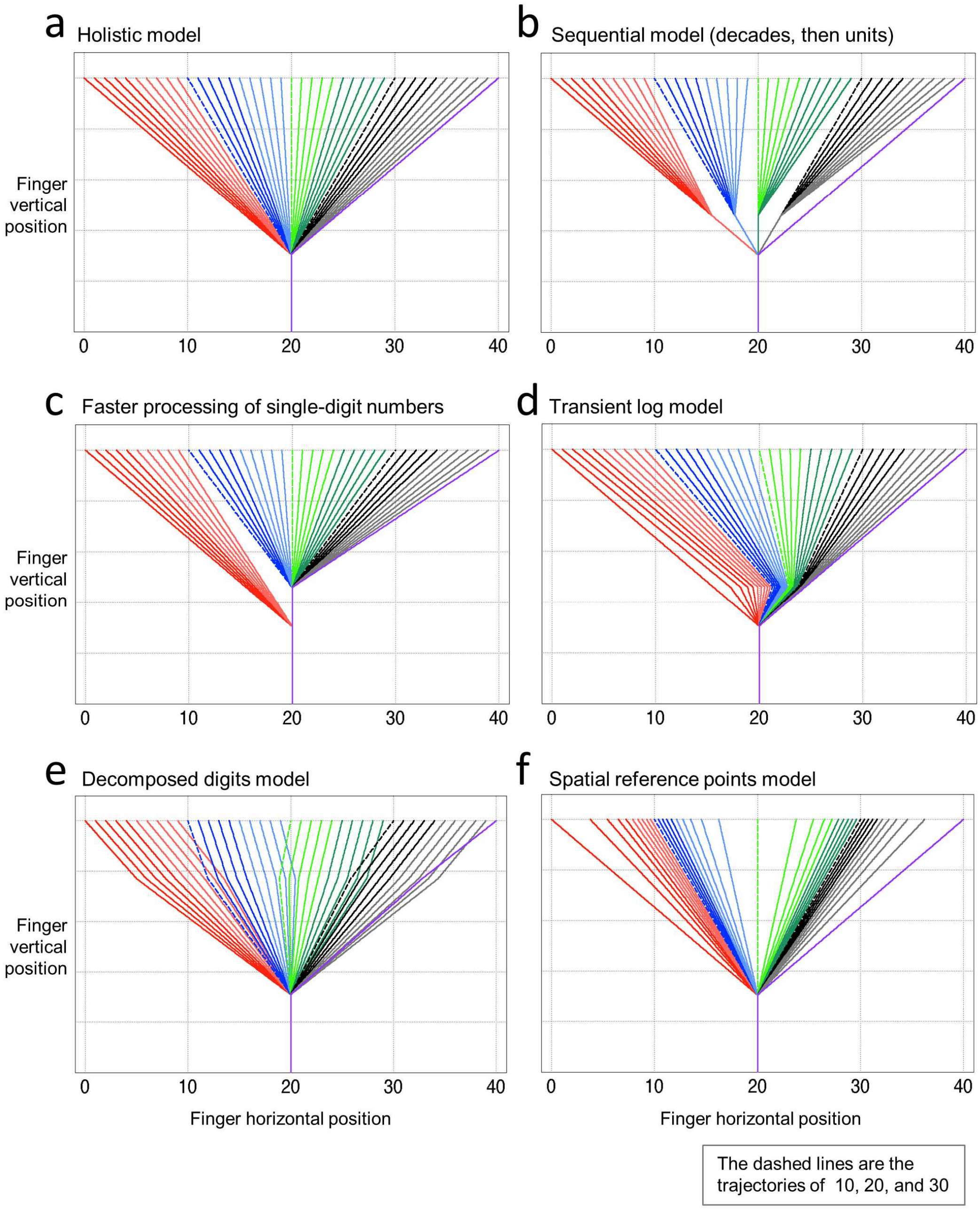

Fig. 2.2. Six idealized models for spatial trajectories. Models are explained in detail in the main text.

6. The *spatial reference points* model (Fig. 2.2f) is specifically concerned with the process of translating a quantity into a spatial position on the visually presented number line. Inspired by previous work on the role of reference points in proportionality judgments (Hollands & Dyre, 2000; Spence, 1990), it assumes that the target's position on the number line is estimated with respect to three reference points: the two ends of the number line (0 and 40)

and its middle (20). The position to which a number is mapped is assumed to be obtained by comparing the relative proportions of the estimated distances to the nearest two reference points (e.g., 7 is positioned by comparing its distances from 0 and from 20). Crucially, these estimated distances are scaled by a compressive function, giving rise to a non-linear bias term which pushes the participant's responses away from the reference points. Previous findings supported this notion (Barth & Paladino, 2011; Sullivan, Juhasz, Slattery, & Barth, 2011). To account for their findings, Barth and Paladino used a power function with the exponent as a free parameter, but in the present study, to avoid over-fitting (Opfer, Siegler, & Young, 2011) we used a log-based function log(*d*+1), where *d* is the linear distance. Thus, the exact position of 27 is calculated by the proportion between its estimated distances from 20 and 40, i.e., between log(7+1) and log(40-27+1), using the formula $\frac{\log(7+1)}{\log(7+1)+\log(13+1)} = .4407$. This proportion is then rescaled within the interval 0-20 to obtain a location on the complete range 0-40, i.e.: 20 + 20 * .4407 = 28.814 (very similar results were obtained with a power function with exponent 0.5, as proposed by Krueger (2010). In fact, the correlation between the log-based and the power-based functions over the integers between 0 and 40 is r = .9994).

The six models are not necessarily mutually exclusive. It is possible that different quantity representations dominate different parts of the trajectory. Two of the models even make this assumption explicitly: both the transient log and the decomposed digits models assume an intermediate representation that is then overridden by a linear representation. Our methodology allows for this possibility because we analyze the finger position in several time points along the trajectory. Another possibility is that two quantity representations co-exist simultaneously. This should result in a finger position that is some weighted average of the two quantity representations. Our methodology allows for this possibility by using regression analyses in which predictors from several theoretical models are put into a single regression model.

2.2. Method

2.2.1. Materials and Procedure

The experiment was performed on an iPad tablet computer[1]. Numbers between 0 and 40 were presented on screen, and the participants were required to point with their index finger at

[1] A demo of the application can be found in http://www.trajtracker.com.

the corresponding position along an unmarked number line. Each target number was presented 10 times, so there were 410 trials, presented in random order.

Each trial began with a black screen with a horizontal number line at the top, marked with the labels 0 and 40 in its ends (see Fig. 2.1). The number line remained on the screen throughout the experiment session. When the participants touched the initiation rectangle, a trial started and a fixation indicator (+) appeared above the middle of the number line. When the participants started moving their finger towards the number line, the fixation symbol was replaced by the target number and the participants moved their finger to what they judged to be the corresponding position on the number line. When the finger crossed the number line the target number disappeared and a green feedback arrow showed where the finger actually hit the number line. The arrow did not show how accurate the response was – its purpose was only to help the participant improve the finger's motor aiming. The participants could then initiate the next trial whenever they wanted to, which was usually immediately.

The following violations were considered as failed trials: lifting the finger in mid-trial, touching the screen with more than one finger, moving the finger backwards, and starting a trial with sideways (rather than upward) movement. Furthermore, to ensure that the experiment provided continuous trajectory information, minimal velocity was enforced except the first 300 ms of each trial. The finger had to reach the number line within two seconds and 1/3 of the vertical distance within one second, with linear interpolation. Furthermore, a minimal finger velocity of 6 mm/sec was required at all times. Failed trials were excluded from all analyses and their target numbers were presented again later in the experiment. Five participants, who had more than 25 failed trials (5.7%), were excluded.

2.2.1.1. Technical Specifications

The experiment used an Apple iPad device and the software was written in Objective-C. The iPad screen size is 197x148mm. The resolution of display and finger tracking was 1024x768 pixels. The device was placed in landscape orientation. The screen background was black throughout the experiment. The number line was white, 844 pixels long (162mm), two pixels wide, and was located 80 pixels below the top of the screen, centered. The target number was shown in Arial bold white font, centered above the number line, and the digits were 10 mm high. The numbers 0 and 40 at the ends of the number line were shown in light grey Helvetica font and were 5 mm high (so they were a little less salient than the target number). The height and width of the fixation cross was 7.7 mm. The feedback arrow was green, 7.7 mm high, and was pointing downwards with its tip touching the number line.

The rectangular area that the participant touched to initiate a trial was dark grey and its size was 60x40 pixels, in landscape orientation. The target onset was triggered when the finger reached a fixed distance from the bottom of the screen (50 pixels here, but 70 px in some subsequent experiments).

2.2.2. Training

The experiment began with a short training that was done in four stages, each stage introducing some of the experiment rules. The first stage of training resembled the experiment procedure described above, with two differences: no minimal velocity was enforced, and no target number appeared. Instead of the target number, a downward-pointing red arrow appeared somewhere above the number line, and the participant was instructed to aim her finger “towards the red arrow”. The second training stage was the same, but it also enforced minimal finger velocity. The minimal velocity was visualized as an upward-moving horizontal line, and the participants were instructed to maintain their finger above the line. In the third training stage, minimal velocity was still enforced but the guiding horizontal line was not shown. The last training stage was identical to the experiment procedure, i.e., the targets were no longer red arrows but numbers between 0 and 40. The participants were shown the positions of 0, 20, and 40 but not of other numbers. In each training stage the experimenter first demonstrated what should be done, and the participant then performed a few training trials.

2.2.3. Participants

21 healthy adults participated voluntarily in the experiment. Ten of them were native speakers of Hebrew, which is read from right to left (RTL), and the rest were left-to-right (LTR) readers – 9 French, one Italian, and one Thai. Numbers in Hebrew are read from left to right, like in English, and are printed using the same characters 0-9. All participants were right-handed, and their mean age was 35;5 (SD=10;7). There was no significant age difference between the LTR and RTL groups ($t_{(19)}$ = .66, two-tailed p = .52).

2.2.4. Data Encoding

Several measures were calculated per trial. The *trial endpoint* is the position in which the finger crossed the number line, encoded using the number line’s scale (0-40; endpoints were out of this range if the participant pointed outside of the number line). The *endpoint bias* is the difference between endpoint and the target number, with positive values indicating rightward bias. The *endpoint error* is the absolute value of endpoint bias. Finally, a trial *movement time* is the time elapsed from the target onset until the finger crossed the number line.

The finger trajectory throughout the trial was recorded as a sequence of x,y coordinates with timestamps attached to each. The finger position was sampled at the highest possible rate provided by the iPad (M=16 ms between subsequent samples, SD=1 ms) and then transformed into a sampling rate of 100Hz using cubic spline interpolation. The fixed sampling rate allows comparing finger coordinates in identical post-target-onset times between different trajectories.

2.2.5. Data Cleanup

Failed trials and trials with outlier endpoints were excluded from all analyses. A trial was considered as failed if one of the experiment’s restrictions were violated, e.g., if the finger was lifted from the screen or was moved too slowly, or if the movement time was less than 200 ms. An outlier endpoint was defined with respect to endpoints of the 10 trials with the same target number, as an endpoint that exceeded the 25^{th} or 75^{th} percentile by more than 1.5 times the inter-quartile range.

2.2.6. Statistical Analysis

Each of the six models was analyzed using a two-stage analysis. The first stage was a set of regression analyses, one per participant. The dependent variable in these regressions was the finger x coordinate, and the predictors depended on the theoretical model being assessed. For example, the transient log model assumes that the logarithmic and linear representations co-exist, so it was assessed by a regression with two predictors – the target number (between 0 and 40) and its logarithm.

Each of these regression analyses was carried out per time point, in 50 ms intervals. This allowed examining how the quantity representation evolves over time. For example, the logarithmic model predicts that the log predictor will be strong in the regressions done for early time points, but that the linear predictor will be strong in late time points.

The second stage checked which of the predictors showed a consistent pattern over all participants – namely, whether the b values, which were obtained for a specific predictor in a specific time point for each of the participants, were significantly different from zero. This was assessed using repeated measures ANOVA which compared the b values with zero values (this was a within-subject factor), with a between-subject factor of language group, and the participant as the random factor. Non-significant b values were also included in this analysis. These ANOVAs were run per predictor and per time point, in 50 ms intervals. One-tailed *p* values were used when the mean b value was positive (which indicates a predicted result), and two-tailed *p* values were used when mean b was negative.

2.3. Results

2.3.1. General performance

The average movement time (from target onset until the finger reached the number line) was 1.11 seconds (SD = .14). The mean rate of endpoint outliers was 4.9% (SD = 1.8%). Excluding outliers, the mean endpoint error was 5.9 mm (SD = 1.59 mm), i.e., 1.45 numerical units on the 162 mm long, 0-40 number line. The mean endpoint bias was -.45 numerical units (SD = .34), i.e., a small leftward bias. The average rate of failed trials was 2.7% (SD = 1.4%). The total of 238 failed trials had the following failure reasons: minimal velocity violation (44.5%), multiple fingers touched the screen (20.2%), finger backward movement (10%), finger lifted from screen (9.7%), trials shorter than 200 ms (8.8%), and starting the trajectory sideways rather than upwards (6.7%).

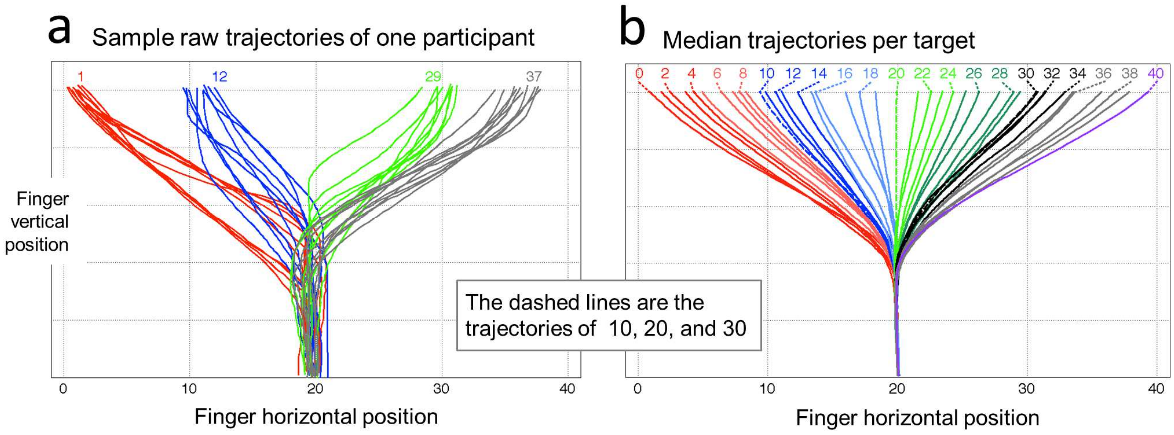

Fig. 2.3. Two depictions of finger trajectories. (a) Spatial depiction of sample trajectories of one participant (finger location on the horizontal and vertical axes) to four distinct target numbers. (b) Spatial depiction of median trajectories for each target number, averaged across all participants.

The LTR group was slightly quicker than the RTL group: the mean movement time was 1.06 seconds in the LTR group vs. 1.18 seconds in the RTL group (unpaired t(19) = 2.15, *p* = .04). The LTR group was also less accurate, with a mean endpoint error of 1.61 vs. 1.27 in the RTL group (t(19) = 2.13, *p* = .05). There were no significant differences between the two groups with respect to the rates of outliers and failed trials (t(19) < 1.25, *p* > .22).

Fig. 2.3a shows a typical example for the shape of trajectories in the experiment (one line per trial). It shows the raw trajectories of one of the participants when responding to the target numbers 1, 12, 29, and 37. Fig. 2.3b shows the trajectories, averaged over all trials and participants. For each target number, a median trajectory was calculated per participant by re-sampling the raw trajectories into equally spaced 201 time points and finding the median coordinate per time point (Santens et al., 2011; Song & Nakayama, 2008a). These median trajectories were then averaged across participants.

2.3.2. Assessment of the models

2.3.2.1. The holistic model

The holistic model assumes that the task is performed by mapping the two-digit quantity to a linearly organized number line. This model was examined using regression analysis in which the dependent variable was the finger x coordinate, linearly transformed to the 0-40 scale of the number line (this variable is hereby denoted as $X_{0\text{-}40}$). There was a single predictor – the two-digit target number (which will be hereby denoted as $N_{0\text{-}40}$). One regression was run per participant and per time point, in 50 ms intervals.

Fig. 2.4a shows the mean b values of $N_{0\text{-}40}$ over all participants in all time points. The b value (hereby denoted as b[$N_{0\text{-}40}$]) gradually increases as the trajectories branch apart. The mean r^2 value starts at 1% (SD = 1.4%) at t=450 ms and rises up to 97.2% (SD 1.7%) at the end of the trajectories.

A between-participant analysis was done by submitting the b[$N_{0\text{-}40}$] values of all participants to repeated measures ANOVA (as described in section 2.2.6) with b versus zero as a within-subject factor, the language group as a between-subject factor, and the participant as the random factor. This showed that b[$N_{0\text{-}40}$] was significantly larger than zero as early as 450 ms post stimulus onset and in all subsequent time points.

These results are in line with the holistic model. The real question, however, is whether the holistic-linear trend would remain strong even when compared with other models. As the next sections will show, the answer to this question is yes.

2.3.2.2. The transient log model

The transient log model assumes that mapping a number to the number line is governed by a logarithmic quantity representation in the early part of the trajectories, but by a linear-holistic quantity representation in the later parts. This model was examined using regression analysis with $X_{0\text{-}40}$ as the dependent variable, and with two predictors: the target number $N_{0\text{-}40}$, and a logarithmic predictor denoted as $log'(N_{0\text{-}40})$, which is log(1+$N_{0\text{-}40}$), linearly transformed so that $log'(0) = 0$ and $log'(40) = 40$ (this transformation was used to allow meaningful comparison between the b values of the logarithmic and linear predictors). One regression was run per participant and per time point, in 50 ms intervals. The resulting b values were compared with zero using repeated measures ANOVA with b versus zero as a within-subject factor, the language group as a between-subject factor, and the participant as the random factor. The transient log model predicts an intermediate stage during which b[log'($N_{0\text{-}40}$)] will be significantly larger than zero.

The results confirm this prediction (Fig. 2.4b). b[log'($N_{0\text{-}40}$)] was significantly larger than zero from 500 ms to 1050 ms, which indicates that a logarithmic quantity representation exists during this intermediate time window and then disappears. A linear representation of quantity also exists, as demonstrated by the finding that b[$N_{0\text{-}40}$] was significantly larger than zero in all time points as of 450 ms. Note that at the time points in which the log predictor was significant, the linear predictor was significant too. This shows that the logarithmic quantity representation does not *precede* the linear quantity representation but exists *in parallel* to it. Finally, b[log'($N_{0\text{-}40}$)] was significantly *smaller* than zero at all time points from 1300 ms and onwards. There was no significant difference between the language groups for any of the two predictors ($F(1,19) < 2.1$, $p > .16$).

The reliable contribution of $log'(N_{0\text{-}40})$ could have an alternative explanation: it could be attributed to a logarithmic quantity representation of each of the digits, rather than to a logarithmic representation of the two-digit target number. According to such an explanation, the reason that $log'(N_{0\text{-}40})$ is a good predictor is its correlation with the logarithms of the decade and unit digits (indeed, the correlation between $log'(N_{0\text{-}40})$ and *log'(decade-digit) + log'(unit-digit)* is high, $r = .95$). To evaluate this alternative explanation, the trajectory data was submitted to regression analysis with $X_{0\text{-}40}$ as the dependent variable and with four predictors: $N_{0\text{-}40}$, $log'(N_{0\text{-}40})$, *log'(decade-digit)*, and *log'(unit-digit)*. The resulting b values were compared with zero using repeated measures ANOVA with b versus zero as a within-subject factor, the language group as a between-subject factor, and the participant as the random factor. The alternative explanation predicts that this analysis will show significant contribution of the single-digit predictors, *log'(decade-digit)* and *log'(unit-digit)*, but the results refuted this prediction: b[log'(decade-digit)] and b[log'(unit-digit)] were not significantly larger than zero in any time point – in fact, they had negative values in all time points later than 650 ms.

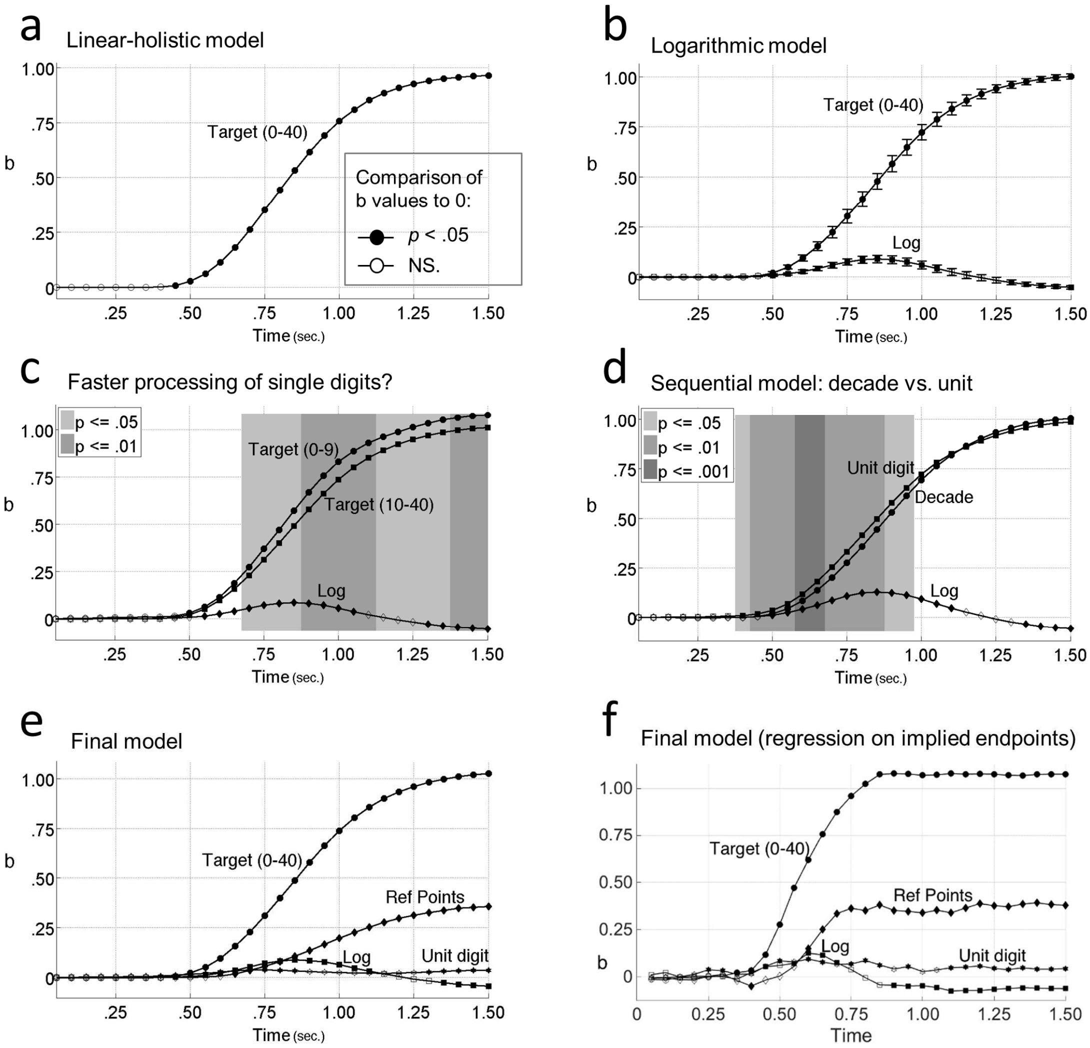

Fig. 2.4. Incremental regression models of the finger trajectories. Each graph results from a multiple linear regression on horizontal finger location ($X_{0\text{-}40}$), performed separately for each subject and each time-point. The regression weights are then averaged over all participants and plotted as a function of time on the x axis. (a) regression with target number; (b) regression with the target and its log, showing a transient logarithmic effect (error bars show one standard error across subjects); (c) separate assessment of one- and two-digit numbers shows no significant difference between the 0-9 and 10-40 predictors in early time points; (d) regression with distinct regressors for unit digits and decades; (e) final regression model with the target, its unit digit, its log, and spatial reference points; (f) The same regression model as in panel e, applied to the implied endpoint of the trajectory. Note that the effects appear earlier than in panel e: the implied endpoint provides a better reflection of the temporal dynamics of processing, because it directly reflects where the subject is aiming at a given time.

Another alternative explanation is that the logarithmic effect results from a spatial or motor bias that is unrelated with the representation of quantities. Such a non-quantity account does not specifically predict a leftward or a rightward bias. To evaluate the specificity of the log function as a predictor, we compared it to a left-to-right mirror of this function. The trajectory data was submitted to regression with $X_{0\text{-}40}$ as the dependent variable and with $N_{0\text{-}40}$ and the mirrored log function as predictors. This mirror function is denoted as $revlog'(N_{0\text{-}40})$ and defined as $revlog'(x) = 40 - log'(40\text{-}x)$. The resulting b values were compared with zero using repeated measures ANOVA with b versus zero as a within-subject factor, the language group as a between-subject factor, and the participant as the random factor. Contrary to the alternative explanation, revlog'($N_{0\text{-}40}$) was not a good predictor of the finger position: the average b[revlog'($N_{0\text{-}40}$)] values were negative in most time points, and the negative values were significantly lower than zero in time points later than 1300 ms. The spatial/motor alternative explanation was also refuted by a control experiment that did not involve numbers but arrows as targets, in which no log effect was found (see below in section 2.3.5).

2.3.2.3. Faster processing of single digits

This model assumes that numbers are mapped to a linearly-organized number line, but single-digit numbers are processed more quickly than two-digit numbers, and consequently the trajectories of single-digit numbers would branch earlier than the trajectories of two-digit numbers. The model was examined using regression analysis with $X_{0\text{-}40}$ as the dependent variable, with the logarithmic predictor $log'(N_{0\text{-}40})$, and with two additional linear predictors: $N_{0\text{-}9}$, which was the target number for single-digit trajectories and as zero for two-digit trajectories, and $N_{10\text{-}40}$, which was the opposite – the target number for two-digit trajectories and zero for single-digit trajectories. One regression was run per participant and per time point, in 50 ms intervals. The resulting b values were compared with zero using repeated measures ANOVA with b versus zero as a within-subject factor, the language group as a between-subject factor, and the participant as the random factor.

The "faster processing of single digits" model assumes a time window during which the trajectories of 0-9 have already branched but the trajectories of 10-40 have not yet branched. Consequently, at any time point during this time window (and even at later time points), the trajectories of 0-9 would be farther apart from each other than the trajectories of 10-40 (see Fig. 2.2c), although the trajectories within each of the two groups would be still linearly organized. Thus, the model predicts an intermediate stage during which b[N0-9] > b[N10-40].

The results (Fig. 2.4c) did not confirm this prediction. A significant difference was found between b[$N_{0\text{-}9}$] and b[$N_{10\text{-}40}$] only in relatively late time point (from 700 ms and onwards,

$F(1,19) > 2.23$, $p < .05$). This finding does not agree with the notion of faster processing of single digits[2]. The ANOVA showed no effect of language group ($F(1,19) < 1.2$, $p > .28$).

2.3.2.4. The sequential model

The sequential model assumes that the quantity representation is decomposed and that the decade digit is processed earlier than the unit digit. The model therefore predicts an early stage during which only the decade digit affects the finger position. This model was examined using a regression analysis with $X_{0\text{-}40}$ as the dependent variable, with the logarithmic predictor $log'(N_{0\text{-}40})$, and with two additional linear predictors: the target number's decade (denoted as D and having the values 0, 10, 20, or 30) and the unit digit (U). The trajectory of target number 40 was excluded from this analysis in order to prevent a possible bias, because there is only one value of U for the decade 40, and the trajectory of 0 was excluded to maintain symmetry (but the results were similar even with the trajectories of 0 and 40 included). One regression was run per participant and per time point, in 50 ms intervals. The resulting b values were compared with zero using repeated measures ANOVA with b versus zero as a within-subject factor, the language group as a between-subject factor, and the participant as the random factor.

The sequential model predicts a time window during which $b[D] > b[U]$. The results (Fig. 2.4d) did not confirm this prediction. In fact, they were the exact opposite: there was an intermediate time window, from 400 ms to 950 ms, during which b[D] values were significantly *smaller* than b[U] ($F(1,19) > 4.8$, two-tailed $p < .04$). The language group did not interact with this decade-unit difference ($F(1,19) < 1.62$, $p > .21$), and the effect was present in both groups (RTL group: from 550 ms to 1000 ms, $F(1,9) > 2.42$, $p \leq .04$; LTR group: from 400 ms to 700 ms, $F(1,9) > 5.76$, $p < .04$), indicating that it was not just due to the group of right-to-left readers treating the rightmost digit before the leftmost one. The faster processing of the unit digit was found also when the regression was run without the $log'(N_{0\text{-}40})$ predictor ($b[D] < b[U]$ from 350 ms to 1000 ms, $F(1,19) > 4.49$, $p < .05$).

These results refute the sequential model. And yet, a parallel model is also not sufficient to fully explain the results. A parallel model assumes similar effects of decades and units on the finger x coordinates, and therefore predicts similar b values for the decade and unit predictors (because the decade predictor is the decade digit multiplied by *10*) – so such a model cannot

[2] In Chapter 3 we will revisit this notion with a slightly different modeling – that smaller numbers, not single digits, are processed faster. We will show that this notion is not only supported by the finding but is also crucial to understand our task.

explain the finding that b[D] < b[U]. The models described in the next two sections, however, provide a possible explanation of this finding.

2.3.2.5. The decomposed digits model

The decomposed digits model assumes that on top of the two-digit quantity, the quantities of each of the digits would also affect the finger position. Such a model would artificially inflate the impact of the unit predictor compared to the decade predictor, because in our regressions the unit predictor *U* is the unit digit itself, whereas the decade predictor *D* is the decade digit multiplied by 10. This model can therefore explain the unit-decade difference found in the regression in the previous section. Note, however, that the data allow us to exclude a strictly serial model in which, for a certain period of time, the two digits are freely floating without any binding to position. If that was the case, the b[U] would have transiently been ten times larger than b[D]. We did not observe such an extreme effect: at the first point where the two regressors became significant (450 ms), the b[U]:b[D] ratio was only 1.64, and it continuously decreased to reach the average value of 0.95 at the end of trajectories. This finding suggests that a more likely interpretation is that, for a transient period, both the single digits and the two-digit quantity are activated in parallel and contribute to the finger trajectory.

Could we provide a more specific test of this decomposed-digits model? The model predicts that the trajectories of number pairs such as 29 and 31 may be reversed, so that, transiently, 29 would be incorrectly mapped to a position to the right of 31. This is because the large difference between the units (9 versus 1) may override the much smaller difference between the decades (2 versus 3) or the whole-number quantity (29 versus 31). Indeed, there is prior evidence that such decade-unit compatibility effects may confuse two-digit number comparison judgments (Macizo et al., 2011; Meyerhoff et al., 2012; Nuerk et al., 2001; Nuerk & Willmes, 2005).

The general prediction is that, for targets around whole decades, trajectories should tend to be reversed, relative to how they would appear if they were simply based on the two-digit number quantity. This prediction was examined by analyzing the residuals of the log + linear regression (see Section 2.3.2.2). The residuals (x_{res}) were calculated per participant as the delta between the x value of the per-target median trajectories and the *x* value predicted by the log + linear regression described above. A median trajectory was created per subject and per target number by calculating the median coordinates for equivalent post-stimulus-onset time points (in 10 ms intervals). Late time points exceeded the movement time of some trajectories; for those, the endpoint was used as the x coordinate. The residuals were calculated with respect to the log

+ linear regression which was run on the median trajectories. The reason for calculating x_{res} based on median rather than raw trajectories was that this allowed pairing together trajectories from corresponding targets in the within-subject ANOVA hereby described.

These residuals were compared in three separate comparisons, centered on each decade: 8-9 vs. 11-12, 18-19 vs. 21-22, and 28-29 vs. 31-32. Each of these comparisons was done using an ANOVA with x_{res} as the dependent variable, two within-subject factors of size (above or below decade) and distance from decade (1 or 2), a between-subject factor of language, and subjects as the random factor. The decomposed digits model predicts that the trajectories would tend to reverse, namely, that $x_{res}(28,29)$ would be larger than $\hat{x}_{res}(31,32)$, and corresponding differences around the decades 20 and 10.

The results confirmed this prediction: $x_{res}(28,29)$ was significantly larger than $x_{res}(31,32)$ in 550 ms ($F(1,19) = 3.83$, one-tailed $p = .03$) and in all subsequent time points ($F(1,19) > 8.01$, one-tailed $p \leq .01$). Similarly, $x_{res}(8,9)$ was larger than $x_{res}(11,12)$ in 550 ms and in all subsequent time points ($F(1,19) > 4.24$, one-tailed $p \leq .03$). The comparison around 20 resulted in a more complicated pattern: the prediction of the decomposed digits model was confirmed only for an early time window of 50 ms – 550 ms, during which $x_{res}(18,19)$ was significantly larger than $x_{res}(21,22)$ ($F(1,19) > 3.66$, one-tailed $p < .04$). However, the results were opposite in the late trajectory parts: $x_{res}(18,19)$ was significantly *smaller* than $x_{res}(21,22)$ in 750 ms and in all subsequent time points ($F(1,19) > 4.45$, two-tailed $p \leq .05$). There was no significant effect of language group in any of these comparisons ($F(1,19) < 3.9$, two-tailed $p > .06$) except a single time point (200 ms in the comparison around the decade 30; $F(1,19) = 5.21$, two-tailed $p = .03$) and no interaction between the language group and the above/below decade factor ($F(1,19) < 3.79$, two-tailed $p > .06$).

In summary, the decomposed digit model accounts for the results in the early time window around 550 ms: the finger trajectories do tend to reverse around all whole decades (though note that this effect was found in the residuals of the linear + log regression; as Fig. 2.3b shows, the effect size was not sufficiently strong to yield a complete reversal of the physical trajectories themselves). However, the pattern in late time windows (from 750 ms) is different: trajectories tend to be reversed around 10 and 30, but 20 has a repulsion effect that pushes the trajectories away (this repulsion effect around 20 is quite visible in Fig. 2.3b). The decomposed digit model cannot explain this pattern, but the next model offers a possible explanation.

2.3.2.6. The spatial reference points model

The spatial reference points model assumes that in order to determine the position to which a number is mapped, the participant estimates the distances between the number and two out of three reference points: the ends of the number line (0 and 40) and its middle (20). The model further assumes that this distance estimation is logarithmic and not linear. This model predicts that trajectories around 10 and 30 would cluster around the whole decade, and trajectories around 20 will be pushed away from 20 (Fig. 2.2f). i.e., the model predicts the pattern of results observed for late time windows in the analysis of residuals described in the previous section.

This model was examined using regression analysis with $X_{0\text{-}40}$ as the dependent variable, and with four predictors: the two-digit target $N_{0\text{-}40}$, the logarithmic predictor $log'(N_{0\text{-}40})$, the unit digit U (to account for possible effect of quantities of the decomposed digits), and a spatial-reference-points-based bias *SRP*. The *SRP* predictor was the delta between the target number and the spatial-reference-points-based estimated position, which was defined like the simulation function in section 2.1.4 (f):

$$\text{For } 0 \leq N \leq 20,\ SRP(N) = 20 * \frac{\log(N+1)}{\log(N+1)+\log(21-N)} - N \qquad [1]$$

$$\text{For } 21 \leq N \leq 40,\ SRP(N) = 20 + 20 * \frac{\log(N-20+1)}{\log(N-20+1)+\log(41-N)} - N$$

The b values from this regression were compared with zero using repeated measures ANOVA with b versus zero as a within-subject factor, the language group as a between-subject factor, and the participant as the random factor.

The results (Fig. 2.4e) support the spatial reference points model, as well as the assumption that the SRP effect occurs in late time windows: b[SRP] was significantly larger than zero in all time points as of 650 ms ($F(1,19) > 4.07$, $p < .03$). As for the other predictors, b[$N_{0\text{-}40}$] was significantly larger than zero in all time points as of 450 ms ($F(1,19) = 3.25$, $p = .04$ at 400 ms, and $F(1,19) > 14.71$, $p < .001$ thereafter). b[log'($N_{0\text{-}40}$)] was significantly larger than zero from 550 ms to 1050 ms ($F(1,19) > 3.8$, $p < .02$). b[U] was larger than zero from 400 ms and onwards (this effect was marginally significant from 900 ms to 1250 ms, $F(1,19) > 1.88$, $p < .1$, and significant in the other time points, $F(1,19) > 3.16$, $p < .05$). There were no significant differences between the language groups for any of the predictors ($F(1,19) < 2.06$, $p > .16$).

Thus, the line of analyses described above indicated that the position to which a number is mapped along the number line is determined by multiple factors: the linear two-digit quantity representation, a logarithmic quantity representation, the decomposed quantity of the unit digit, and a spatial-reference-point-based bias.

2.3.3. Limitations of the spatial reference points model

The final regression model, described in the previous section, showed that the endpoints (and the finger x coordinates in the final trajectory parts) depart from a strictly linear organization along the number line. This bias is captured in the model by the spatial-reference-point function SRP and by the logarithmic predictor (which is significant but with negative values, and therefore cannot reflect logarithmic quantity representation). Although the SRP predictor was significant and the regression r^2 values were high, we believe that the SRP function we used is not the ideal explanation for the endpoint bias. One reason for this belief is the fact that the log predictor was significant with negative values, which is not explained by any theoretical model. Another way to look into this issue is by comparing the actual endpoint biases with the prediction of the SRP function. As Fig. 2.5a shows, the SRP function only partially resembles the observed endpoint biases. Notably, there seems to be an overall leftward bias (mean bias = -.45), which is not predicted by the spatial reference points model. This bias was consistent across participants (comparing the participants' mean endpoint bias versus zero, $t(20) = -6.23$, two-tailed $p < .001$).

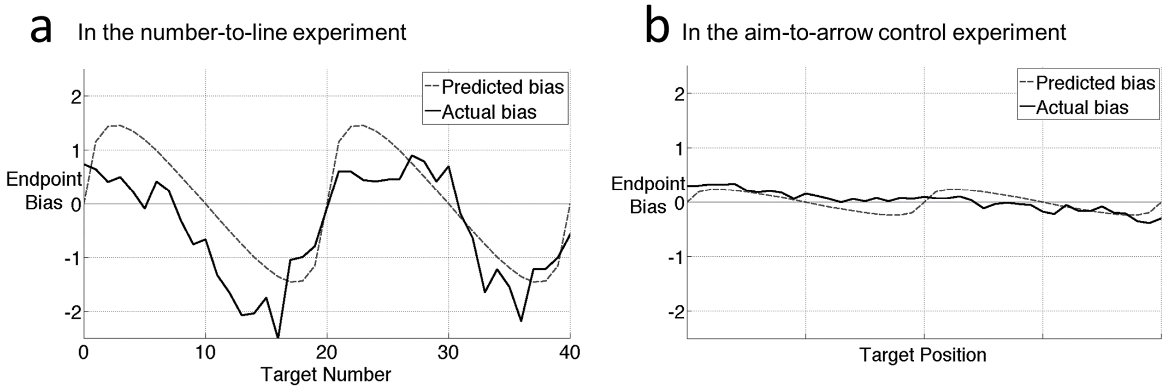

Fig. 2.5. Endpoint biases (averaged over participants) compared to the prediction of the spatial reference points model. The "predicted bias" line shows the prediction of the spatial reference points model, linearly rescaled to fit the actually observed average bias. The y axis specifies the bias using the 0-40 scale.

2.3.4. Clarifying the time course of access to quantity

Most of the analyses throughout this study were based on the finger x coordinates, and we believe that the results showed it to be a powerful measure of the underlying quantity representation. However, the finger coordinates offer poor temporal granularity. The reason is that the finger position is slow in responding to cognitive changes, because even after the participant changes her cognitive representation of the finger's target position, there are still two things that must happen before the finger coordinate reflects this change: first, the finger must change its direction towards the new target position. Second, once the direction changes, it still takes time for the finger position to change: in essence, finger position is the time integral of direction and therefore smoothes out its fine-grained temporal variations.

We did not find a good way to eliminate the time it takes to change the finger direction, but there is a way to overcome the second factor – the time the finger spends moving in the new direction until its position changes. To overcome this factor, we used the finger's *implied endpoint* at each point along the trajectory rather than its x coordinate. The implied endpoint is the position along the number line that the finger would reach if it keeps moving in its current direction. First, the x and y coordinates were separately smoothed with Gaussian smoothing (σ = 20 ms). The current direction (θ_t) was then defined as the direction vector between the finger x,y coordinates at times *t - 10* ms and *t*. Implied endpoints were cropped so not to exceed the number line by more than 5% its length on each side (i.e., to the range [-2, 42] for the number line length of 40), and were undefined when the finger moved sideways ($|\theta| > 80°$).

The regression of the final model was executed again, and this time the dependent variable was the implied endpoint. The predictors were the same as before: the two-digit target $N_{0\text{-}40}$, the logarithmic predictor $log'(N_{0\text{-}40})$, the unit digit *U*, and the spatial-reference-points-based bias *SRP*.

The implied-endpoint-based regression (Fig. 2.4f) showed similar trends to those found in the x-coordinate-based regression: the linear factor was significant throughout the trajectory, the log factor in an early time window, and the SRP factor in late time windows. Importantly, the implied-endpoint-based regression indeed revealed earlier effects than the x-coordinate-based regression. b[$N_{0\text{-}40}$] was significantly larger than zero at all time points beginning at 350 ms. b[log'($N_{0\text{-}40}$)] was significantly larger than zero as early as 400 ms and remained significant until 700 ms. b[U] was significantly larger than zero from 250 ms and onwards in most time points, and b[SRP] was significantly larger than zero at all time points beginning at 600 ms. Thus, the implied endpoint analysis seems to provide a more accurate picture of how the cognitive quantity representation evolves over time.

We also investigated another measure that could have provided a more accurate estimate of finger direction and therefore of temporal flow, the time derivative of the finger x coordinate.

However, this measure ($\frac{dx}{dt}$) turned out to be less accurate than the implied endpoint – its regression revealed the same four factors, but at later time points than the implied endpoint regression.

2.3.5. Numerical or spatial effects? Control experiment

Two of the findings described above could have alternative explanations that focus on motor factors rather than numerical processes. One such finding is the bias of endpoints from linear organization: we suggested that this bias is related to the cognitive representation of quantity or position, i.e., it is a bias in the way numbers are mapped to a planned position along the number line. An alternative explanation could be that the bias originates in the processes that guide the finger to this target position. The second finding is the transient log effect: an alternative explanation, mentioned in the end of section 2.3.2.2, attributes this effect to spatial or motor processes.

To assess these possibilities, we administered a control experiment, in which the target finger position was indexed non-numerically by an arrow. Importantly, no numbers were presented in this control task. If the spatial-reference-points bias originates in a quantity representation, no corresponding bias should be observed in this control task. If, however, the spatial-reference-points bias originates in non-numeric mechanisms, we expect to find a similar bias in the control experiment too. Similarly, if the log effect originates in a spatial/motor process, a similar effect should be observed in the control task too.

2.3.5.1. Participants

Ten healthy right-handed adults participated voluntarily in this experiment. They were all native Hebrew speakers. Their mean age was 34;3 (SD = 12;8).

2.3.5.2. Method

The method was similar to the number-to-position experiment, with a single difference: the target stimulus was not a number, but a downward-pointing red arrow placed at a specific position along the top line. The participants were instructed “to move their finger towards the arrow”. Thus, this experiment was conducted exactly like the third training stage of the number-to-position experiment (see section 2.2.2).

Each target arrow could appear in one of 41 positions (corresponding with the positions of the numbers 0-40), and each position was presented four times, i.e., there were 164 trials in the experiment. No trials were defined as outliers because the number of trials per position (4) was insufficient for outlier analysis.

2.3.5.3. Results

The average movement time was 762 ms (SD = 167 ms). The mean endpoint error (rescaled to 0-40, to allow for comparison with the number-to-position experiment) was .39 (SD = .1). The mean endpoint bias was .02 (SD = .1). The average rate of failed trials was 1.5% (SD = 1.8%). Thus, the aim-to-arrow task was performed faster than the number-to-position task, more accurately, and with fewer errors.

To assess the spatial reference points model, the trajectory data was submitted to regression analysis similar to the regressions reported for the number-to-position experiment. The dependent variable was $X_{0\text{-}40}$ and there were two predictors: the position of the target arrow along the line $N_{0\text{-}40}$, and the spatial-reference-point-based bias function SRP (detailed in section 2.3.2.6). The regression b values were compared with zero using t-test.

The results (Fig. 2.6) showed that b[$N_{0\text{-}40}$] was significantly larger than zero in all time points as of 250 ms ($F(1,9) > 3.12$, $p < .01$), indicating a linear trend that begins even earlier than in the number-to-position experiment. The spatial reference points bias also had a significant effect: b[SRP] was significantly larger than zero in all time points from 300 ms to 800 ms (much earlier than the value of 700 ms observed in the numerical task). The peak SRP effect size was at 450 ms (b[SRP] = .138), and then the SRP effect decreased and in the last part of the trajectories it was very small (b[SRP] < .02) and non-significant. Indeed, the organization of endpoints was almost perfectly linear (Fig. 2.5b). This pattern is quite different from the pattern observed in the number-to-position experiment, in which the SRP effect began in later time points, and continuously increased as the fingers approached the number line.

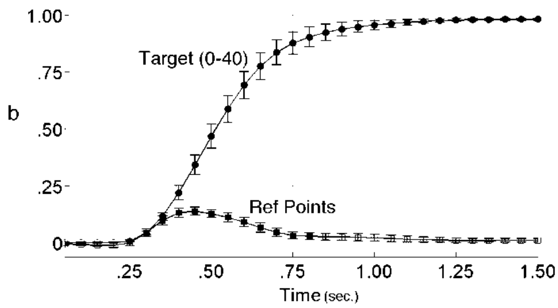

Fig. 2.6. Control experiment, where the subject was asked to point to a flashed arrow: the effect of spatial reference points is present at an earlier moment, suggesting that it arises from a non-numerical level of representation. The error bars show one standard error across participants.

To assess the transient log model, namely, the possibility that some spatial/motor process caused the log effect in the number-to-position task, the trajectory data in the aim-to-arrow task was submitted to a second regression analysis, which was similar to the regression described above, with a single difference – the addition of a third predictor, the logarithmic predictor $log'(N_{0\text{-}40})$. The regression b values were compared with zero using t-test. This analysis showed that the logarithmic predictor had no significant positive effect in any time window. In fact, $b[log'(N_{0\text{-}40})]$ was negative in all time points from 300 ms and onwards, and had significantly negative values from 350 ms until 750 ms ($t_{(9)} < -2.87$, two-tailed $p < .02$). Thus, the logarithmic trend in the number-to-position experiment should not be attributed to spatial or motor factors.

2.3.5.4. Discussion of the arrows task

The aim-to-arrow task showed that the trajectories deviate from a purely linear organization during an intermediate time window, and that the spatial reference points bias function can account for some of this deviation from linearity.

Why is the SRP bias observed only during an intermediate time window and then disappears? Most likely, as the finger approaches the target arrow, the participant can compare the finger position with the position of the target arrow (which is still visible on screen), and can readjust the finger trajectories to eliminate the bias.

Whether this explanation is correct or not, the results show a spatial reference points bias in a task that does not involve quantity estimation of numbers. It is therefore a plausible assumption that the SRP bias in the number-to-position task, unlike the logarithmic bias, originates, at least in part, in mechanisms unrelated to the quantity representation of numbers.

2.4. Discussion of Chapter 2

This research aimed to clarify the processes involved in converting two-digit Arabic numbers into quantities and then into spatial coordinates. We investigated which cognitive representations are activated during this encoding process, either transiently or not. We used the number-to-position task and tracked the finger trajectories throughout each trial. An analysis of the factors influencing finger movement at various points in time revealed the underlying representations at various stages during a trial. Different predictors were used to assess five different theoretical models of quantity representation, and one model that concerns the way these quantities are mapped to spatial positions.

The findings suggest a multi-stage process that involves both holistic and decomposed quantity representations, with four factors affecting finger movement. These factors are now discussed in turn. The two measures of finger movement (x coordinate and implied endpoint) yielded very similar results, but we focus primarily on the implied endpoint regressions because they provided a more accurate timing of the underlying cognitive processes.

2.4.1. Linear representation

The strongest predictor of finger movement was the two-digit target number. This linear quantity was a reliable predictor of the implied endpoint at all time points starting at 400 ms following stimulus onset, and until the end of the trial. This finding suggests that a linear representation of the two-digit quantity (either holistic or decomposed) is quickly accessed and dominates the finger movement, as requested by the task. Assuming that it takes approximately 110-120 ms from motor intention to finger movement (Rammsayer & Stahl, 2007; Jaśkowski et al., 2007), our findings suggest that an intention is activated by 280-290 ms. Previous estimates, based on event-related potentials, suggest that digit identification takes place at about 160 ms, and that a quantity representation of single-digit numerals starts activating at 174 ms and is maximally activated approximately 210 ms after target onset (Dehaene, 1996). Based on this earlier study, the series of stages dominating the present task, possible organized in a cascade, are likely to be: identification (~160 ms), quantity (~170-210 ms), representation of the (linear) target location (~290 ms) and first finger deviation towards it (~400 ms). On top of this process there could be additional, faster or more automatic processing routes that process single digits (as is indicated by the finding of an early contribution of the units digit, see section 2.4.4 below). Such automatic processing is in line with previous studies (Pisella et al., 2000).

2.4.2. Transient logarithmic representation

The second factor that predicted finger location was the logarithm of the two-digit target number. This factor was a reliable predictor of the implied endpoint from 450 ms until 750 ms post stimulus onset. It indicates that a compressive quantity representation exists during an intermediate time window. We cannot conclude that the quantity representation was strictly logarithmic although the regression predictor was a log function, because several other compressive functions resemble the log function (e.g. a power function with exponent 0.5) and may have accounted for the results just as well.

The finding that the log factor started early and then disappeared suggests that the activation of a compressive representation is automatic rather than the result of conscious reasoning. Indeed, compressive quantity encoding was previously shown in educated adults in several paradigms (Anobile et al., 2012; Dehaene & Marques, 2002; Piazza et al., 2004; Viarouge et al., 2010). In the number-to-position task, however, it was shown only for young children (Berteletti et al., 2010; Booth & Siegler, 2006; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003) and for uneducated adults (Dehaene et al., 2008). The current research extends these previous findings and shows that educated adults use a compressive quantity scale even in the context of the number-to-position task. Earlier developmental and anthropological studies suggested that a few years of education suffice to move away from the innate compressive "number sense" that we share with animals (Dehaene et al., 1998; Gallistel & Gelman, 1992) and develop a linear sense of number (Booth & Siegler, 2006; Dehaene et al., 2008; Siegler & Booth, 2004; Siegler & Opfer, 2003). Nevertheless, the present findings confirm that an intuitive representation of numbers on a logarithmic scale remains dormant even in educated adults (Viarouge et al., 2010). Indeed, in agreement with previous studies, we found that linear and compressive quantity representations co-exist in the same individuals (Anobile et al., 2012; Lourenco & Longo, 2009; Viarouge et al., 2010).

The nature of the quantity scale is a topic of long-lasting debate between two different views. Some researchers showed how speed and accuracy decrease logarithmically as numbers become larger or closer, and suggested that these findings provided evidence for a compressive quantity scale (Brysbaert, 1995; Dehaene et al., 1990). A different interpretation of such findings, however, was offered by the scalar variability model, which proposes that quantities are encoded using a linear scale but the noise surrounding the quantity representation increases with number size (Brannon, Wusthoff, Gallistel, & Gibbon, 2001; Cordes et al., 2001; Gallistel & Gelman, 1992; Whalen, Gallistel, & Gelman, 1999; but see Dehaene, 2001, 2003 for a discussion). Both the logarithmic model and the scalar variability model hold that it is harder to discriminate between large numbers than between small numbers, and therefore the two models are quite hard to separate, as they make very similar predictions about experimental measures such as reaction time, accuracy, and discriminability of numbers, and as we shall see in Chapter 3 – even about finger trajectories.

2.4.3. Holistic two-digit quantity representation

The finding of a logarithmic contribution to finger position indicates that the quantity representation is not only compressive but also holistic. A decomposed model could have explained the logarithmic factor as an artifact of logarithmic encoding of the single-digit quantities, but this alternative explanation was explicitly tested and ruled out, as we found a better fit of finger position with a log function of the whole 2-digit number. Thus, the results supports a holistic model, in agreement with previous studies (Dehaene et al., 1990; Ganor-Stern et al., 2009; Reynvoet & Brysbaert, 1999; Zhang & Wang, 2005; Zhou et al., 2008).

The results also do not support a sequential model, according to which the decade digit is processed before the unit digit. No time window was found in which the effect of the decade digit on the finger movement was larger than that of the unit digit. The results are therefore in accord with previous studies that showed parallel processing of two-digit numbers (Friedmann, Dotan, & Rahamim, 2010; Meyerhoff et al., 2012; Moeller, Fischer, et al., 2009). We also found no evidence that single-digit numbers are processed faster than two-digit numbers, as might be suggested by their simpler notation or higher frequency (Dehaene & Mehler, 1992): trajectories of single digit numbers did not branch apart earlier than trajectories of two-digit numbers (but see Chapter 3 for further investigation of this point).

Although the present study presents strong evidence in favor of a holistic processing of 2-digit numerals, this does not mean that numbers cannot be represented in a decomposed manner. As we reviewed in the introduction, other studies have presented evidence for decomposed processing. Subjects seems to strategically choose to process two-digit quantities holistically or in a decomposed manner, with different contexts facilitating different representations (Ganor-Stern et al., 2009; Greenwald et al., 2003; Reynvoet & Brysbaert, 1999; Zhang & Wang, 2005; Zhou et al., 2008). Some paradigms, such as the number-to-position task and the linear-distribution judgment task used by Viarouge et al. (2010), may encourage estimation and therefore facilitate holistic processing. Conversely, exact processing of several multi-digit stimuli may encourage decomposition strategies. Indeed, decomposed processing was often revealed when subjects had to compare two 2-digit numbers (Meyerhoff et al., 2012; Moeller, Fischer, et al., 2009; Nuerk & Willmes, 2005). A holistic strategy in this task would require encoding two separate 2-digit quantities almost simultaneously, which may be difficult. The number-to-position paradigm is simpler than number comparison because it presents a single target number per trial. In accord with this view, holistic processing was found in other tasks that showed only a single 2-digit number at a time (Dehaene et al., 1990; Ganor-Stern et al., 2009; Reynvoet & Brysbaert, 1999; Zhang & Wang, 2005; Zhou et al., 2008), whereas studies that presented more complicated stimuli – numbers with four or six digits – revealed that the digits can be processed sequentially (Hinrichs et al., 1982; Meyerhoff et al., 2012). Chapter 4 will examine the issue of decomposed processing in more detail.

2.4.4. Effect of unit digit

A third factor influencing finger position was the unit digit, which was a reliable predictor of the implied endpoint from 300 ms post stimulus onset. The unit digit effect is also shown by the finding of a trajectory bias that corresponded with the unit digit: trajectories of target numbers with a small unit digit (1 or 2) were biased to the left compared to trajectories with a large unit digit (8 or 9). Three models can account for these findings: decomposed encoding of single-digit quantities, sequential processing of the 2-digit numbers (first the unit digit and then the decade digit), or transposition of the two digits. All these models focus on decomposed processing of the two digits, and the first model also assumes decomposed single-digit quantities.

The decomposed quantities model assumes that on top of the two-digit quantity, the single-digit quantities are encoded too, and thus the finger is influenced by their mean value. It could be objected that the results showed only an independent contribution of the unit digit and not of the decade digit. Note, however, that with the regression approach, we cannot independently estimate the effects of units u, decades d, and the whole number (=10d+u), as these three variables are linearly dependent. All we can therefore conclude is that the effect of the unit digit is, initially at least, larger than predicted by the equation 10d+u, and this is compatible with an additional contribution of the mean of d and u.

The second model assumes that the two digits are processed sequentially in a reversed order, first the unit digit, then the decade digit. As a result, the unit digit contributes to the quantity before the decade digit does. Thus, for a certain period of time the overall quantity – whether if holistic or decomposed – over-represents the value of unit digit compared with the decade digit.

The third model that can account for the results is a transposition model. This model assumes a transient stage during which the digits are already identified but are not yet bound to their relative positions (Friedmann, Dotan, & Rahamim, 2010), thus creating illusory conjunctions (Treisman & Schmidt, 1982). During this transient stage, both the target quantity (10d+u) and the transposed quantity (10u+d) would be activated (e.g., presenting 28 activates both quantities

28 and 82), either as holistic or decomposed quantities, thus enhancing the overall effect of u on the finger position.

Finally, note that the unit digit effect was relatively small: in the x-coordinate regression, the maximal mean b[U] that was significantly larger than zero was only .039. This peak happened at 750 ms following target onset, and the contributions of the other predictors in that time point were much larger (b[N_{0-40}] = .31, b[log'(N_{0-40})] = .07; the log predictor later reached a peak b value of .088, in 850 ms). Thus, the unit digit effect may indeed originate in decomposed quantity representation, but the more dominant quantity representation in this task is still the holistic one. The issue of decade-unit processing is examined in detail in Chapter 4.

2.4.5. Spatial bias

The last factor to influence finger position was a spatial-reference-point bias function (SRP), which was a reliable predictor of the implied endpoint from 600 ms post stimulus onset and in all later time points – even the endpoints were biased away from a purely linear organization (Fig. 2.5a), in agreement with previous studies (Barth & Paladino, 2011; Sullivan et al., 2011). This factor suggests that the target position in the number-to-position task is obtained using a non-linear estimation of the distances to three fixed reference points: the left end, middle, and right end of the number line.

The SRP bias function was also a significant factor in the aim-to-arrow task, although this task does not involve any numbers or quantities. This finding suggests that the SRP factor originates – at least in part – in a spatial/motor process rather than in the quantity representation or in the process that creates it from the Arabic number. A comparison of the SRP factor between the two tasks is in line with the assumption that this factor reflects a position-estimation error: in the aim-to-arrow task, the estimation error is expected to be larger when the finger is far from the target arrow, and indeed the SRP bias factor was observed early on in the trajectory. Conversely, the number-to-position task never presents the target position, so the position-estimation process continues throughout the trajectory, and correspondingly, the SRP bias was found late in the trajectory and even in its endpoint.

The spatial reference point model was able to account for much of the bias from linear organization in the number-to-position task, and yet the results did not fit the model perfectly. Two major findings indicate that the spatial reference points model should be amended to fully account for the bias observed in the present study: the existence of a global leftward endpoint bias, and the negative contribution of the log predictor to the final regression model (Fig. 2.4e), a finding that is not explained by any theoretical model yet. In Chapter 3, we propose a model that may capture this set of biases in a better way than the SRP bias function.

2.4.6. The successive stages of converting a number to a position

Organizing these factors along a timeline clarifies the process performed in the number-to-position task. When the two-digit target number is presented, the participants first create a transient quantity representation of the unit digit (or, alternatively, a quantity representation of the transposed number, e.g., the quantity 52 upon seeing the target number 25). This representation is activated surprisingly quickly, as it affects the finger direction (implied endpoint) as early as 250 ms after the target number was presented. This finding is however not incompatible with earlier ERP studies, which indicate significant quantity effects as early as 174 ms after target onset (Dehaene, 1996), and with the finding that digit comparison can be performed above chance level within 230 ms from the stimulus onset (Milosavljevic, Madsen, Koch, & Rangel, 2011).

Shortly afterwards, two separate representations of the two-digit quantity are created: a holistic logarithmic representation and a linear representation (either holistic, or decomposed with the unit and decade digits contributing in almost exact 1:10 ratio). The log and linear representations must be active at about 300 ms, since they start affecting the finger direction 400 ms after the target onset. The linear representation remains until the end of the trial, but the log representation is transient: 750 ms after the target onset, it no longer affects the implied endpoint.

Finally, as their finger approaches the target line, the participants start adopting a spatial strategy of transforming the two-digit quantity representation into a precise location on the number line. This strategy, which has a measurable effect 600 ms after the target onset, relies on three reference points (the left end, middle, and right end of the line), and results in a bias that pushes the finger trajectories away from these reference points.

3. On the origins of logarithmic number-to-position mapping°

Abstract. We present a detailed experimental and theoretical dissection of the processing stages that underlie the number-to-position task. When adults map the position of two-digit numbers on a line, their final mapping is essentially linear, but when monitoring the finger trajectories, the intermediate finger location shows a transient logarithmic mapping. Here we identify the origins of this log effect: small numbers are processed faster than large numbers, so the finger deviates towards the target position earlier for small numbers than for large numbers. When the trajectories are aligned on finger deviation onset, the log effect disappears. The small-number advantage and the log effect are enhanced in dual-task setting and are further enhanced when the delay between the two tasks is shortened, suggesting that these effects originate from a central stage of quantification and decision making. We also report cases of logarithmic mapping – by children and by a brain-injured individual – which cannot be explained by faster responding to small numbers. We show that these findings are captured by an ideal-observer model of the number-to-position mapping task, comprising 3 distinct stages: (1) a quantification stage, whose duration is influenced by both exact and approximate representations of numerical quantity; (2) a Bayesian accumulation-of-evidence stage, leading to a decision about the target location; and (3) a pointing stage.

3.1. Introduction

In Chapter 2 we saw how the process of understanding the quantities represented by two-digit numbers can be explored using the number-to-position task, and how trajectory tracking can serve to gain an insight into the successive stages of this process. The main finding in Chapter 2 was that the finger position was correlated with the two-digit target number, but there was a transient time window in which the finger position was affected by an additional contribution of the logarithm of the target. This observation suggested that the quantities were encoded by two distinct systems: an exact linear representation, where all numbers are equally well represented, and an approximate representation where small numbers are represented more precisely than larger ones. This conclusion was in accord with studies that found compressive quantity representation in other tasks (Anobile et al., 2012; Berteletti et al., 2010; Booth & Siegler, 2006; Dehaene et al., 2008; Dehaene & Marques, 2002; Lourenco & Longo, 2009; Núñez et al., 2011; Opfer & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003; Viarouge et al., 2010). Mathematically, the approximate representation can be described as a

° This chapter is an article published as Dotan, D., & Dehaene, S. (2016). On the origins of logarithmic number-to-position mapping. *Psychological Review*, *123*(6), 637–666. doi:10.1037/rev0000038. The text here is identical with the published article, except reformatting and removing some parts that would, if remained, repeat other sections of this dissertation. The chapter has supplementary material in Appendix A.

logarithmic number line with fixed variance, as suggested by neural recordings and brain imaging data (Nieder & Miller, 2003; Piazza et al., 2004). As previously noted (Dehaene, 1997), an equally accurate model of behavioral data can be obtained by postulating a linear number line with scalar variability (standard deviation proportional to number; for discussion, see Cicchini, Anobile, & Burr, 2014; Dehaene, 2007). As shorthand, we refer to these two representations simply as “approximate”, referring to the fact that they both show an increasing uncertainty as the numbers get larger.

Our goal in the present chapter was to clarify the theoretical reasons why a logarithmic effect arises even in adults, who know perfectly well that they should point to the linear location of the numbers. In particular, we designed new experiments exploring the hypothesis of a dual representation of quantity. We reasoned that, if there are two distinct representations of number, respectively exact and approximate, then we might be able to interfere with one of them and therefore transiently enhance the influence of the other. We relied on the method introduced by Anobile et al. (2012), who used quantity-to-position mapping in a dual-task setting. In the critical condition, participants estimated a number of dots and responded by marking a position on a line, while simultaneously performing a secondary task of color pattern judgment. This manipulation made their mapping more logarithmic. This pattern could be explained as a psychological refractory period (PRP) effect in which the secondary task competed with the exact linear quantification process for central resources, while leaving approximate quantification intact. As a result, the log effect was facilitated while the linear representation was reduced. The log-linear dissociation can therefore support a model of dual quantity representation. We aimed to replicate these findings with two-digit symbolic numbers, using our continuous number-to-position paradigm.

We also assessed a new theoretical interpretation that has recently arisen for the log effect in number-to-position tasks (Cicchini et al., 2014). This interpretation rests on a single quantity representation with differential variability – large quantities are represented with greater noise than small quantities. The idea is that the log effect results from a Bayesian process that combines this fuzzy quantity representation with prior knowledge (Fischer & Whitney, 2014; Jazayeri & Shadlen, 2010). Because large quantities are fuzzier than small quantities, they are estimated with lower confidence, and the Bayesian decision process assigns them a smaller weight relatively to prior knowledge. The decision is therefore slower (and the effect of prior biases is stronger) for larger target numbers than for smaller target numbers, and this is what gives rise to the logarithmic effect. In a dual-task setting, interference from the secondary task reduces even further the amount of evidence that can be extracted from the quantity representation per unit of time, and therefore the logarithmic effect is increased.

Note that differential variability between small and large numbers can take many forms: one possibility is scalar variability (linear mapping of numerical quantities, and linear relation between the noise level and the target number), but the model can accept almost any form of differential encoding of small and large numbers. Thus, a compressive scale for number (e.g., logarithmic) with fixed variability would lead to similar results. Furthermore, when the stimuli are sets of dots (as was the case in Anobile et al., 2012), differential variability may arise from the assumption that the noise in the subitizing range (1-3) is lower than in the non-subitizing range (> 4) (Cicchini et al, 2014).

Crucially, according to this model, logarithmic mapping can be obtained even if the internal quantity scale is not logarithmic. Although it was initially argued that logarithmic behavior in the number-to-position task implies an internal logarithmic representation (Booth & Siegler, 2006; Dehaene et al., 2008; Siegler & Booth, 2004; Siegler & Opfer, 2003, and in Chapter 2), Cicchini et al.'s model shows that this is not the case. In particular, as previously argued, there is a near-complete behavioral equivalence between the log and the scalar variability models of approximate number representation (Dehaene, 2007).

Cicchini et al. (2014) further showed that the prior in the Bayesian decision process need not be fixed. Indeed, they discovered a new empirical finding that suggests that the prior is adjusted on a trial-by-trial basis: judgments are strongly affected by the quantity presented on the immediately previous trial. Nevertheless, whether the prior is fixed or is updated after each trial, what really accounts for the log effect in Bayesian decision models is differential variability. Accordingly, a recent study has shown a logarithmic effect in quantity-to-position mapping even in the first trial of an experiment, when prior trial information was not yet available (Kim & Opfer, 2015).

The experiments and equations presented in Cicchini et al. (2014) capture only the participants’ ultimate response location in a number-to-position task, and remain silent about the sequence of processing stages that ultimate leads to this decision. In the present study, we wish to extend this model to account for the detailed within-trial dynamics of the number-to-position task. Our goal, indeed, is to obtain a detailed theory of the successive stages leading to a decision in the number-to-position task. We will show that an ideal-observer theory can account for our main finding that the mapping to position shows a logarithmic trend when the trial starts but becomes fully linear when the finger reaches the number line. The intuition behind this model can be specified succinctly: assuming that the decision to move is based on a Bayesian decision process, with a progressive accumulation of evidence arising from the target, then differential variability should affect the *processing time* of the target. Large target numbers, which are represented with higher variability, are quantified more slowly than small target numbers (hereby, "small-number advantage"), so the Bayesian prior is overridden more slowly for large target numbers. As a result, at each post-stimulus time point, small-target trials are in a more advanced stage of processing than large-target trials, which means that the finger trajectories for small targets are farther apart from each other than the trajectories for larger numbers. These differential distances between the trajectories appear as logarithmic effect when analyzing a specific time point.

We term this dynamic version of Cicchini et al.'s (2014) model *the differential encoding time model*. In the final section, we present a precise mathematical model and simulations of this idea. Note that the differential encoding time model conforms to the two main assumptions of Cicchini et al. (2014): (1) The target position is determined by a Bayesian decision process, with a prior that is affected by previous trials; (2) The log effect results from differential variability for small versus large numbers, which causes differential overriding of the prior by the present-trial quantity.

3.2. Experiment 3.1: Number-to-Position Mapping with Dual Task

In Experiment 3.1, the participants mapped two-digit numbers between 0 and 40 to the corresponding positions on a number line. Each participant performed the task in three conditions, administered in three separate blocks. The first condition involved a single task: the participants mapped numbers to positions, with no other manipulation (like in Chapter 2).

The second condition involved dual-tasking: subjects performed the number-to-position mapping parallel to a distracter task. Like Anobile et al. (2012) we used a color-detection task, which in our case was color naming. We hoped this task would maximize the interference effect, because it is not only attention demanding but also involves verbal output, which may selectively interfere with the linear quantity representation. One possible reason for such a selective interference rests on the assumption that Arabic numbers can be encoded as quantities by both hemispheres, but only the left hemisphere houses a verbal representation of numbers (Cohen & Dehaene, 2000). When the verbal system is occupied, approximate quantity may still be perceived without verbal mediation (Dehaene & Cohen, 1991; Dehaene et al., 2008; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). Under this hypothesis, verbal interference should increase the relative weight of the non-verbal parietal circuit that encode approximate quantities. The dual-representation model thus predicts that the color naming condition should enhance the transient logarithmic effect.

The third, control condition, was number naming: the participants said aloud the number while pointing to the corresponding position. This condition does not divert attention from the target number and, if anything, should enhance the exact linear representation.

3.2.1. Method

3.2.1.1. Participants

Eighteen right-handed adults, aged 27;8 ± 6;5, with no reported learning disabilities or color blindness, were compensated for participation. Their mother tongue was Hebrew. For comparison, we also reanalyzed the data of the 21 right-handed participants reported in Chapter 2 – 10 Hebrew speakers, 9 French, one Italian, and one Thai, aged 35;5 ± 10;7, who performed the number-to-position task silently. Digital numbers in Hebrew are written like in English, and in our number-to-position paradigm Hebrew participants and left-to-right readers were found to exhibit similar patterns of results (Chapter 2).

3.2.1.2. Procedure

In each of the three conditions (silent, color naming, and number naming), each number 0-40 was presented 6 times (246 trials) in random order. In the two naming conditions the participants were told that the two tasks (number-to-position and naming) were equally important but that they should first attend to the naming task and then to the number-to-position task. The three conditions were blocked and were administered in random order (3 participants per presentation order). The silent condition was as described in Section 2.2. In the naming conditions, while moving the finger the participants also said aloud (in Hebrew) the target number or a color name. One color per trial – white, yellow, orange, pink, red, blue, or green – was indicated by two horizontal stripes that appeared simultaneously with the target number, surrounding it. The oral responses were tape recorded and trials with semantic or phonological errors were excluded. The speech onset time was defined as the first time point in which the voice level, sampled at 20 Hz, exceeded a threshold level for a consecutive period of 200 ms.

This threshold was configured per experiment session to match environment noise and the participants' speech volume.

The training procedure was as described in Section 2.2.2, with additional training phases for reviewing the color names and for adapting to the speech onset limits.

3.2.1.3. Trajectory analysis

Trajectories were analyzed using the two-stage regression analysis described in Section 2.2.6. Here, the dependent variable in the regressions was the implied endpoint of the median trajectories (iEP_{med}). The predictors in the regressions were the target number ($N_{0\text{-}40}$), log'($N_{0\text{-}40}$), the unit digit (U), and the SRP bias function (Section 2.3.2.6). The 2nd stage analysis compared the regression b values to zero using t-test. The reported *p* values are one-tailed when mean[b] > 0 and two-tailed when mean[b] < 0.

3.2.1.4. ANOVA

The speed of performing the number-to-position task varied a lot between individuals. Our goal in the present study was not to explain these inter-individual differences, but to focus on the within-subject factors that affect people's behavior in the number-to-position task. For this reason, in all ANOVA's in this study – most of which concern reaction times – we use repeated measures design and report effect sizes as partial η^2, a measure that is independent of the between-subject variance. To maintain standardization, we also report η^2 for one-way ANOVAs, and generalized η^2 (Bakeman, 2005; Olejnik & Algina, 2003), denoted η_G^2, for ANOVA with several factors. In case of an ANOVA effect for which df=1 and the effect direction has a clear prediction, we used the corresponding t test and one-tailed *p* values.

3.2.2. Results

3.2.2.1. General performance

Table 3.1. General performance measures in Experiment 3.1

Measure	Silent	Color naming	Number naming
Failed trials (%)	3 ± 2.4	22.1 ± 8 ***	14.4 ± 10.3 ***
Invalid speech onset (%) [a]	–	10.6 ± 5.3	11.2 ± 9
Naming error (%) [b]	–	2.6 ± 2	.04 ± .13
Minimal velocity violation (%)	1.4 ± 2.1	7.6 ± 5.8 ***	1.8 ± 3.3
Other errors (%)	1.6 ± 1.6	1.3 ± .9	1.4 ± 1.3
Endpoint outliers (%)	4.6 ± 1.5	5.6 ± 1.9 +	4.7 ± 1.4
Movement time (ms)	1102 ± 154	1398 ± 131 ***	1208 ± 132 ***
Endpoint bias (0-40 scale)	-.65 ± .45	-.68 ± .46	-.58 ± .39
Endpoint error (0-40 scale)	1.7 ± .42	2.1 ± 0.7 ***	1.74 ± 0.4
Speech onset time (ms)	–	898 ± 101 *** c	695 ± 90

Note. The standard deviations refer to between-subject variance of the per-subject means. Speech onset was compared between the two naming conditions.

[a] Invalid speech onset: the verbal response was too slow, too fast, or no response was made.

[b] Naming error: semantic or phonological

[c] Speech onset time was compared between the color naming and number naming conditions.

Paired t-test vs. the silent condition: + one-tailed $p < .1$ ** $p < .01$ *** $p < .001$

Table 3.1 shows that number-to-position mapping was more difficult in the color naming condition than in the silent condition: the participants were less accurate (larger endpoint error), slower, and had more failed trials. Thus, the color naming manipulation was clearly effective. The number naming manipulation had a weaker effect: a smaller difference was observed in movement time and failed trial rate, and accuracy was similar to the silent condition. The participants' unanimous subjective impression was that color naming was considerably harder than the two other conditions.

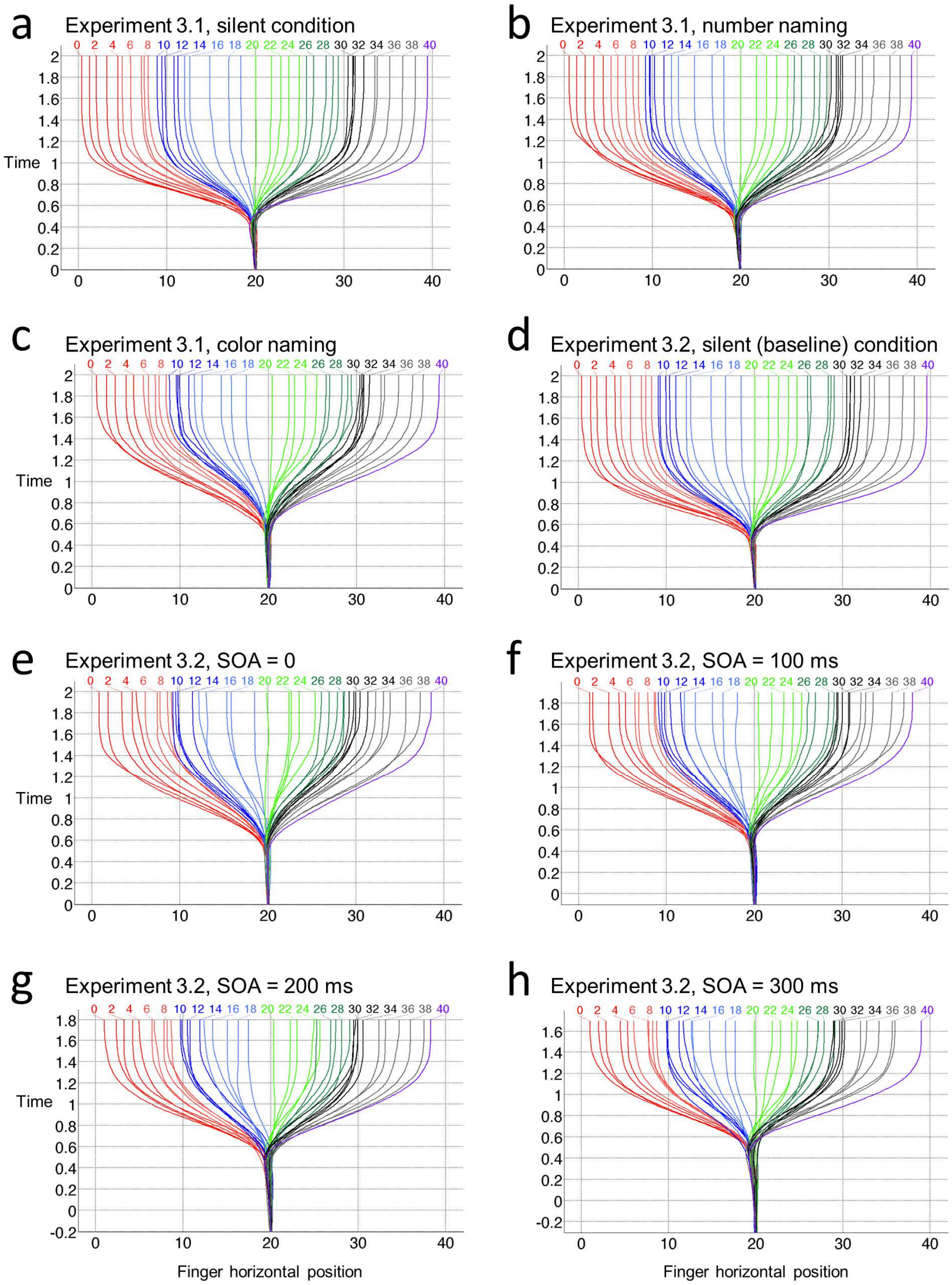

Fig. 3.1. Median trajectories per target in Experiments 3.1 and 3.2. A median trajectory was created by re-sampling each trajectory into equally-spaced time points, finding the per-subject median coordinates per time point, and averaging these medians over participants. Median trajectories shorter than 2 s were extended using the endpoint. Note that in Experiment 3.2 the number sometimes appeared after the color (panels f-h); the bottom of each of these panels is aligned to the beginning of the trial (color onset), and time=0 indicates the number onset.

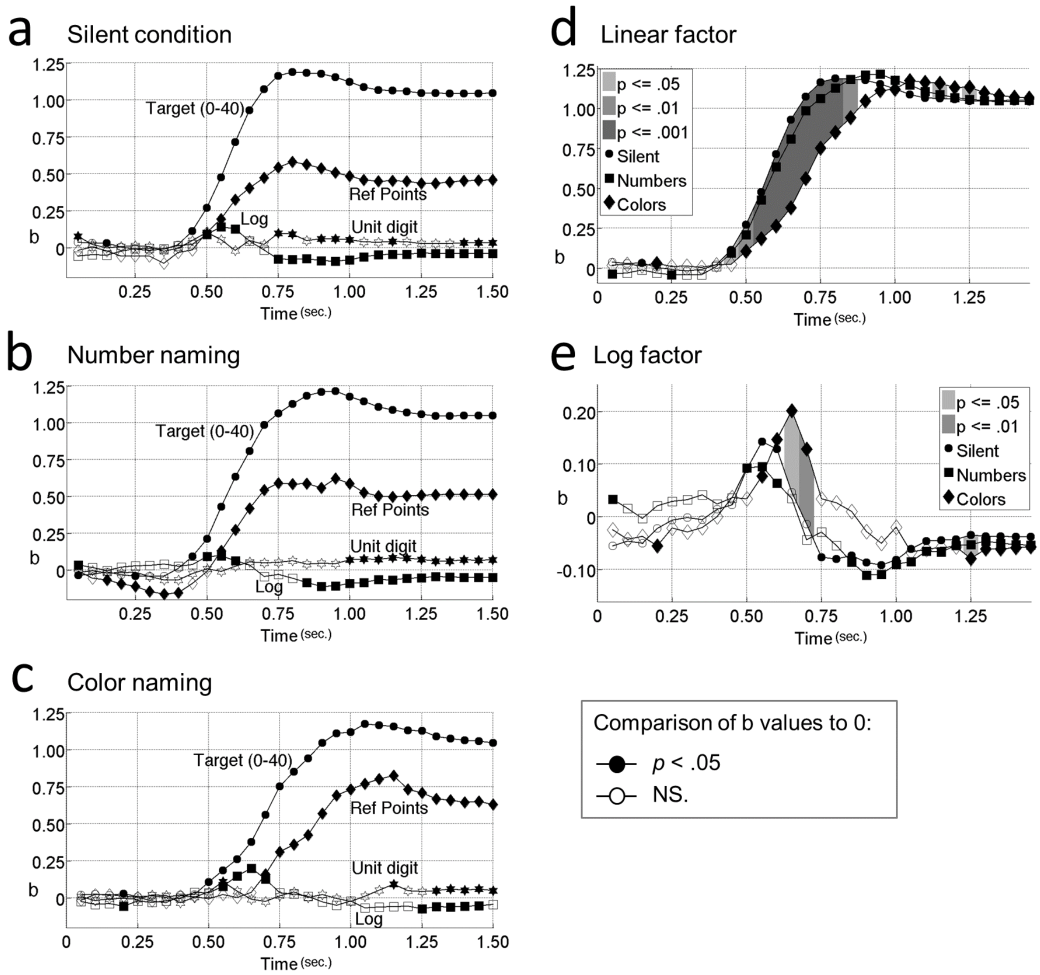

Fig. 3.2. Time course of the effects in Experiment 3.1. All panels show b values of regressions on the implied endpoint of the median trajectory (iEP_{med}). One regression was run per time point, participant, and condition. The b values were averaged over participants and plotted as a function of time. In this and all subsequent regression figures, the b values were compared to zero (t-test), and a black dot indicates a significant b value. (a-c) The b values per experimental condition. (d) The b values of the linear factor $b[N_{0\text{-}40}]$ in all three conditions: the effect of the linear factor arises faster in the silent condition than during color naming (the shaded area indicates a significant difference). (e) The b values of the logarithmic factor $b[\log'(N_{0\text{-}40})]$, showing a slightly stronger effect in color naming than in the silent condition.

The participants' median trajectories are presented in Fig. 3.1a-c. The trajectory data was submitted to the two-stage regression analysis described above in the Methods. All four predictors showed significant effects in all conditions (Fig. 3.2a-c and Table 3.2). The silent condition replicated the results from Chapter 2, including the approximate time window for the significant effect of the log regressor (500-600 ms here, 450-750 ms in Chapter 2). The only essential difference was that here we did not observe an early contribution of the unit digit;

instead, both decades and unit digits arose simultaneously as significant regressors, giving rise to a main effect of the linear value of the 2-digit target number (Fig. 3.2). The pattern of significant effects in the two naming conditions was similar to the silent condition, but in the color naming condition the factors were observed in later time points, in accord with the slower finger movement in this condition.

Table 3.2. Experiment 3.1: Time windows (ms post stimulus onset) in which the regression b values were significantly different from zero ($p \le .05$)

Factor	Silent	Color naming	Number naming
$b[N_{0\text{-}40}] > 0$	450-end	200,500-end	450-end
$b[\log'(N_{0\text{-}40})] > 0$	500-600	550-700	500-600
$b[\log'(N_{0\text{-}40})] < 0$	750-end	1250-end [a]	850-end
$b[U] > 0$	750-end [b]	550, 1150, 1300-end	1000-end
$b[SRP] > 0$	550-end	700-end	550-end
$b[SRP] < 0$	None	None	150-400

[a] $p < .05$ in 1250-1450, 1650-1700, and $.05 < p < .07$ in the other time points.

[b] $p < .05$ in 750-800, 900-1000, 1400-1500, and $.05 < p \le .08$ in the other time points.

3.2.2.2. Assessment of the dual representation model

3.2.2.2.1. Color naming interferes with the linear factor and enhances the logarithmic factor

The regression b values of the log and linear factors in the silent condition were compared, per time point, versus the color naming condition using a paired t-test (see Fig. 3.2). This comparison confirmed the crossover interaction between the log and linear factors: as predicted, the color naming manipulation enhanced the log factor and reduced the linear factor. The linear factor in the color naming condition was significantly smaller than in the silent condition from 450 ms to 850 ms ($b[N_{0\text{-}40}]_{colors} < b[N_{0\text{-}40}]_{silent}$, $t(17) > 1.75$, one-tailed $p < .05$). The pattern was reversed for the log factor: a significant difference $b[\log'(N_{0\text{-}40})]_{colors} > b[\log'(N_{0\text{-}40})]_{silent}$ was observed from 650 to 750 ms ($t(17) > 2.1$, one-tailed $p \le .05$; the difference $b[\log'(N_{0\text{-}40})]_{colors} < b[\log'(N_{0\text{-}40})]_{silent}$ from 500 ms to 550 ms did not reach significance, $t(17) < 1.4$, one-tailed $p > .09$). This dissociation supports the predicted enhancement of the approximate representation, relative to the exact representation, during dual-task interference.

This influence of color naming can be interpreted in two ways – either as facilitating the approximate quantity representation and weakening the linear representation, or as delaying the linear representation (i.e., the difference between the silent and color curves in Fig. 3.2d can be viewed as either vertical or horizontal). If we accept the delay model, the delay size can be estimated from Fig. 3.2d as ~50 ms around movement onset (~450-500 ms post stimulus onset), increasing to ~200 ms when crossing the $b = 1$ threshold (at ~670-870 ms). Fig. 3.2e suggests that color naming may have slightly delayed the influence of the logarithmic factor too, but this delay was much smaller and never exceeded ~50 ms. The results are therefore compatible with the hypothesis that the linear quantity representation was delayed, which left the stage for the log representation to have a larger effect on the finger movement.

Given these apparent delays, we also attempted to compute a time-independent per-participant index of the peak log effect size. This index, denoted $b[\log'(N_{0\text{-}40})]_{global}$, was defined as the 75th percentile of $b[\log'(N_{0\text{-}40})]$ between 450 ms and 750 ms (the time window in which a significant log effect was found in Chapter 2; we used 75th percentile rather than the peak b value to increase the robustness to noise and outliers). The $b[\log'(N_{0\text{-}40})]_{global}$ was larger in color naming ($b = 0.20 \pm 0.10$) than in the silent condition ($b = 0.15 \pm 0.11$; $t(17) = 2.1$, one-tailed $p = .03$, Cohen's $d = 0.47$), confirming that the color naming manipulation enhanced the participants' log effect.

When comparing regression b values in different conditions or at different time points, a potential confounding factor may be that the b values are affected by the global variance among trajectories σ(iEP), and that this variance may differ between conditions. However, this explanation cannot account for the present results, because we found a *larger* log effect size in the color naming, whereas σ(iEP) in equivalent time points was smaller in this condition. To completely rule out the alternative interpretation, we re-ran the log effect size analysis using the regression β values, which are not affected by the overall implied endpoints variance[3]. This analysis too showed a larger log effect size in color naming ($\beta[\log'(N_{0\text{-}40})]_{global/silent} = 0.22 \pm 0.22$, $\beta[\log'(N_{0\text{-}40})]_{global/colors} = 0.32 \pm 0.18$, $t(17) = 1.83$, one-tailed $p = .04$).

[3] A regression analysis results in a regression formula $Predicted(y) = const + \Sigma b_i x_i$. These b values are informative when the predictors x_i and the dependent variable y are specified using a meaningful scale, as is the case in the present study. However, the b_i values are sensitive to the scale in which x_i and y are specified, and consequently they are typically not comparable with each other or across datasets. This comparability issue can be solved by standardizing the predictors and the dependent variable using linear transformation into a common scale with mean = 0 and σ = 1. Denoting the transformed variables x_i' and y', the regression formula would now be $Predicted(y') = \Sigma \beta_i x_i'$, where $\beta_i = b_i * \sigma(x_i) / \sigma(y)$. Unlike b values, the β values are comparable with each other because all x_i' are specified using the same scale. More importantly for the present issue, even β values from different regressions are comparable, because the dependent variables too are specified using a fixed scale.

3.2.2.2.2. Control condition: Number naming

The number naming results were very similar to the silent condition (Fig. 3.2). The linear factor $b[N_{0\text{-}40}]$ was slightly smaller in number naming than in the silent condition, but this difference was significant only in two time points (650 ms and 750 ms, $t(17) > 2.47$, two-tailed $p \leq .03$; in all other time points, $t(17) < 1.77$, $p > .09$). The log factor did not show a clear trend: $b[\log'(N_{0\text{-}40})]$ was stronger in the silent condition than in number naming at some time points and weaker at other time points, with a significant effect only in two time points (350 ms and 750 ms, $t(17) > 1.88$, $p < .04$). The global log effect size was similar in number naming ($b[\log'(N_{0\text{-}40})]_{global} = 0.12 \pm 0.13$) and in the silent condition ($b[\log'(N_{0\text{-}40})]_{global} = 0.15 \pm 0.11$; $t(17) = 1.12$, two-tailed $p = .25$). Thus, number naming, unlike color naming, did not facilitate the log factor. Analyzing the results in terms of delay shows that number naming caused only a small delay of ~ 10-20 ms in the linear factor and no delay in the log factor.

3.2.2.3. Dependency on prior trials

To assess the possibility that the participants' performance was affected by perseverations from previous trials, as described in Cicchini et al. (2014), the trajectory data was submitted to regression analysis with the four predictors described above ($N_{0\text{-}40}$, $\log'(N_{0\text{-}40})$, U, and SRP), to which we added the values of the target numbers in each of the last 15 trials (predictors denoted N-1, N-2, … N-15). The regression was run on the raw, unaveraged trials and the dependent variable was iEP. One regression was run per condition and participant in 50 ms intervals. Per predictor, condition, and time point, the participants' b values were compared to zero using t-test (Fig. 3.3a-c). These regressions showed a significant effect of the last 2 or 3 trials, which decreased around 500 ms as the finger began to point to the target quantity of the current trial.

To examine the relative effect of perseveration from each of the previous trials, we calculated the mean b value of each of the predictors N-1 to N-10 over the time range 0-600 ms. This was done for the three conditions in Experiment 3.1 and for the data from Chapter 2. We observed an exponentially decreasing contribution of previous targets (Fig. 3.3d). This pattern is consistent with the notion of a Bayesian process (Cicchini et al., 2014), according to which the finger is initially guided by an expectation or "prior" based on past trials, which gets constantly updated as the new target gradually overrides the expectation generated from older trials. The prior appears to decay roughly exponentially across trials N-1, N-2, N-3 etc., and in this respect, the phenomenon bears similarity to perseverations observed in many brain-lesioned patients (Cohen & Dehaene, 1998).

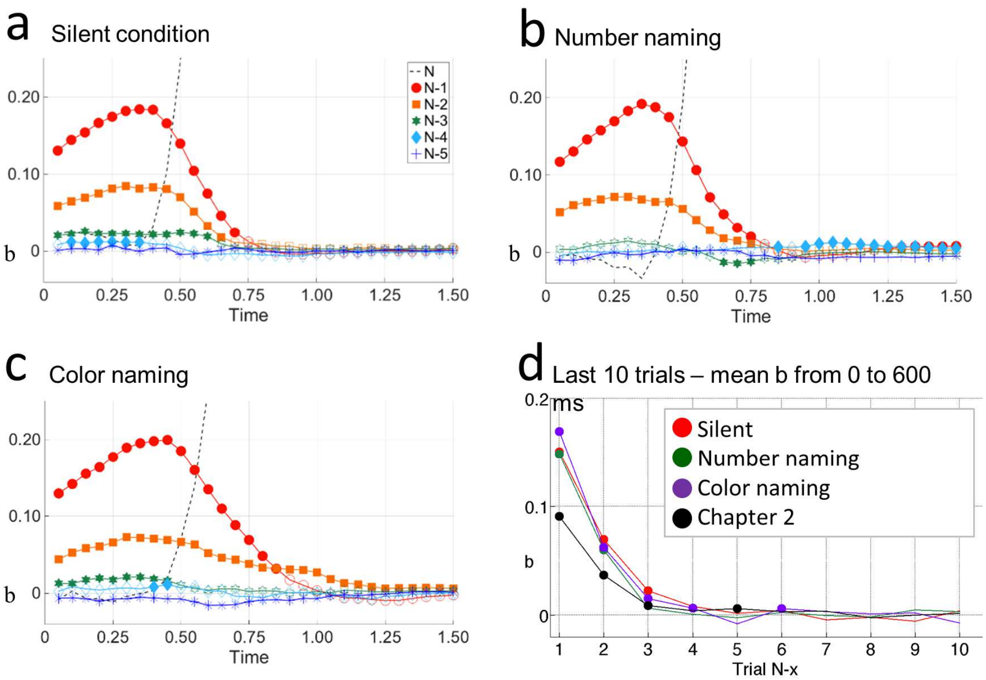

Fig. 3.3. Influence of the prior targets on current finger trajectory in Experiment 3.1. (a-c) Influence of the current target N and the past 5 targets (N-1 to N-5) on the implied endpoint, as measured by regression (same type of plot as in Fig. 3.2). (d) Mean b value over 0-600 ms, for each of the past 10 targets (N-1 to N-10), showing an exponentially decreasing influence of prior targets.

3.2.2.4. Assessment of the differential encoding time model

The differential encoding time model stipulates that the log pattern occurs because the finger deviates towards the desired location at an earlier time point for smaller target numbers (small-number advantage). As a result, in several post-stimulus-onset time points, trials with small target numbers are in a more advanced stage of processing (and finger movement) than trials with large targets, so the trajectories of small-target trials are farther apart from each other, giving rise to a log effect in the regression.

3.2.2.4.1. Identifying the onset of horizontal movement

To assess the differential encoding time model, we first calculated the onset time of the finger's horizontal movement on each trial. To determine the horizontal movement onset per trial, we used an algorithm that aimed to identify the time point where the finger horizontal velocity started building up. A typical horizontal velocity profile of a trial consists of one or more velocity peaks (which may reflect several successive movement plans), but as every experimental measure it is also affected by jitter and random movements. Our goal was to find the onset of the earliest non-random velocity peak. To identify non-random peaks, we first estimated the participant's individual level of "motor noise" based on the distribution of horizontal velocities during the time window 0-250 ms (assuming that before 250 ms, movement is not yet affected by the target number; see Appendix A for a justification of this assumption). We considered only velocity peaks that were significantly higher than this motor noise, and found the onset of the earliest of these peaks – as long as the onset occurred after 250 ms.

The specific algorithm was as follows. To calculate the horizontal velocity along each trajectory, we first applied Gaussian smoothing with $\sigma = 20$ ms to the finger x coordinates, and then computed the derivative of the smoothed coordinates. To determine the horizontal movement onset per trial, we first looked for a significant peak of the x velocity profile – the highest x velocity that exceeded the top 1 percentile of the participant's velocity distribution on the first 250 ms of all trials. The onset time of this peak x velocity was defined as the latest time point where the x velocity remained lower than 5% of the peak velocity (if velocity never got below this threshold from 250 ms onwards, no onset was found and the peak was ignored). To detect cases in which there was evidence for several successive movements (several velocity peaks), we checked if there was, earlier to the detected movement onset, another significant velocity peak, and reapplied the algorithm to detect this peak's onset. This procedure was applied recursively until no further velocity peak was detected. Visual inspection indicated that, for the vast majority of the trials, the algorithm was in excellent agreement with our subjective perception of the movement onset.

The algorithm failed to find the movement onset when the peak velocity was too low to reach significance, or when the above 5% criterion was never met in the time window from 250 ms post onset until 100 ms before the finger reached the number line. Such failures amounted to 19%, 18.3%, and 13.6% of the trials in the silent condition, number naming, and color naming, respectively. The horizontal movement onset time of these trials was coded manually whenever possible (the encoder was blind to the target number and saw only if it was smaller or larger than 20). After manual encoding, onset information was available for 97.9% of the trials. In the Chapter 2 data, the algorithm failed to find the onset of 11.4% of the trials, and after manual encoding the onset information was available for 99.2% of the trials.

Fig. 3.4a shows the mean horizontal movement onset times per target number and experimental condition. In the analyses of horizontal movement onsets (detailed below), we excluded trials with target number between 15 and 25, in which the target was close to the center of the screen and the horizontal movement was too small for reliable onset detection. We also excluded trials with endpoint outliers (as explained in Section 2.2.5).

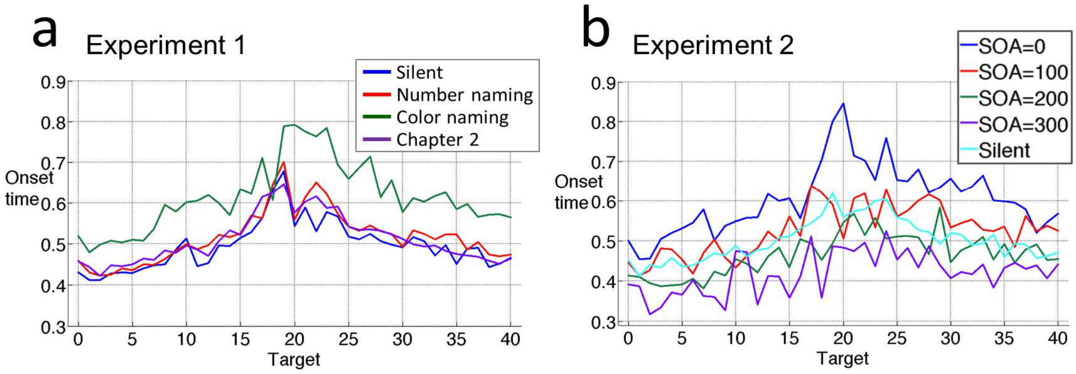

Fig. 3.4. The mean onset time of horizontal movements, averaged over all participants, as a function of target number. (a) Onset time per condition in Experiment 3.1 and in Chapter 2. (b) Onset time per SOA in Experiment 3.2 (t=0 is the target number onset time). Targets = 15-25 are plotted here but were excluded from all analyses.

3.2.2.4.2. The factors affecting the horizontal movement onset

The differential encoding time model predicts that the onset times should be earlier for smaller numbers (the small-number advantage effect), and that color naming should enhance the small-number advantage. To examine this assumption, the onset times were submitted to three-way repeated measures ANOVA with the subject as the random factor and with 3 within-subject factors: the experimental condition, the target side (< 20, left; or > 20, right), and a numeric factor given by the absolute distance between the target number and 20. Two separate ANOVA’s were run: one compared color naming with the silent condition, and another compared number naming with the silent condition.

3.2.2.4.2.1. Color naming versus the silent condition

A significant main effect of condition ($F(1,17) = 114.1$, $p < .001$, $\eta_p^2 = .87$, $\eta_G^2 = .38$) reflected the dual-task interference: movement onset in color naming was delayed by 111 ms relative to the silent condition.

A significant main effect of side ($t(17) = 3.99$, one-tailed $p < .001$, $\eta_p^2 = .48$, $\eta_G^2 = .11$) confirmed the small-number advantage: movement onset was earlier for small numbers than for large numbers (mean delay = 49 ms), as predicted by the differential encoding time model. The differential encoding time model also predicts that color naming would facilitate the small-number advantage, and this was indeed the case: the small-number advantage in color naming (62 ms) was larger than in the silent condition (36 ms), and the Condition x Side interaction was significant (t(17) = 1.99, one-tailed p = .03, η_p^2 = .19, η_G^2 = .008). Thus, the predictions of the differential encoding time model were fully confirmed.

A significant main effect of distance (F(1,17) = 97.4, p < .001, η_p^2 = .85, η_G^2 = .15) showed that movement onset was earlier as the target number became closer to either end of the number line – a pattern clearly observable in Fig. 3.4a. To analyze the interactions with the distance factor, we first examined their direction by calculating the distance effect in the various conditions. The movement onset time was submitted to regression analysis with distance = |target-20| as a single predictor – one regression per participant, condition, and side. The distance effect in color naming (average b[distance] = -9.92 ms) was stronger than in the silent condition (b[distance] = -7.24 ms). The three-way ANOVA showed that this difference was significant (Distance x Condition interaction: t(17) = 2.68, one-tailed p = .01, η_p^2 = .0.3, η_G^2 = .0.01). The distance effect was also marginally stronger for numbers < 20 (b[distance] = -9.49 ms) than for numbers > 20 (b[distance] = -7.67 ms; Distance x Side interaction: t(17) = 1.43, one-tailed p = .08, η_p^2 = .11, η_G^2 = .001). This Distance x Side interaction is predictable by both logarithmic and scalar variability models, which attribute the distance effect to the target quantity: such models predict a stronger distance effect when the ratios between the quantities are larger, as is the case for targets < 20 compared with targets > 20. The three-way Condition x Side x Distance interaction was not significant (F(1,17) < 0.01, p = .98).

3.2.2.4.2.2. Number naming versus the silent condition

A significant main effect of condition (F(1,17) = 12.43, p = .003, η_p^2 = .42, η_G^2 = .02) reflected a dual-task interference, although smaller than in color naming: movement onset in number naming was delayed by 18 ms relative to the silent condition.

A significant small-number advantage was observed (41 ms, main effect of side: t(17) = 3.14, one-tailed p = .005, η_p^2 = .37, η_G^2 = .10). The small-number advantage did not differ significantly between number naming (45 ms) and the silent condition (36 ms; Condition x Side interaction: F(1,17) = 0.53, p = .48).

The main effect of distance was significant (F(1,17) = 81.7, p < .001, η_p^2 = .83, η_G^2 = .13) and this effect too did not interact with condition (F(1,17) = 0.45, p = .51). The direction of the Distance x Side interaction was examined using the same method we described above to analyze the color naming condition. This analysis showed that as predicted, the distance effect for numbers < 20 (b[distance] = -9.0 ms) was marginally larger than the distance effect for numbers > 20 (b[distance] = -7.24 ms, Distance x Side interaction: t(17) = 1.64, one-tailed

p = .06, η_p^2 = .14, η_G^2 = .004). The three-way Condition x Side x Distance interaction was not significant (F(1,17) = 0.28, p = .60).

3.2.2.4.3. Differential encoding times as the reason for the log effect

The differential encoding time model attributes the transient log effect (Fig. 3.2) to earlier horizontal movement onset times in small-target trajectories than in large-target trajectories. If these differences in movement onset times were eliminated, the model predicts that the transient log effect would disappear. To eliminate onset time differences, we aligned each trial's trajectory data to its horizontal movement onset time. The aligned trajectories (excluding trials with no movement onset information) were submitted to regression analysis similar to the one described in the "Assessment of the dual representation model" section above, with iEP as the dependent variable and with four predictors: $N_{0\text{-}40}$, log'[$N_{0\text{-}40}$], the unit digit U, and SRP. One regression was run per condition, participant, and post-horizontal-movement-onset time point in 50 ms intervals. Per predictor, condition, and time point, the participants' b values were compared with zero using t-test. A significant positive contribution of b[$N_{0\text{-}40}$] was found in all conditions and in all time points (Fig. 3.5). b[$N_{0\text{-}40}$] was significant even at the time of horizontal movement onset (t=0), and within 50 ms it reached a considerable effect in all conditions (over participants, mean b > 0.38). This indicates that when the finger horizontal movement started, the participants already had a linear quantity representation of the 2-digit number. Crucially, the log factor b[log'($N_{0\text{-}40}$)] no longer showed any significant positive effect in any experimental condition, excluding a short time window (150-250 ms) in the Chapter 2 data, in which there was a minor log effect (b[log'($N_{0\text{-}40}$)] ≤ 0.05; Fig. 3.5d). Thus, controlling for the movement onset time eliminated the log effect, as predicted by the differential encoding time model.

The elimination of the log effect cannot be attributed to the fact that the aligned regression was run only on a subset of the trials (those for which we could identify the movement onset): when the same regression was run on the same subset of trials without aligning trajectories by their onset time, the log factor b[log'($N_{0\text{-}40}$)] was significantly larger than zero in each of the 3 conditions during at least 250 ms, with peak b[log'($N_{0\text{-}40}$)] ≥ 0.11 (average over participants).

These findings indicate that the transient log effect in this task, both in the silent single-task condition and in the dual-task conditions, can be fully explained by differential horizontal movement onsets per target.

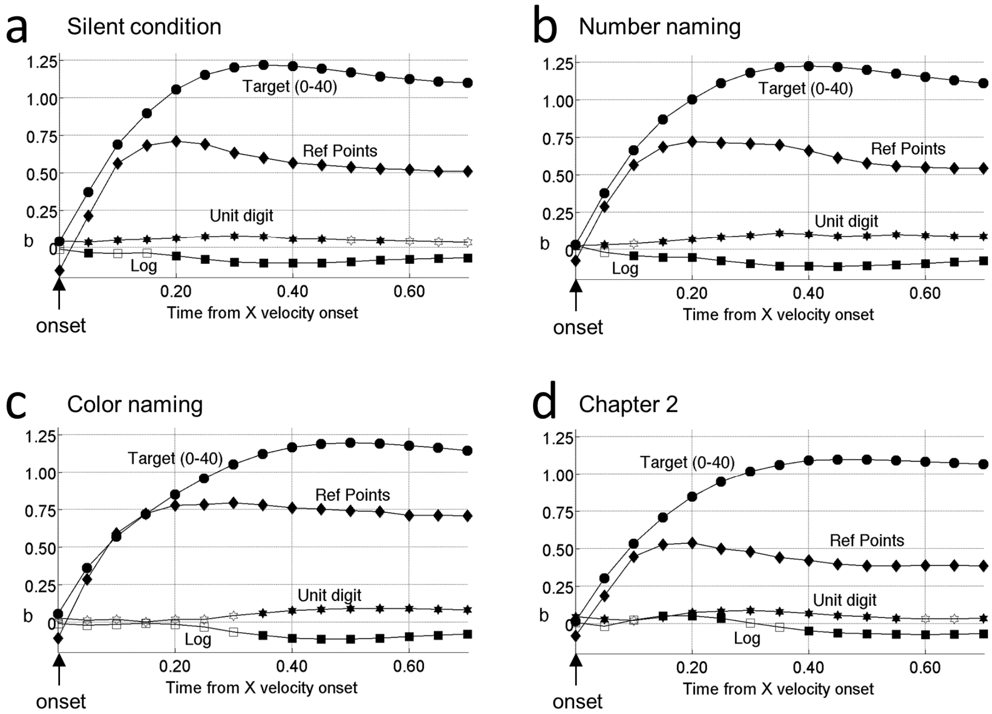

Fig. 3.5. Time course of the effects in Experiment 3.1 after alignment on horizontal movement onset time. The figure shows the regression b values (dependent variable: iEP) per condition and time point in Experiment 3.1 and, for comparison purposes, in Chapter 2, averaged over participants. The x axis indicates the time after the initiation of horizontal movement.

3.2.3. Discussion of Experiment 3.1

The silent condition in Experiment 3.1 replicated the results of Chapter 2: the analysis of trajectories showed a strong linear effect and a transient logarithmic effect. The color naming condition confirmed the prediction that dual-tasking makes the number-to-position mapping more logarithmic: the regression analysis showed a decreased (or delayed) linear factor and an enhanced log factor. This is similar to the results previously found when the quantities were presented non-symbolically (Anobile et al., 2012).

The log-linear dissociation was initially taken (in Chapter 2) as direct evidence for separate log and linear quantity representations, with the linear representation being more sensitive to interference from the dual task – presumably due to competition of resources between the color naming task and linear quantity encoding mechanisms. However, the analysis of movement onsets suggests a simpler explanation: the decision to start moving the finger is earlier for smaller target numbers, thus the trajectories fan out more quickly for smaller number than for larger numbers, and this induces a transient log effect in the regressions. The dual task (color naming) further enhances this differential delay in movement onset as a function of target size, and consequently increases the log effect. Thus, the differential movement time model fully accounts not only for the dissociation between the silent and color naming conditions, but also for the log effect in each of the conditions. Indeed, when the horizontal movement onset time was controlled for (by aligning each trial to its movement onset time), the log effect was eliminated, and with it the difference between the conditions.

Our findings are consistent with the idea that, early in the trial, before the participants obtain evidence from the target, they move their finger in accordance to prior knowledge. In our task, participants are asked to initially point towards the midpoint of the line, which happens to be the optimal prior given the flat distribution of target numbers. Furthermore, their pointing is also influenced in part by the distribution of previous targets: when the previous targets are large, the finger is slightly displaced towards the right side, and vice-versa. This effect is essentially a replication of Cicchini et al.’s (2014) finding of a prior-trial effect, although in our case the effect (1) showed an exponentially decreasing influence of several recent targets, and (2) influenced only the initial part of the next trial’s finger trajectory, not the final endpoint.

The aligned-by-movement-onset analysis also showed that the unit and decade digits affected finger movement in an almost accurate 1:10 ratio throughout the trial, indicating that the decade and unit quantities were assigned very accurate relative weights. This finding is interesting because it suggests parallel rather than sequential processing of the two digits: if one of the digits was processed before the other, its effect on movement should have been larger than implied by the 1:10 ratio. The absence of such deviation from the 1:10 ratio suggests either that the decade and unit quantities were processed in parallel, or that the decision to initiate finger movement was delayed until a complete two-digit quantity was constructed.

Last, the analysis of movement onsets revealed a strong distance effect that was not predicted by any of the models: the movement onset was much earlier for targets close to the ends of the number line and delayed for targets near the middle (Fig. 3.4). The origins of this effect are discussed in Experiment 3.5.

Methodologically, these results indicate that data from the number-to-position paradigm should be analyzed with caution. Regression analyses of stimulus-aligned finger trajectories, as performed in Chapter 2, show log and linear patterns at different times, yet this does not necessarily reflect directly the underlying internal representations. Rather, movement-aligned analysis suggests that this pattern may reflect the differential durations of a pre-movement stage of intention buildup.

An alternative interpretation of the small-number advantage is in terms of a motor rather than a numeric effect. According to this interpretation, the faster deviation to small numbers would not result from their magnitude but from their location on the left side on the number line. Purely motoric reasons, including for instance the types of muscle activity required to push the finger left or right, may make leftward movements faster than rightward movements. We refuted this hypothesis, however, with two control experiments, which are reported fully in Appendix A. In one experiment, the silent condition of Experiment 3.1 was replicated with a group of left-handed participants. The task required these participants for the cognitive operation as in Experiment 3.1, but for a reversed muscle operation. The motor hypothesis therefore predicts that the left-handed participants would deviate more quickly towards the right side (large numbers), i.e., a large-number advantage. However, the findings were exactly the opposite: the left-handed participants showed a small-number advantage just like the right-handed group. In a second control experiment, a group of right-handed participants pointed to the same 41 locations as in the number-to-position task, but the target location was now indicated explicitly and non-numerically by an arrow placed at the target location. Thus, the set of required responses in this task was as in Experiment 3.1, but the decision process did not involve numbers. The findings showed that the participants in fact deviated slightly faster to the right than to the left, i.e. the opposite of the bias we observed in the numerical experiments. Taken together, these control experiments clearly refute the motor hypothesis and support our interpretation of the small-number advantage as a numerical effect.

3.3. Experiment 3.2: Manipulating the Color-Number SOA

Experiment 3.2 was designed to replicate the dual-task interference effect observed in Experiment 3.1 within the better-controlled setting of a psychological refractory period design (Pashler, 1984, 1994). In Experiment 3.1, the three conditions were very different from each other: one condition was a single task, and the two other conditions were dual-tasks involving naming of words from different categories (numbers and colors), which could trigger different cognitive processes (Bachoud-Lévi & Dupoux, 2003; Bormann, Seyboth, Umarovaa, & Weillera, in press; Cohen, Verstichel, & Dehaene, 1997; Dotan & Friedmann, 2015; Marangolo, Nasti, & Zorzi, 2004; Marangolo, Piras, & Fias, 2005). Experiment 3.2 therefore used the classic PRP manipulation of SOA between two fixed tasks. Only the color naming task was used, but the SOA between the onset of the color and the target number was manipulated. We assumed that decreasing the SOA would increase the temporal overlap between the central decision stages of the two tasks, thus imposing a decision bottleneck (Sigman & Dehaene, 2005). Thus, the effect of shortening the SOA would be similar to adding the dual-task in the first place. Consequently, we predicted an increased log effect for shorter SOAs, which according to the differential encoding time model should be entirely reducible to a differential delay of movement onset for different numerical targets.

3.3.1. Method

Twenty right-handed adults (age 26;2 ± 4;0) with no reported cognitive deficits or color blindness were compensated for participation. Their mother tongue was Hebrew.

One experimental block was a replication of the color naming condition in Experiment 3.1. In 3 other blocks the color stripes still appeared when the finger started moving, but the onset of the target number was delayed by 100 ms, 200 ms, or 300 ms. Each participant performed all blocks and was randomly assigned to one of four block presentation orders (0-100-200-300, 100-0-300-200, 300-200-100-0, or 200-300-0-100). Each number between 0 and 40 was presented twice per block (82 trials). The participants also performed silent number-to-position mapping (identical with the silent condition in Experiment 3.1) as a fifth block, which was administered last and presented each number 4 times (for 5 participants) or 6 times (for the other participants).

The horizontal movement onset time was calculated per trial using the method described above (Section 3.2.2.4.1), excluding trials with target numbers 15-25. The automatic algorithm succeeded finding the onset of 90.8% of the trials (88.9%, 89.9%, 91.6%, and 92.6% per SOA condition) and 84.5% of the trials in the silent control condition. For the remaining trials, horizontal movement onset was encoded manually, after which 98.7% of the trials (and 98.6% of the control trials) had movement onset information. The other trials were excluded from the onset-related analyses. The onset analyses described below were also run while excluding the trials with manual onset encoding, and the results were essentially the same.

3.3.2. Results

3.3.2.1. Comparison of the conditions using trial-level measures

Table 3.3 shows the basic performance measures in this experiment. Each of these measures was compared across the four SOA conditions using repeated measures ANOVA with the per-subject mean as the dependent variable. There were no significant differences between the SOA's in endpoint bias ($F(3,57) = 1.67$, $p = .18$) and endpoint error ($F(3,57) = 1.66$, $p = .19$), but there were differences in movement time ($F(3,57) = 3.65$, $p = .02$, $\eta_p^2 = .16$, $\eta^2 = .03$) and failed trial rate ($F(3,57) = 3.98$, $p = .01$, $\eta_p^2 = .17$, $\eta^2 = .06$). The results were essentially the same when the ANOVA was run with the SOA as a numeric factor.

Table 3.3. General performance measures in Experiment 3.2.

Measure	Silent	0 ms	100 ms	200 ms	300 ms
Failed trials (%)	2.6 ± 2.0	19.6 ± 10.2	21.4 ± 10.7	16.5 ± 10.8	14.4 ± 12.3
Movement time (ms) [a]	1180 ± 162	1414 ± 152	1410 ± 143	1428 ± 129	1476 ± 155
Endpoint bias (0-40 scale)	-.66 ± .46	-.67 ± .72	-.57 ± .56	-.61 ± .67	-.78 ± .92
Endpoint error (0-40 scale)	1.7 ± .43	2.49 ± 1.1	2.41 ± .82	2.27 ± .98	2.43 ± 1.16
Speech onset time (ms) [a]	–	878 ± 106	871 ± 140	811 ± 138	802 ± 153
Horizontal movement onset time (ms) [b]	496 ± 42	597 ± 86	525 ± 83	467 ± 75	420 ± 75

Note. Standard deviations refer to between-subject variance of the per-subject means.

[a] The movement time and the speech onset time are indicated with respect to the color onset time.

[b] The horizontal movement onset time is indicated with respect to the number onset time.

We continued with a classical PRP analysis, which consists in examining how the reaction times in the two tasks were affected by the SOA manipulation. The two RT measures are the speech onset time for the naming task and the movement onset time for the pointing task.

The speech onset times of color naming were significantly different between the SOA conditions (one-way repeated measures ANOVA, $F(3,57) = 10.9$, $p < .001$, $\eta_p^2 = .36$, $\eta^2 = .06$). They were longer in SOA=100 than in SOA=200 (paired $t(19) = 4.0$, one-tailed $p < .001$, Cohen's $d = 0.10$), but were similar between SOA’s 0-100 and 200-300 (paired $t(19) < 0.54$, one-tailed $p > .6$).

The horizontal movement onset time too was significantly different between the SOA conditions (one-way repeated measures ANOVA, $F(3,57) = 91.93$, $p < .001$, $\eta_p^2 = .83$, $\eta^2 = .42$; for all pairs of adjacent SOAs, paired $t(19) \geq 3.0$, $p < .001$, Cohen's d > 1.1). Table 3.3 shows that each increase of the SOA by 100 ms decreased the horizontal movement onset time by ~50-70 ms. Note, however, that this added delay is significantly smaller than the 100-ms spacing of the SOA conditions (when comparing onset times after adding 100 ms to the earlier SOA, paired $t(19) > 2.56$, $p < .02$, Cohen's d > 0.57 for all adjacent SOAs). In many PRP experiments, a 1:1 relation between SOA shortening and secondary-task delay is obtained (Pashler, 1984, 1994; Sigman & Dehaene, 2005). The fact that it was not obtained here suggests that interference was not complete and that there was partial resource sharing (Tombu & Jolicoeur, 2002) or inter-trial variability in the prioritizing of the two tasks, as also confirmed by the above finding that color naming too was significantly delayed by shortening the SOAs.

3.3.2.2. Regression analysis of the trajectories

The trajectory data was submitted to regression analysis with iEP as the dependent variable and with the predictors introduced in Experiment 3.1: N_{0-40}, log'(N_{0-40}), the unit digit U, the spatial-reference-points-based bias function SRP, and the target number of the previous trial N-1. One regression was run per SOA (and for the silent condition), participant, and time point, in 50 ms intervals. The per-subject regression b values of each SOA, time point, and predictor were compared versus zero using t-test. The pattern of factors we observed in Experiment 3.1 was replicated for all four SOA's (Fig. 3.6a-d): dominant linear factor, transient logarithmic factor, SRP contribution in the late trajectory parts, and an effect of the previous trial in early trajectory parts.

We then examined the effect of SOA on the linear factor (Fig. 3.6e). The per-subject b[N_{0-40}] values were first compared using a repeated measures ANOVA with a factor of SOA (one ANOVA per time point, starting from 150 ms). A significant difference between SOA's was found from 550 ms to 900 ms ($F(3,57) > 4.48$, $p < .01$, $.19 < \eta_p^2 < .34$, $.03 < \eta^2 < .11$). A comparison of b[N_{0-40}] between each pair of adjacent SOA's using paired t-test showed that for SOA's from 0 to 200 ms, the difference was in the predicted direction, i.e., decreasing the SOA resulted in a reduced linear factor: we found a significant difference b[N_{0-40}]$_{/SOA=100}$ > b[N_{0-40}]$_{/SOA=0}$ from 700 ms to 900 ms ($t(19) > 1.89$, one-tailed $p < .04$, 0.42 < Cohen's d < 0.56), and b[N_{0-40}]$_{/SOA=200}$ > b[N_{0-40}]$_{/SOA=100}$ from 550 ms to 750 ms ($t(19) > 1.82$, one-tailed $p < .05$, 0.41 < Cohen's d < 0.74). There was no significant difference between SOA's 200 ms and 300 ms at any time point ($t(19) < 0.84$, one-tailed $p \geq .21$); in fact, as Fig. 3.6e clearly shows, the linear factor was almost identical for these two SOA values. Thus, as predicted, decreasing the SOA (and thereby extending the time overlap between the two tasks) caused an increasing interference with the linear factor of the number-to-position task, which can be interpreted as a delayed onset of this factor. The shape of this effect, with an absence of a difference between the longer SOAs (200 and 300 ms), is classical for the PRP effect (Pashler, 1984, 1994; Sigman & Dehaene, 2005). It suggests that the central competition between the two tasks lasted no more than 200 ms, and therefore reached a floor level for SOA of 200 ms and beyond.

The effect of SOA on the log factor was examined in a similar manner (Fig. 3.6f). No significant SOA effect on b[log'(N_{0-40})] was found in any time point: a per-time-point repeated measures ANOVA, starting from 150 ms, with SOA as a within-subject factor and the subject as the random factor, showed no significant difference ($F(3,57) < 2.15$, $p > .10$). Thus, whereas in Experiment 3.1 we observed significant effects on both the log and linear factors but in opposite directions, in Experiment 3.2 shortening the SOA reduced the linear factor while keeping the log factor almost unchanged.

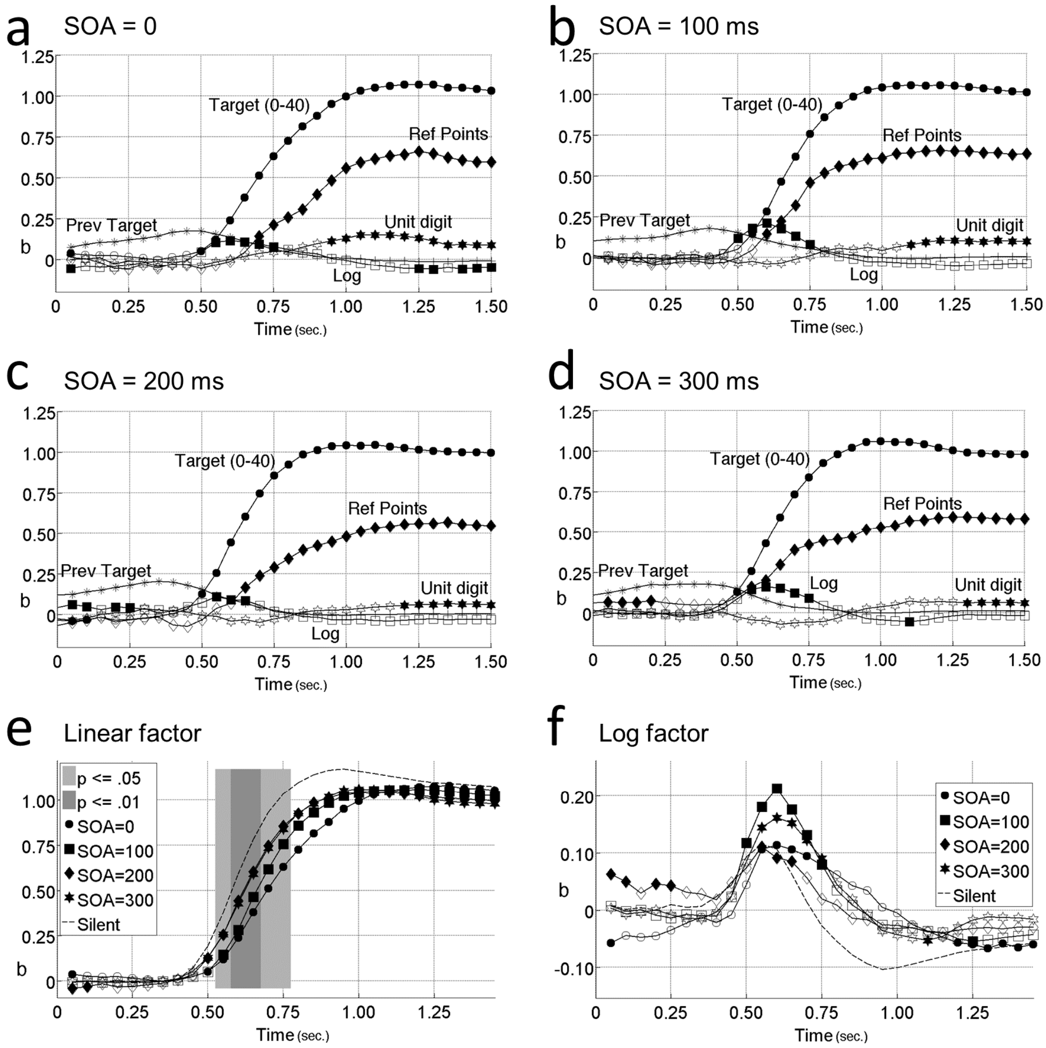

Fig. 3.6. Time course of the effects in Experiment 3.2. Note that the different experimental conditions are horizontally aligned to the target number onset, not the color onset. (a-d) Regression factors per SOA. (e) The linear factor $b[N_{0\text{-}40}]$ per SOA. Grey areas show a time window of 200 ms during which $b[N_{0\text{-}40}]_{/SOA=100} < b[N_{0\text{-}40}]_{/SOA=200}$. A similar difference $b[N_{0\text{-}40}]_{/SOA=0} < b[N_{0\text{-}40}]_{/SOA=100}$ was found in a slightly later time point, 700 ms to 900 ms. (f) The log factor $b[\log'(N_{0\text{-}40})]$ showed no significant differences among SOA's.

The interaction between the log and linear factor was evaluated using two-way repeated measures ANOVA with the regression b values as the dependent variable, between-subject factors of regression predictor (log, linear) and SOA, and the subject as the random factor. One ANOVA was run per time point, starting from 150 ms. A significant interaction was found from 600 ms to 850 ms ($F(3,57) > 3.16$, $p \leq .03$, except $p = .06$ in time point 650 ms; $.12 < \eta_p^2 < .16$, $.02 < \eta^2 < .06$), confirming that the SOA manipulation affected the linear and log factor differently.

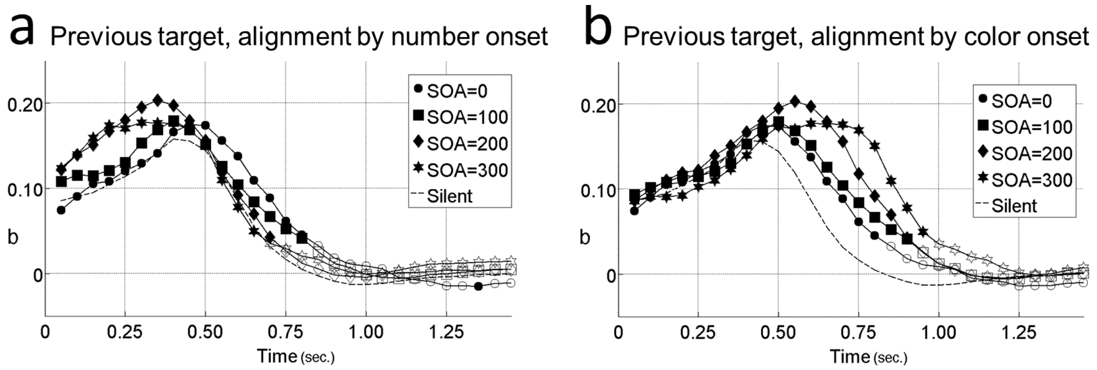

Fig. 3.7. The prior-target factor b[N-1] in Experiment 3.2, with the SOA conditions aligned by the number onset (a) or by the color onset (b). The prior effect is initially independent of the number onset time, but its decay is linked to the number onset.

We assumed that the effect of prior from the previous trial would initially be independent of the new number presented, and consequently independent of SOA. Indeed, when aligning the SOA conditions to the beginning of the trial, i.e., to the color onset rather than to the number onset (Fig. 3.7b), no significant differences in b[N-1] were found between SOA's until 550 ms (repeated measures ANOVA per time point, with b[N-1] as the dependent variable, SOA as a within-subject factor, and the subject as the random factor: $F(3,57) < 1.63$, $p > .19$). In later time points, from 600 ms to 900 ms (the downhill part of the b[N-1] curve), a significant difference was found between the SOA conditions (from 600 ms to 900 ms, $F(3,57) > 3.15$, $p \leq .03$, $.14 < \eta_p^2 < .42$, $.06 < \eta^2 < .18$; between 650 ms and 850 ms, $F(3,57) > 5.51$, $p \leq .002$). This late between-SOA difference almost disappeared when the conditions were aligned to the number onset rather than to the color onset (Fig. 3.7a; from 400 ms to 1000 ms, $F(3,57) > 2.44$, $p > .07$, except two time points, 650-700 ms, in which $p = .05$). Thus, the initial effect of b[N-1] was triggered by the color onset, whereas its decay was linked to the number onset. These findings suggest that finger movement is initially affected by the prior from previous trial/s, and this effect decays as the prior is overridden by the new number presented.

3.3.2.3. Differential encoding times as the reason for the log effect

The differential encoding time model assumes that the log effect occurs because the horizontal movement onset time is different for different target numbers. Once these onset differences are eliminated by aligning trajectories to their movement onset, the regression analysis should show no logarithmic effect. To examine this prediction, the trajectory data was submitted to regression analysis after aligning each trajectory to the trial's horizontal movement onset time. The dependent variable was iEP and the predictors were $N_{0\text{-}40}$, log'($N_{0\text{-}40}$), the unit digit U, and SRP. One regression was run per SOA, participant, and time point in 50 ms intervals. Per predictor, SOA, and time point, the participants' b values were compared with zero using t-test (Fig. 3.8). The linear factor b[$N_{0\text{-}40}$] in these regressions showed a virtually identical pattern for all SOA's (Fig. 3.8a). A per-time point repeated measures ANOVA, with SOA as a single within-subject factor and the subject as a random factor, showed no difference in b[$N_{0\text{-}40}$] between SOA's at any time point from 50 ms ($F(3,57) < 2.21$, $p > .09$), and only a minor difference at t = 0 ($F(3,57) = 3.7$, $p = .02$, $\eta_p^2 = .16$, $\eta^2 = .12$; the b values per SOA at t=0 were 0.04, 0, -0.02 and -0.02). The log factor too showed no significant difference between SOA conditions (a per-time point repeated measures ANOVA, with SOA as a single within-subject factor and the subject as a random factor, $F(3,57) < 1.31$, $p > .28$). In fact, the log factor showed no significant positive contribution in any of the conditions (Fig. 3.8b). Thus, as in Experiment 3.1, the differences in horizontal movement onset times fully accounted for the log effect as well as for the differences between the four SOA conditions, including the log-linear dissociation.

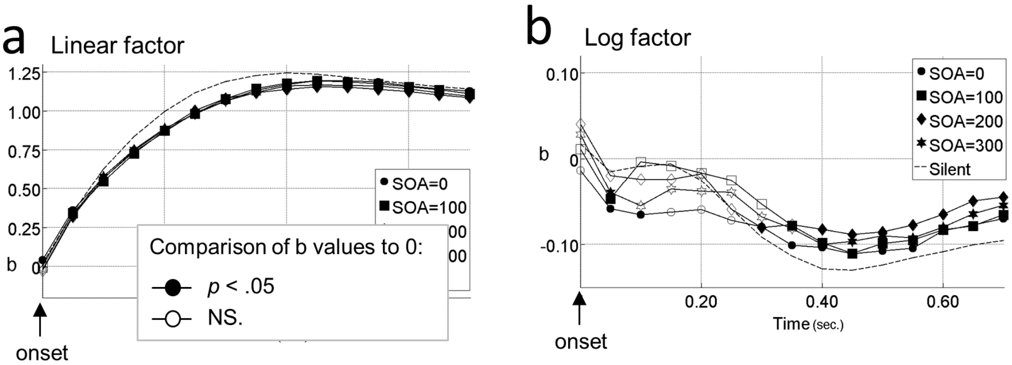

Fig. 3.8. Time course of the effects in Experiment 3.2 after alignment on horizontal movement onset time. Here, b[$N_{0\text{-}40}$] and b[log'($N_{0\text{-}40}$)] no longer show any difference between the conditions.

3.3.2.4. Factors affecting horizontal movement onset

We next examined how the target number and SOA affect the horizontal movement onset times (Fig. 3.4b). Similarly to Experiment 3.1, the onset times, specified as the time since the target number appeared on screen, were analyzed using repeated measures ANOVA with the subject as the random factor and with 3 within-subject factors: the target side (< 20, left; or > 20, right) and two numeric factors – the SOA and the absolute distance between the target number and 20. To minimize noise, as well as to resolve the problem of missing data in 13 participant-SOA-target combinations, the distance factor grouped each set of 3 adjacent target numbers, resulting in 5 levels of this factor: 6-8, 9-11, 12-14, 15-17, and 18-20.

A main effect of SOA ($F(1,19) = 80.53$, $p < .001$, $\eta_p^2 = .81$, $\eta_G^2 = .24$) mirrored the SOA effect that was earlier observed in the trial-level PRP analysis: decreasing the SOA created some delay in the movement onset, indicating that the dual task interference was not complete and that there was partial resource sharing with the naming task.

A main effect of side ($F(1,19) = 17.15$, $p < .001$, $\eta_p^2 = .50$, $\eta_G^2 = .14$) reaffirmed the small-number advantage: as predicted by the differential encoding time model, onset times were earlier for small target numbers (< 15) than for large target numbers (> 25). We then examined whether the small-number advantage interacted with SOA. The small-number advantage was calculated per SOA as the delta between mean movement onsets on the left and right sides. The differential encoding time model predicts an increasing small-number advantage for smaller SOAs (i.e., for larger overlap between the two tasks). Indeed, averaged over participants, the small-number advantage was 79 ms, 85 ms, 65 ms, and 54 ms for SOA = 0, 100, 200, 300 respectively, and the three-way ANOVA showed that this difference between SOA conditions was significant (Side x SOA interaction: $t(19) = 1.74$, one-tailed $p = .05$, $\eta_p^2 = .16$, $\eta_G^2 = .004$).

A significant main effect of distance-from-20 ($F(1,19) = 22.86$, $p < .001$, $\eta_p^2 = .58$, $\eta_G^2 = .06$) showed that movement onset was delayed for target numbers closer to the middle of the number line. To examine whether this distance effect was sensitive to the SOA manipulation, we calculated the distance effect per participant and SOA as the slope of the onset-per-target function. This was done using regression analysis with the movement onset time as the dependent variables and with two predictors: the target number side (-1 or 1) and its absolute distance from 20. The resulting b[distance] from this regression reflects the distance effect; its values for SOA's 0, 100, 200, and 300 were -10.1 ms, -6.6 ms, -5.4 ms, and -4.1 ms, respectively (average over participants), namely, decreasing the SOA continuously increased the distance effect. The three-way ANOVA showed that this effect of SOA on the distance effect was significant (Distance x SOA interaction: $F(1,19) = 16.48$, $p < .001$, $\eta_p^2 = .48$, $\eta_G^2 = .01$).

There was no significant Distance x Side interaction ($F(1,19) = 0.51$, $p = .48$) and no three-way interaction (SOA x Side x Distance, $F(1,19) = 0.13$, $p = .73$).

3.3.3. Discussion of Experiment 3.2

Experiment 3.2 used the color naming dual task and manipulated the color-number SOA. The analysis of trajectories replicated the dissociation between the log and linear factors that was observed in Experiment 3.1: decreasing the SOA decreased (or delayed) the linear factor in the participants' mapping to positions (implied endpoints), while leaving the log factor almost unchanged. In this respect, the effect of shortening the color-number SOA, a manipulation that presumably makes the experiment harder, was similar to the effect of adding the distracter task in the first place.

The dual representation model can explain these findings as a selective interference of the color naming task with the exact-linear quantity representation, but not with the approximate representation. However, again the differential encoding time model offers a simpler account of the findings. Inter-trial differences in the horizontal movement onset times can fully account for the log effect: when the onset times were controlled for (by aligning each trajectory to the trial's movement onset time), the log effect in the regression analyses completely disappeared, and so did the inter-SOA differences in the linear factor.

Experiment 3.2 also reaffirmed the main assumptions of the differential encoding time model, namely, that horizontal movement onset was earlier for smaller numbers, and that this small-number advantage was increased when increasing the level of interference from color naming (by shortening the color-number SOA).

The use of a PRP design allowed exploring the nature of the interference between the color naming and number-to-position tasks. Several observations in the pattern of delays were compatible with a partial PRP effect. First, *both* tasks were delayed by the interference; in particular, the RT of the color naming task was not constant (as should have been the case if this task was systematically prioritized over the number task), but became slower at shorter SOAs. Second, while the onset of responses to the number task was also delayed at short SOA, the amount of this delay was not compatible with a full PRP effect. The number task was not delayed by a full 100 ms whenever the SOA decreased by this amount, but rather, by about 50%-70% of that value. Third, the size of the target number influenced the horizontal movement onset time of task 2, but crucially this effect was not additive with SOA (as predicted by a rigid delay of task 2 due to a full PRP effect; Pashler et al., 1984, 1994) but was enhanced at short SOAs. All these findings indicate that color naming was not fully prioritized over finger pointing, which is perhaps not surprising given that participants were required to start moving the finger in order to make the target appear, and were therefore already "launched" in the number-to-position task.

The above observations are compatible with either a partial resource sharing model (Tombu & Jolicoeur, 2002), according to which both decisions are computed in parallel and are jointly slowed by dual-task interference, or by a rigid delay model (Pashler, 1984, 1994; Sigman & Dehaene, 2005) with random prioritization of one task or the other (Sigman & Dehaene, 2006). The latter interpretation predicts that our trials are a mixture of two trials types, depending on whether the central decision does color first and number second, or vice-versa. However, given the variability in task performance, this bimodal distribution model cannot be distinguished from the single distribution predicted by partial resource sharing.

3.4. Experiment 3.3: 0-100 Number Line

Experiments 3.1 and 3.2 supported the differential encoding time model: small numbers are encoded faster than large numbers, thereby inducing the transient log effect in the implied endpoints. The model stipulates that the reason for the small-number advantage is that quantity encoding is noisier for large quantities than for small quantities (differential variability), and the greater noise causes slower processing. However, an alternative account is that single-digit numbers are processed faster than two-digit numbers – i.e., what we observed in Experiments 3.1 and 3.2 was not a small-number advantage but a single-digit advantage.

In the setting of Experiments 3.1 and 3.2, the two models are hard to tease apart, because over the range of target numbers that were analyzed for movement onset (0-14 and 26-40) most of the small numbers were single digits. To dissociate between the small-number advantage model and the single-digit advantage model, Experiment 3.3 used a longer number line (0-100) which allows excluding from the analysis the single-digit numbers and consequently the possibility for a confounding factor.

3.4.1. Method

Seventeen right-handed adults (aged 26;10 ± 5;2) with no reported cognitive deficits were compensated for participation. Their mother tongue was Hebrew. The experiment was performed like the silent condition in Experiment 3.1, except that the number line extended from 0 to 100 (rather than from 0 to 40). Each number between 0 and 100 was presented 4 times, i.e., 404 non-failed trials per participant. The horizontal movement onset time was encoded as described in Section 3.2.2.4.1, while excluding trials with target numbers 39-61. Automatic onset encoding succeeded for 82.8% of the trials, and manual encoding increased this to 97.8%. The analyses of onset times (described below) were also performed without the manually encoded trials and the results were essentially the same.

3.4.2. Results

The rate of failed trials in this experiment was 2.9% ± 2.3%. The mean movement time was 1191 ± 204 ms, the endpoint bias was -0.25 ± 1.13 numerical units, the endpoint error was 4.69 ± 1.87 numerical units, and the horizontal movement onset time was 444 ± 113 ms (all standard deviations refer to the between-subject variance of the per-subject means). The median trajectories are presented in Fig. 3.9a.

The trajectory data was submitted to regression analysis with iEP as the dependent variable and with five predictors: $N_{0\text{-}100}$, log'($N_{0\text{-}100}$), the unit digit U, the spatial-reference-points-based bias function SRP, and the target number of the previous trial N-1. One regression was run per participant and time point in 50 ms intervals. The per-subject regression b values of each predictor and time point were compared with zero using t-test. The results (Fig. 3.9b) replicated the previous experiments: dominant linear factor, transient logarithmic factor, SRP contribution in the late trajectory parts, and an effect of the previous trial in early trajectory parts. When the regressions were re-run after aligning each trajectory to the trial's horizontal movement time, the log factor disappeared (Fig. 3.9c), as predicted by the differential encoding time model.

The small-number advantage was found in this experiment too, even when we analyzed only the two-digit numbers (Fig. 3.9d): the horizontal movement onset of targets in the range 10-38 was shorter than that of targets in the range 62-90 by 22 ± 54 ms (the standard deviation refers to the between-subject variance of the per-subject means). To examine this difference statistically, the onset times were submitted to repeated measures ANOVA with within-subject factors of side (smaller or larger than 50) and distance from middle (|target-50|) and the subject as the random factor. Distance was a numeric factor and it grouped each set of 3 adjacent targets so the factor had 9 levels (12-14 to 36-38). A significant main effect of Side ($t(16) = 1.84$, one-tailed $p = .04$, $\eta_p^2 = .17$, $\eta_G^2 = .04$) confirmed the small-number advantage within two-digit numbers, and thus refuted the "single-digit advantage" hypothesis. The Distance effect was also significant (F(1,16) = 68.1, p < .001, η_p^2 = .81, η_G^2 = .18), with later onset times close to the middle of the number line, and there was no Side x Distance interaction (F(1,16) = 1.84, p = .19). Similar results were obtained when single digits were included in the analysis: targets 0-39 had shorter movement onsets than 61-100 by 23 ± 55 ms. The Side x Distance ANOVA showed significant main effects of Side (t(16) = 1.87, one-tailed p = .04, η_p^2 = .18, η_G^2 = .05) and Distance (F(1,16) = 69.2, p < .001, η_p^2 = .81, η_G^2 = .21), with no interaction (F(1,16) = 0.91, p = .35).

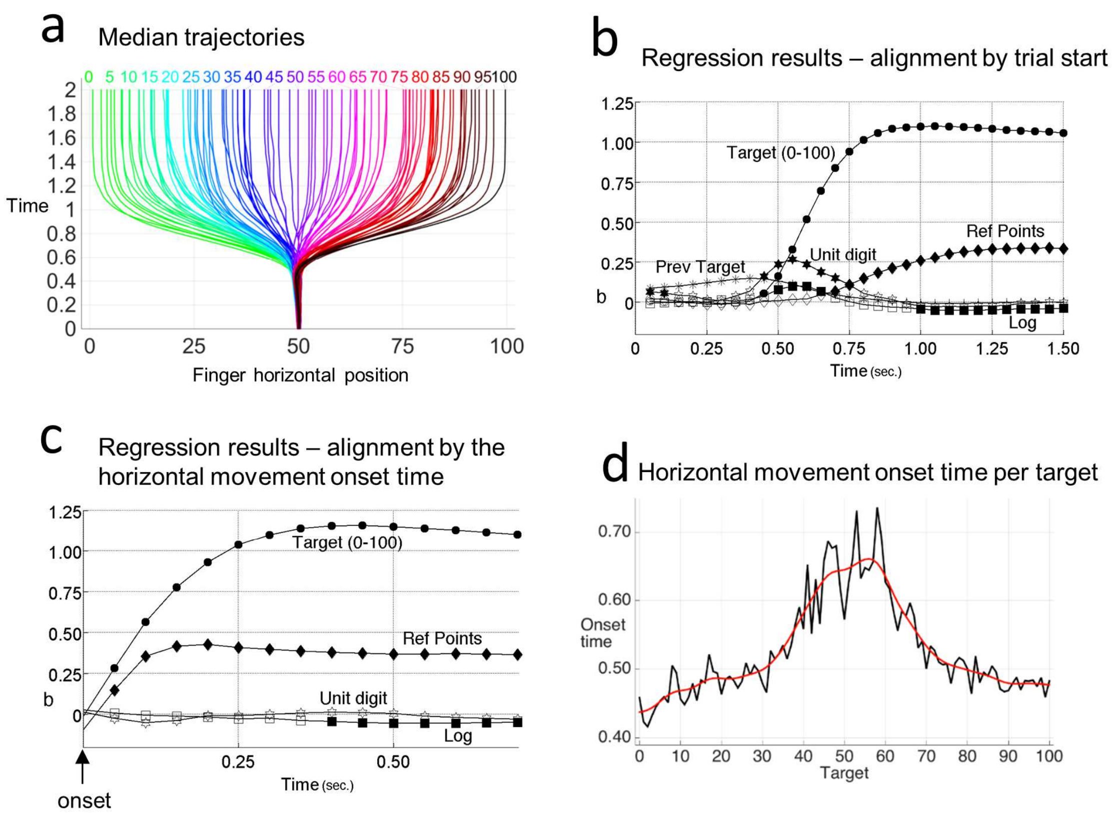

Fig. 3.9. Results of Experiment 3.3 (0-100 number line). (a) Median trajectories, created by re-sampling each trajectory into equally-spaced time points, finding the per subject+target median coordinates in each time point, and averaging these medians per target number. (b-c) Regression b values (dependent variable: iEP), averaged over participants. In (b), the trials were aligned to the trial start time and a significant transient log effect appeared. In (c), the trials were aligned to the horizontal movement onset time. This eliminated the log effect, as predicted by the differential onset time model. (d) Mean horizontal movement onset time per target. The black line is the average over trials and participants. The red line is the same data after Gaussian smoothing with σ = 3. Crucially, a significant small-number advantage was found not only over all targets but also within two-digit numbers, contrary to the notion that it originated only in processing speed differences between single-digit and two-digit numbers.

To characterize the distance effect, the horizontal movement onset times were regressed with three predictors: the target side (left = -1, right = 1), its distance from the middle of the line, and log(distance), linearly transformed to 0-50. The side effect was significant (b = -27.78 ms, t(6161) = 6.59, one-tailed $p < .001$). The log(distance) effect was significant (b = -5.28 ms, t(6161) = 9.39, one-tailed $p < .001$) and much stronger than the linear distance effect (b = -0.68 ms, t(6161) = 1.75, one-tailed $p = .04$), in accord with number comparison studies (Cantlon & Brannon, 2006; Dehaene, 1989; Dehaene et al., 1990).

3.4.3. Discussion of Experiment 3.3

Experiment 3.3 showed a small-number advantage, earlier onset of horizontal movement for smaller targets than for large targets, even within two-digit numbers. Thus, the small-number advantage cannot be discarded as faster processing of single digits; it is a genuine phenomenon in processing of two-digit numbers.

Experiment 3.3 also replicated the other major findings of our previous experiments: the regressions showed a strong linear factor, a transient log factor (which was eliminated when aligning trajectories by the movement onset time), and a spatial-reference-points effect in the late trajectory parts. The replication of these findings using a 0-100 number line confirms that they do not reflect strategies specific for the 0-40 range (e.g., trying to memorize the positions of decade boundaries – a strategy overtly used by several participants in the 0-40 experiments, but not in the 0-100 experiment).

Interestingly, whereas our previous experiments showed that the decade digit was processed parallel to the unit digit (Experiments 3.1 and 3.2) or slightly after it (Chapter 2), in Experiment 3.3 the regressions showed a strong effect of the unit digit. Although absent from the aligned-by-onset regressions, this effect suggests decomposed processing of the unit quantity. The exact nature of this decomposed processing cannot be unambiguously determined by the present experiment, and will be further discussed in Chapter 4. For a discussion of possible interpretations of the unit digit effect, see Section 2.4.4.

3.5. Non-Transient Logarithmic Effects

The differential encoding time model attributes the logarithmic mapping to delayed horizontal movement onset in trials with large target numbers. Presumably, the effect of this delay will not last forever: eventually, even the large-target trajectories catch up with the small-target trajectories, and the differences in horizontal movement onset become irrelevant as other factors start governing the finger movement. Thus, the differential encoding time model can account only for a transient logarithmic effect, which disappears in late trajectory parts. Indeed, this was the pattern observed in Experiments 3.1, 3.2, and 3.3. Several other studies, however, reported non-transient logarithmic effects, which were observed even in the endpoints – in children (Berteletti et al., 2010; Booth & Siegler, 2006; Opfer & Siegler, 2007; Siegler & Booth, 2004) and in a brain-injured adult (described in Chapter 9).

We hypothesized that the differential encoding time model will not be able to explain such non-transient logarithmic effects. To test this prediction, Experiment 3.4 examined the number-to-position mapping of 4th grade children. We also re-analyzed the number-to-position mapping data of ZN (described in detail in Chapter 9), a brain-injured adult who showed a logarithmic effect in the trajectory endpoints. We examined whether the log effect in these cases would be observed even when the trajectories are aligned by the movement onset time.

3.5.1. Experiment 3.4: Fourth Grade Children

3.5.1.1. Method

Forty-three Hebrew-speaking 4th grade children (aged 9;9 ± 0;4), recruited from a single elementary school in Tel Aviv, participated voluntarily in this experiment, with written informed consent of their parents. They performed the silent number-to-position mapping task described in Experiment 3.1. Each number between 0 and 40 was presented 4 times.

Visual inspection of the results suggested that the children's trajectory data was noisier than the adults'. We therefore calculated several per-participant quality measures and excluded participants with especially noisy data. Two measures were based on the finger's initial direction θ_0. This direction is presumably independent of the target number, and may reflect a bias, noise, or over-relying on prior trials, all of which could potentially disrupt the trajectory analysis. The value of θ_0 was calculated per trial using regression analysis with x coordinate as the dependent variable and y coordinate as the predictor, over all time points (in 10 ms intervals) from 0 to 160 ms, or from 0 to 100 ms if the first regression was non-significant. We excluded one participant whose $\sigma(\theta_0)$ was an outlier (higher than the participants' 75th percentile by at least 150% the inter-quartile range), and 4 participants whose mean θ_0 was an outlier to the left or to the right (mean(θ_0) higher than the participants' 75th percentile or lower than their 25th percentile by at least 150% the inter-quartile range). We also excluded 3 participants who had low correlation (r < .6) between the endpoints and the target number. For the remaining 35 children (aged

9;8 ± 0;4), the horizontal movement onset time was encoded per trial as described above (Section 3.2.2.4.1), excluding target numbers 15-25. The encoding succeeded for 63.9% of the trials automatically and for 87.3% of the trials after manual encoding.

3.5.1.2. Results.

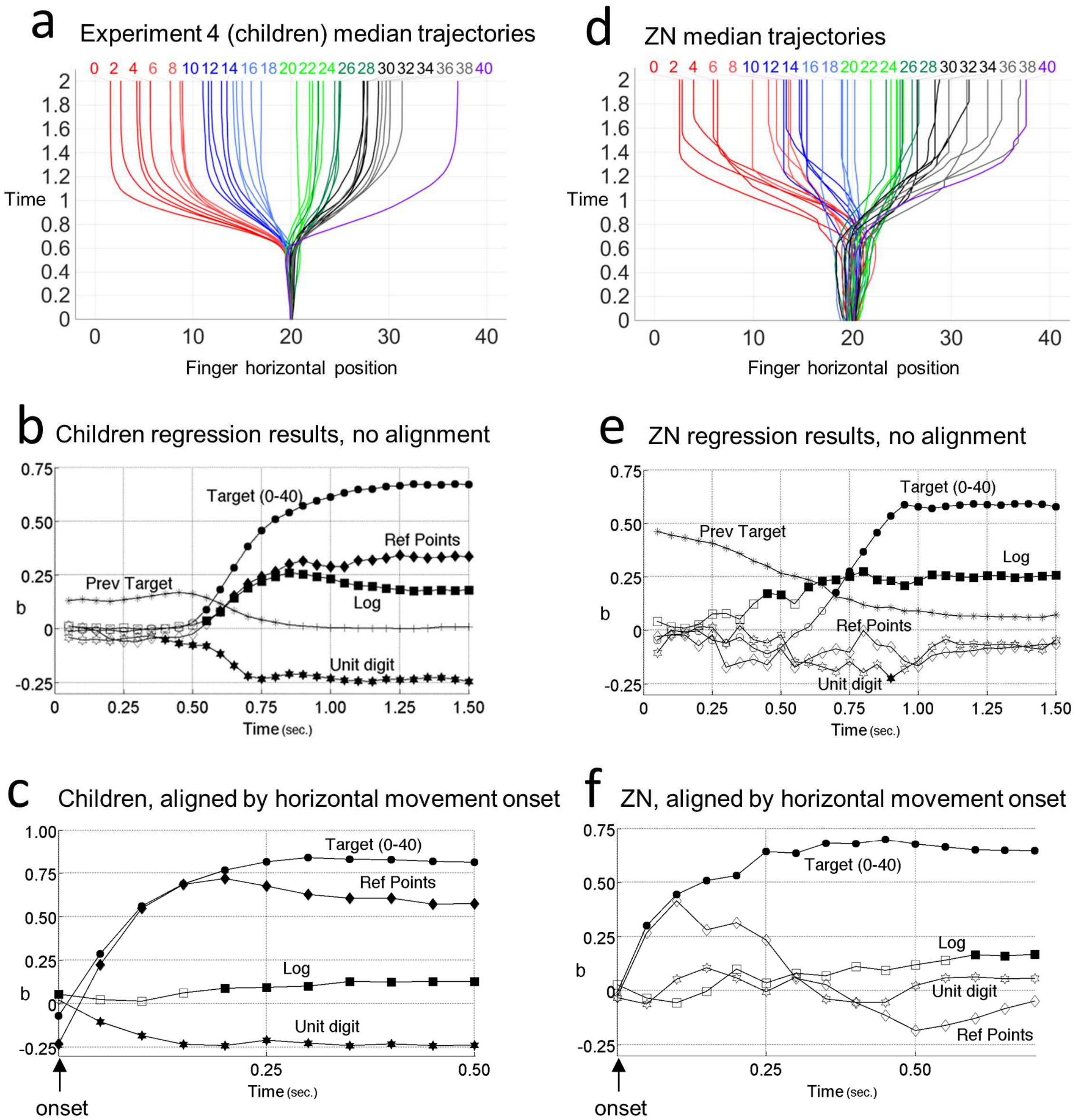

Fig. 3.10. Median trajectories and the regression b values in Experiments 3.4 (4th grade children) and the data of the brain-injured aphasic patient ZN. (a,d) The median trajectories. (b,e) Regression b values, with the trajectories aligned by the target onset. (c,f) The b values of the regression after aligning each trial to its horizontal movement onset time. A significant log effect was found both in Experiment 3.4 and in ZN's data. This log effect cannot result from different movement onset times per trial, because the alignment by onset controlled for this factor.

The median trajectories are presented in Fig. 3.10a. The trajectory data was submitted to regression analysis with the iEP as the dependent variable and with 5 predictors: $N_{0\text{-}40}$, log'($N_{0\text{-}40}$), the unit digit U, SRP, and the previous target N-1. One regression was run per time point, in 50 ms intervals. These regressions (Fig. 3.10b) showed a strong log effect that lasted until the end of the trial and was observed even in the endpoints (see the endpoints in Fig. 3.10a).

The trajectory data was then submitted to a similar regression in which each trajectory was aligned to the trial's horizontal movement onset time, and the N-1 predictor was removed (Fig. 3.10c). This alignment eliminated the log factor from the initial trajectory parts, but a significant log factor was still observed in the late trajectory parts (from 200 ms post movement onset time) and in the endpoints – a finding that is not predicted by the differential encoding time model.

3.5.2. Reanalysis of Patient ZN's Data

ZN was a 73 years old man who was recovering from a stroke. He was diagnosed with aphasia, severe apraxia of speech, impaired comprehension, dyslexia, dysgraphia, agrammatism, and a selective deficit in converting multi-digit numbers to their verbal representation (but not to quantity). In Chapter 9 we describe in detail his performance in several number processing tasks, including the iPad-based number-to-position task, which he performed like the silent condition in Experiment 3.1, with each number between 0 and 40 being presented 4 times. To re-analyze ZN's data, we encoded the horizontal movement onset time of each trial using the method described above (Section 3.2.2.4.1), excluding target numbers 15-25. This encoding succeeded for 63.3% of the trials automatically and for 95.8% of the trials after manual encoding.

ZN's trajectories are presented in Fig. 3.10d. They were submitted to regression analysis with iEP as the dependent variable and with 5 predictors: $N_{0\text{-}40}$, log'($N_{0\text{-}40}$), the unit digit U, SRP, and the previous target N-1. One regression was run per time point, in 50 ms intervals. This regression (Fig. 3.10e) showed a strong log effect that lasted to the end of the trial and was observed even in the endpoints.

The trajectory data was then submitted to a similar regression in which each trajectory was aligned to the trial's movement onset time, and the N-1 predictor was removed (Fig. 3.10f). In line with the differential encoding time model, the log factor was eliminated from the initial trajectory parts. However, contrary to the prediction of the differential encoding time model, a clear log effect was observed in the late trajectory part of the aligned-by-onset regressions (from 600 ms post movement onset time).

3.5.3. Discussion of Experiment 3.4 and Patient ZN's Data

The main finding from the data of the 4th grade children (Experiment 3.4) and of patient ZN was a non-transient log effect, which was observed in late trajectory parts and in the endpoints. This log effect was not eliminated even when we aligned each trajectory to the trial's horizontal movement onset time. Thus, the log effect cannot be explained by pre-movement differential processing durations, as suggested by the differential encoding time model. We also found no evidence that the log effect in Experiment 3.4 could be explained by quantity-dependent weighting of prior trials. In this respect, our results were different from Cicchini et al. (2014): although both studies found logarithmic effect in the endpoints, we did not replicate their finding of larger prior weight for large-target trials. This difference could be related to the fact that we used symbolic targets, while they used a non-symbolic display (sets of dots).

How should we explain, then, the log effect in the performance of ZN and of the 4th grade children? We think that two classes of explanations remain tenable. The first class of explanations reverts to the notion of dual quantity representation – linear-exact and approximate. The late log effect would result from amplified approximate representation and decreased exact-linear representation (the *early* log effect may result either from amplified approximate representation or from differential encoding times). The difference between the performance patterns of children and adults in the number-to-position task would then indicate a conceptual log-to-linear shift, as suggested in previous studies (Dehaene et al., 2008; Opfer & Siegler, 2007). Does this log-to-linear shift truly result from a *change* in the quantity representation, which begins as approximate and gradually becomes linear with maturation or education? Or perhaps the log-to-linear shift reflects the addition of a *separate* linear-exact representation on top of the approximate representation, and consistent inhibition of the approximate representation by the exact? The finding of logarithmic mapping in the performance of ZN support the latter possibility: ZN worked as an engineer for many years, and reported being extremely fluent with numbers, so it seems unlikely that his quantity representation remained approximate throughout the years. It also seems unlikely that his brain injury transformed the now-linear quantity representation back into approximate. It seems more likely that his logarithmic mapping reflects an approximate representation that was dormant in his cognitive system and re-emerged following a selective impairment to the linear-exact representation.

The second class of explanations for the late log effect is a variant of the differential encoding time model. It assumes that in children and in patient ZN, unlike in adults, the initial decision to move is based on insufficient evidence. Even if the participants understand the linear requirement of the task and intend to move to the linear position of the target, they may err if the decision process is fed with exceedingly noisy evidence. The participant may then stop short of making the proper inference and start moving based on a partial approximate numerical representation. Since this representation is more precise for small than for large numbers, the movement will be more accurate (more systematically away from the default response) for small than for large numbers, resulting in a log effect. In the discussion of this chapter, we verify this property in a precise mathematical model of the task. In adults, this log bias, if it exists at all, would be quickly compensated by new adjustments of finger position even after the onset of the first horizontal movement, resulting only in a transient log effect. If such a correction is impossible, however, then the log effect will remain sustained.

At present, we cannot decide between those two interpretations. However, the behavioral finding of logarithmic mapping in children is in accord with several previous developmental studies that used number-to-position mapping without tracking trajectories. These previous studies found logarithmic mapping only until second grade (Opfer & Siegler, 2007) or an earlier age (Berteletti et al., 2010; Booth & Siegler, 2006; Siegler & Booth, 2004), whereas here we found a log effect even in 4^{th} grade children, i.e., in a group that was at least two years older. It is possible that our paradigm, which requires a time-limited response and minimal finger velocity, was more demanding than the paradigms used in these previous studies, and therefore increased the logarithmic effect. Such interpretation seems plausible given that, in Experiments 3.1 and 3.2, we found that increasing task demands increases the log effect.

A peculiar finding in the children data, which was not observed in any of the adult experiments, is a strong negative effect of the unit predictor in the regressions (Fig. 3.10b,c). This could mean that the unit effect was either reduced or delayed relatively to the decade effect. However, interpreting this finding as delayed processing of the unit quantity seems unlikely, because $b[U] < 0$ continues throughout the trial (i.e., the unit digit never catches up with the decade digit). The $b[U] < 0$ can therefore be explained in two ways: either the decade and unit quantities were not encoded in 1:10 ratio but with under-representation of the unit digit; or the unit digit was completely ignored in some trials, resulting in lower $b[U]$ in the regression analysis. Importantly, both explanations suggest that even as late as 4^{th} grade, the processing of two-digit numbers is not fully automated. Previous studies pointed to the log-to-linear shift as one kind of cognitive progress that happens during maturation or education (Berteletti et al., 2010; Dehaene et al., 2008; Opfer & Siegler, 2007); the data from Experiment 3.4 suggests that the assigning proportional weights to the decade and unit quantity may be another cognitive ability that develops with age or education.

3.6. Experiment 3.5: Validating the Movement Onset Detection Algorithm

In all experiments so far, the horizontal movement onset was calculated based on the finger's horizontal velocity profile. To make sure that the onset-detection algorithm did not create some statistical artifact, we administered the number-to-position mapping experiment using a slightly modified paradigm: the participants started moving their finger only after the target number appeared on screen (hereby, stimulus-then-move paradigm). This is the method used in many trajectory-tracking experiments (e.g., Finkbeiner et al., 2008; Santens et al., 2011; Song & Nakayama, 2008a, 2008b, 2009). While the stimulus-then-move paradigm does not allow for continuous monitoring of cognitive processes at early time points, it has the advantage that the movement onset time can be measured directly rather than calculated statistically.

3.6.1. Method

Twenty right-handed participants aged 28;11 ± 6;11 were compensated for participation. Their mother tongue was Hebrew and they had no reported cognitive disorders. The method was similar to the silent condition in Experiment 3.1, except the way a trial was initiated. When the participants touched the initiation rectangle, a fixation cross appeared, and was replaced by the target number after a random duration between 500 - 1500 ms. The participants were instructed to move their finger as soon as the target number appeared, but not before that. The movement onset time was registered as the time from stimulus onset until the finger reached the $y = 50$ pixels coordinate (measured from the bottom of the screen). Movement onset lower than 100 ms or higher than 1000 ms resulted in a failed trial. Each number between 0 and 40 appeared 4 times, i.e., 164 non-failed trials per participant.

3.6.2. Results

The rate of failed trials was 3.17% ± 2.26%. The failures were due to moving the finger too early (13.9%) or too late (44%), to violation of the minimal-velocity policy (32%), or to lifting the finger in mid-trial (10.1%). The movement onset time was 623 ± 139 ms, and the movement time (from movement onset until reaching the number line) was 529 ± 110 ms. The endpoint bias was -0.52 ± 0.47 numerical units and the endpoint error was 1.68 ± 0.44 numerical units (all σ refer to the between-subject variance of the per-subject means).

Fig. 3.11a shows the median trajectories in this experiment. The trajectories are clearly different from the previous experiments: whereas in the movement-triggers-stimulus paradigm the finger initial movement was towards the middle of the number line, in the present experiment the movement was typically aimed more or less directly towards the target number, right from the start. This suggests that the finger movement started only after an initial decision was made about the quantity and the corresponding target position. Note that this pattern is not the result of averaging several trials – it is observed in single trials too (Fig. 3.11b).

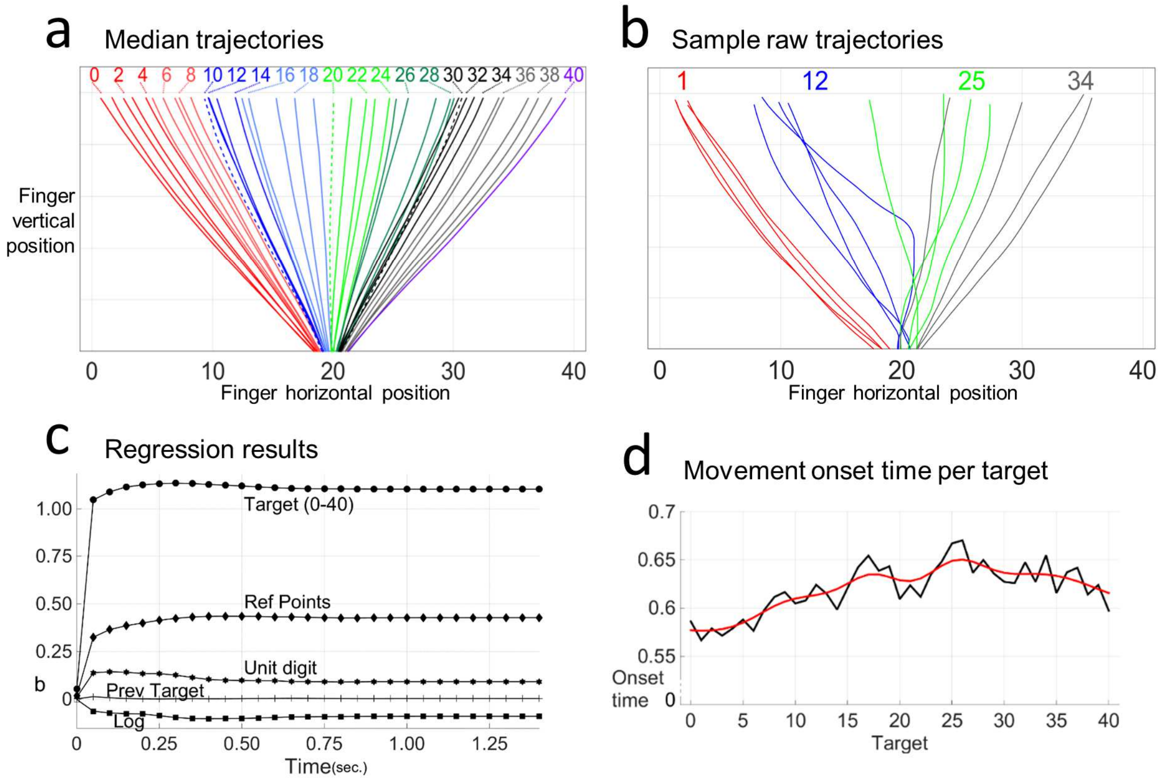

Fig. 3.11. Results of Experiment 3.5 (stimulus-then-move paradigm). (a) The median trajectories, averaged over participants. (b) Sample raw trajectories of one participant to four specific target numbers. In panels (a-b) the y axis reflects the iPad screen vertical dimension, so we can see that the finer moves towards the target number right from the start. (c) Regression b values. (d) Movement onset times per target number. The black line is the average over trials and participants. The red line is the same data after Gaussian smoothing with σ = 2. Onset times were shorter for targets < 20 than for targets > 20, and were shorter near the ends of the number line than around the middle.

The trajectory data was submitted to regression analysis with iEP as the dependent variable and with five predictors: $N_{0\text{-}40}$, log'($N_{0\text{-}40}$), the unit digit U, the spatial-reference-points-based bias function SRP, and the target number of the previous trial N-1. One regression was run per participant and time point in 50 ms intervals. The per-subject regression b values of each time point and predictor were compared versus zero using t-test. A strong effect of the target number $N_{0\text{-}40}$ was found from the time of movement onset (Fig. 3.11c), confirming that the finger aimed more or less towards the target number right from the start. The log effect did not make a positive significant contribution at any time point. There was a clear effect of the spatial reference points predictor, and there was a ~10% over-representation of the unit digit relatively to the decade digit (reflected by the positive contribution of the U predictor). Unlike the previous experiments, no contribution of the previous-target predictor N-1 was found at any time point – i.e., by the time a decision was made to move the finger, the present-trial quantity has completely overridden the prior trial effect.

The critical analysis in this experiment is that of the movement onset times per target (Fig. 3.11d). The onset times (excluding target = 20) were submitted to repeated measures ANOVA with a between-subject factor of side (left, right) and a numeric between-subject factor of distance from 20. A main effect of side ($F(1,19) = 28.36$, $p < .001$, $\eta_p^2 = .60$, $\eta_G^2 = .02$) confirmed the small-number advantage: movement onsets for numbers < 20 (mean = 606 ms) were shorter than for numbers > 20 (mean = 634 ms). A main effect of distance ($F(1,19) = 39.74$, $p < .001$, $\eta_p^2 = .68$, $\eta_G^2 = .02$) replicated the findings in previous experiments: movement onset was shorter when the target number was closer to the ends of the number line. The Side x Distance interaction was significant too ($F(1,19) = 19.03$, $p < .001$, $\eta_p^2 = .50$, $\eta_G^2 = .01$).

A comparison of Fig. 3.11d with Fig. 3.4 shows that the movement onset times in the present experiment (move-then-stimulus-paradigm) were longer than the times detected by our onset-detection algorithm in the movement-triggers-stimulus experiments. This difference was confirmed by a within-participant analysis: Thirteen of the 20 participants in Experiment 3.5 also performed the silent 0-40 experiment in the movement-triggers-stimulus paradigm. The movement onset times of these participants in the stimulus-then-move paradigm (620 ± 84 ms) were longer than the onset times detected in the movement-triggers-stimulus paradigm (448 ± 46 ms; paired $t(12) = 6.05$, two-tailed $p < .001$, Cohen's $d = 1.68$; targets 15-25 were excluded from this analysis).

3.6.3. Discussion of Experiment 3.5

The stimulus-then-move paradigm replicated the major effects found in the movement-triggers-stimulus paradigm. In the regression analysis, the finger movement was dominated by the linear quantity representation, with no logarithmic effect – similarly to the aligned-by-movement-onset regressions in Experiments 3.1-3.3. This provides further support to the differential encoding time model: when the movement onset is controlled for – either statistically, as in Experiments 3.1, 3.2, and 3.3, or methodologically, as in the present experiment – the log effect completely vanishes.

The detailed analysis of movement onsets fully replicated the pattern observed in the silent conditions in Experiments 3.1, 3.2, and 3.3: earlier onsets for target numbers on the left side (small-number advantage), and a distance effect such that onset times are later close to the middle of the number line. The replication of these effects with the stimulus-then-move paradigm confirms that these are genuine effects that do not result from a statistical artifact of the onset detection algorithm. This is especially important with respect to the distance effect: the onset detection algorithm relies on the horizontal velocity, and may consequently detect earlier movement onsets when the horizontal velocity is higher, which is typically the case when the target number is closer to any end of the number line. The replication of the distance effect in Experiment 3.5, in which the movement onset was measured directly rather than calculated, refutes the statistical artifact interpretation and shows that the distance effect has a cognitive origin. Note also that an analogous distance effect was observed by Cicchini et al., (2014, Fig. 3B): their analysis showed higher previous-trial-weights for targets close to the middle of the number line.

How can we explain this distance effect? One possible explanation is inspired by models suggesting that the trigger to change a motor action is the existence of an internal comparison between the action which is intended and the action which is currently being executed (Charles, King, & Dehaene, 2014; Fishbach, Roy, Bastianen, Miller, & Houk, 2007). In Experiments 3.1 to 3.4, participants are asked to initially point towards the middle of the line. Even when the finger is initially at rest (Experiment 3.5), the motor system might encode a default action of pointing towards the optimal location given the distribution of target numbers, which is again the middle of the number line. As the target-induced intention-to-move builds up, the intention-movement comparison mechanism would predict that the difference between the planned location and the middle of the number line must cross a fixed threshold before the finger starts moving towards the target. What we described in this chapter as "movement onset" would thus reflect the first decision to change the motor action. The duration of this decision process would be affected by the difference between the default action location (the middle of the number line) and the target number: the farther the target is from the middle of the number line, the larger this difference and therefore, the faster the decision threshold is reached – namely, earlier movement onset time.

Methodologically, Experiment 3.5 sheds some light on the similarities and differences between the stimulus-then-move paradigm (StM) of Experiment 3.5 and the movement-triggers-stimulus paradigm (MTS) of the previous experiments. The StM paradigm may have the advantage of a clearer separation between the two stages involved in this task – the decision stage, whose duration can be directly measured by the movement onset time, and the pointing stage, which is reflected by the finger trajectories. The StM paradigm also seems to allow for less noisy measurement of movement onsets, as the ANOVA's on movement onset times resulted in much stronger effects in Experiment 3.5 than in the previous experiments. The MTS paradigm, however, may be superior in its sensitivity to early processes: the onset times we detected in the MTS paradigm were much shorter than the onset times measured in the StM paradigm. One possible reason for this could be that initiating a movement takes longer than changing the direction of an existing movement (Pisella et al., 2000). Another possibility is that the longer onsets in Experiment 3.5 resulted from the relatively relaxed limit on movement initiation (up to one second from the stimulus onset). Shortening this limit would probably encourage earlier finger movement. Indeed, some implementations of the stimulus-then-move paradigm required participants to initiate movement as quickly as 200-350 ms from the “go” signal (Finkbeiner et al., 2014; Finkbeiner & Friedman, 2011). Such short time limits could make the StM paradigm more similar to the MTS paradigm – presumably at the cost of less reliable measurement of movement onset and the duration of the decision stage.

3.7. Discussion of Chapter 3

3.7.1. Understanding the Number-to-Position Task

In a series of experiments, we investigated how two-digit Arabic numbers are encoded as quantities in a number-to-position mapping task, which forces participants to convert a numeral into a quantity. To analyze the series of stages involved in this task, we obtained a nearly-continuous measurement of finger position, and we used a dual-task setting to perturb specific stages. In Experiment 3.1, the distraction was manipulated by introducing a simultaneous color-naming distracter task and comparing it with the single-task condition. In Experiment 3.2 we administered only the dual task, and the distraction was manipulated by changing the SOA of the target color and number. An analysis of the finger trajectories showed similar patterns in both experiments: in the experimental conditions with high distraction (color naming in Experiment 3.1, shorter SOA's in Experiment 3.2) the participants' number-to-position mapping became less linear, and in Experiment 3.1 also more logarithmic – a clear dissociation between the log and linear factors.

A careful analysis of the finger movement, however, showed that this log-linear dissociation cannot be taken as direct evidence for two distinct quantity representations, because a simpler interpretation can account for the results. This interpretation assumes that the finger horizontal movement onset is earlier for smaller target numbers, presumably because their quantity representation is less fuzzy than that of large numbers, which results in faster encoding of small numbers. As a result, the trajectories fan out more quickly for smaller number than for larger numbers, and this induces a transient log effect in the regressions. The interference from color naming further enhances this small-number advantage, thereby increasing the log effect. This interpretation is supported by the finding that the horizontal movement onset time is increased for larger numbers. As shown by Experiment 3.3, this small-number advantage cannot be dismissed as a difference between processing single-digit numbers and two-digit numbers. The interpretation is further supported by the finding that aligning the trajectories on movement onset times completely eliminated the logarithmic effect, revealing only a linear mapping of numbers to positions.

Our best interpretation of the data is that the number-to-position mapping task involves separate processes of quantification, decision by evidence accumulation, and pointing (Fig. 3.12a). The quantification process converts the two-digit number into a quantity representation. The decision process maps the quantity representation to a planned position. The pointing process aims the finger to the planned position.

The duration of the decision stage is affected by at least two factors: (1) Number size: large numbers take longer to process than small numbers, presumably because of differential variability in the output of the quantification process (in line with previous studies, e.g., Brysbaert, 1995; Li & Cai, 2014; Schwarz & Eiselt, 2009). (2) Distraction (here induced by the color-naming dual task), which delays the accumulation of evidence arising from the target number (again in line with previous studies of decision making, e.g., Sigman & Dehaene, 2005). Because of partial resource sharing, these two factors interact, so the size of this dual-task delay may also depend on number size, with large quantities suffering from a larger delay than small quantities.

3.7.2. A Mathematical Model of the Number-to-Position Task

In order to flesh out those ideas, we now present an explicit mathematical model of the number-to-position task. The model provides a "rational" or "ideal observer" analysis, i.e., it examines how any rational agent should endeavor to perform this task if it is endowed with exact and/or approximate representations of number. As we will see, such an optimal observer closely predicts human behavior.

We adopt here the same assumptions as in a previous mathematical model of several numerical-decision tasks (Dehaene, 2007). First, at the quantification stage, the quantity associated with the target number is encoded as a time series of independent and identically distributed noisy samples s_t, which are sampled from an internal random distribution. Second, at the decision stage, based on these samples, the posterior distribution over all possible target locations is continuously updated, until a threshold level is achieved and the model commits to a specific location. Third, at the pointing stage, the planned location is used to guide the finger motor movement. We now present detailed equations for each step.

Number representation. Following Dehaene (Dehaene, 2007), we assume that within each of the two quantity representation systems, the target number *T* is represented at any given time step *t* by a noisy sample *s(t)* (see Table 3.4 for a legend of all the notations used here and throughout this mathematical modeling section). The successive samples *s(t), s(t+1),* etc., are assumed to be independently and identically distributed (i.i.d) according to a Gaussian distribution

$$p(s|T) = \frac{1}{\sigma(T)\sqrt{2\pi}} e^{-\frac{(s-c(T))^2}{2\sigma(T)^2}} = Gaussian\big(s, \mu = c(T), \sigma = \sigma(T)\big) \qquad [2]$$

As this expression indicates, the samples *s* are centered on the value *c(T)*, which is a strictly increasing function of target number T representing the hypothesized internal scale for numerical quantity (e.g., linear or logarithmic). $\sigma(T)$, which may also vary as a function of *T*, is the standard deviation of the noise on this representation. The choice of functions *c(T)* and $\sigma(T)$ defines the nature of the internal representation of numbers. For an approximate representation, we may assume either a linear code with scalar variability, i.e. $c(T) = T + 1$

and $\sigma(T) = k_1(T+1)$; or a log-Gaussian coding with fixed variability, i.e. $c(T) = \log(T+1)$ and $\sigma(T) = k_1$. In both cases, k_1 is a constant, and the +1 term avoids singularity when the target is 0. For an exact representation, we take $c(T) = T$ and $\sigma(T) = k_2$ (where k_2 is another constant).

Table 3.4. Notations used for modeling.

Notation	Meaning
T	A target number presented in the experiment
s_{approx}, s_{exact}	A quantity sample sent from the quantification mechanisms (approximate, exact) to the decision process
n	A possible target number (this notation is used mostly for enumeration over all possible targets)
r	A response (decision on a target number) considered by the participant
$\hat{r}$	The response decided by the Bayesian decision process
λ	The slope of the linear distribution of target numbers, as perceived by the participant. Actual targets were distributed evenly (λ=0), but the participants did not know that and may consider various λ values, in distribution denoted p(λ).
Gaussian(x,μ,σ)	The probability to get a value *x* given a Gaussian distribution with mean μ and standard deviation σ
c(T), σ(T)	The mean and standard deviation of a Gaussian distribution of sample quantities given a target number T
Subscripts	
X_t	The value of X at time point t within a trial
X_i	The value of X at trial i
Constant parameters in the model	
k_1	Scaling factor for the approximate quantity representation standard deviation
k_2	Standard deviation of the exact quantity representation
k_3	Forgetting factor: the probability to keep the prior distribution p(λ), the perceived target bias (1-k_3 is the probability to revert to a flat prior)
θ	Posterior probability threshold for deviating the finger
τ_{approx}, τ_{exact}	The time (within a trial) in which the quantity samples s_{approx}, s_{exact} start arriving in the decision process.

In the following, we assume, for maximal generality, that exact and approximate representations co-exist, are activated in parallel, and generate independent samples. At any time t, the information available for decision is therefore comprised of the two sets of samples from time=0 to time=t, i.e., $\{s_{exact}(t')\}_{t'\leq t}$ and $\{s_{approx}(t')\}_{t'\leq t}$.

Accumulation of evidence. By definition, the ideal observer computes, for every possible response location, the posterior probability that this location is the correct one given the set of past samples. In the number-to-position task, there are as many response locations as there are target numbers, and therefore the inference is equivalent to inferring the likelihood of the current target number being *n*, given the set of past samples until time=t. Using Bayes' theorem, we get

$$posterior_t(n) \equiv p(n|past\ samples) \propto p(\{s_{exact}(t')\}_{t'\leq t}|n) \quad p\big(\{s_{approx}(t')\}_{t'\leq t}\big|n\big) \quad p(n) \qquad [3]$$

(note that this equation makes uses of the symbol α meaning "proportional to" – this is because, for simplicity, the denominator in Bayes's rule has been omitted; it is implicitly assumed that the posterior probabilities are normalized by a multiplicative constant in order to sum to 1 at each time step *t*).

In equation [3], $p(n)$ is the prior distribution of target numbers. In the simplest ideal-observer version of the model, the prior is supposed to be flat, in agreement with the fact that, in our experiments, all target numbers in the proposed range are equally likely. Thus, $p(n) = \frac{1}{n_{targets}}$, where $n_{targets}$ is the number of possible targets ($n_{targets} = max - min + 1$). Further below, we consider more complex options for the prior.

Given the independence of successive samples, the model reduces to a simple updating rule. Starting from the prior *p(n)*, on each time step *t* the optimal observer model receives two new random samples – $s_{exact}(t)$ and $s_{approx}(t)$ – and uses them to update the posterior probability that the correct response is *n*, using the equation

$$posterior_t(n) \propto posterior_{t-1}(n)\, p\big(s_{approx}(t)\big|n\big)\, p(s_{exact}(t)|n) \qquad [4]$$

(again up to a multiplicative constant, such that the posterior probabilities always sum to 1).

Simulating the random walk inherent to equation [4] requires expensive computations (generating many trials with random samples at each time step). For a faster, deterministic approximation, we can replace each of the two random multiplicands $p\big(s_{approx}(t)\big|n\big)$ and $p(s_{exact}(t)|n)$ by their time-independent expected value. For a trial with target number T, the expected value of s_{exact} is:

$$E\big(p(s_{exact}|n)\big) = \int_{-\infty}^{\infty} p(s_{exact}|n)\, p(s_{exact}|T)\, ds_{exact} \qquad [5]$$

The expected value of s_{approx} is calculated with the same formula, replacing s_{exact} by s_{approx}. Although this equation looks symmetrical, note that *T* represents the target number that was actually presented in the trial, whereas n represents the participant's enumeration of all possible target numbers.

The product of two Gaussians is itself a Gaussian, so formula [5] yields

$$E\left(p(s_{exact}|n)\right) = Gaussian\left(c(n), \mu = c(T), \sigma = \sqrt{\sigma(n)^2 + \sigma(T)^2}\right) \qquad [6]$$

and similarly for s_{approx}.

By plugging into this equation the parameters for approximate and exact representation, and by replacing both $p\left(s_{approx}(t)\middle|n\right)$ and $p(s_{exact}(t)|n)$ in equation [4] by their expected values according to equation [6], we obtain a deterministic approximation of the updating rule for the posterior, given that the target number is *T*:

For log-Gaussian coding:

$$posterior_t(n|T) \propto posterior_{t-1}(n|T)\ Gaussian(log(n+1), \mu=log(T+1), \sigma=k_1\sqrt{2})\ Gaussian(n, \mu=T, \sigma=k_2\sqrt{2}) \qquad [7]$$

And for linear scalar variability coding:

$$posterior_t(n|T) \propto posterior_{t-1}(n|T)\ Gaussian(n, \mu=T, \sigma=k_1\sqrt{n^2+T^2})\ Gaussian(n, \mu=T, \sigma=k_2\sqrt{2}) \qquad [8]$$

Numerically, equations [7] and [8] yield virtually identical results, thus demonstrating the near-complete behavioral equivalence of the log-Gaussian and scalar variability models (Dehaene, 2007). In the following simulations, we therefore adopt only the log-Gaussian model (equation [7]).

Simulations presented in Fig. 3.12b illustrate how the posterior evolves in the course of the trial for two specific target numbers. Initially, the distribution is flat, and then it evolves to an increasingly sharp peak centered on the target number. Indeed, equation [7] clearly shows that the "bump" in the posterior distribution is always centered at the appropriate target location on the number line, i.e. the highest posterior probability is reached for *n* = T. However, the sharpening of the posterior is faster for small than for large numbers.

Cost function and decision. The above equations specify how the posterior probability distribution of the correct numerical response evolves with time, but not how participants transform this distribution into an intention to move. In any Bayesian decision task, the optimal use of the posterior distribution depends on the cost function imposed by the experimental setting (Maloney & Zhang, 2010). Here, as the task requires minimizing the distance between the finger location and the actual target location on the number line, we stipulate a quadratic cost function:

$$cost(r) \propto (r - T)^2 \quad [9]$$

where *T* the actual target number and *r* is the subject's intended numerical response. At any time t, we assume that participants pick up, out of all possible responses r, the one that minimizes the expected cost:

$$\hat{r} = \underset{r}{\operatorname{argmin}}(\mathrm{E}(\mathrm{cost}(\mathrm{r}))) = \underset{r}{\operatorname{argmin}}(\textstyle\sum_n posterior_t(n|T)\ (r - n)^2) \quad [10]$$

The solution of this equation is the mean of the numbers *n*, weighted by their posterior probability:

$$\hat{r} = \textstyle\sum_n posterior_t(n|T) \cdot n \quad [11]$$

This equation has the following experimental implications: (1) In the absence of information about the target, given that all targets are equiprobable, participants initially point to the center of the number line, i.e., the location that minimizes the quadratic error; (2) As increasingly precise evidence is gathered about the target value, the intended response location deviates progressively from this mid-point value.

Movement. For each target number, the model specifies the subject's optimal intended response at each time step. To compare these *numerical* estimates with the *motor* trajectories recorded, we need to model how a numerical intention is translated into a finger trajectory. A complete model would entail answering each of the following theoretical issues: (1) *When* does the finger move? Do participants wait until a threshold amount of evidence is accrued, or is the evidence continuously passed on to the motor system? (2) *How* does the finger move? Is a target direction programmed once per trial, and then translated into a velocity profile? Is the direction updated continuously? Or is it revised only at discrete times, e.g., whenever the anticipated finger location deviates from the intended location by a sufficient amount (Fishbach et al., 2007), as suggest by previous "change-of-mind" results (Resulaj, Kiani, Wolpert, & Shadlen, 2009)?

Answering these questions is clearly beyond the present research program. Here, we present simulations of the simplest possible model. Based on prior research on decision making (Gold & Shadlen, 2001), we assume that the decision to move is based on the accumulation of evidence towards a fixed probability threshold θ, i.e., movement starts whenever the posterior probability of one of the target locations exceeds this threshold value. At this moment, the movement process sends the finger to the location that minimizes the average square error, as described above. Finally, movement is implemented with the typical bell-shaped velocity profile characterizing limb motion (Flash & Hogan, 1985; Friedman et al., 2013).

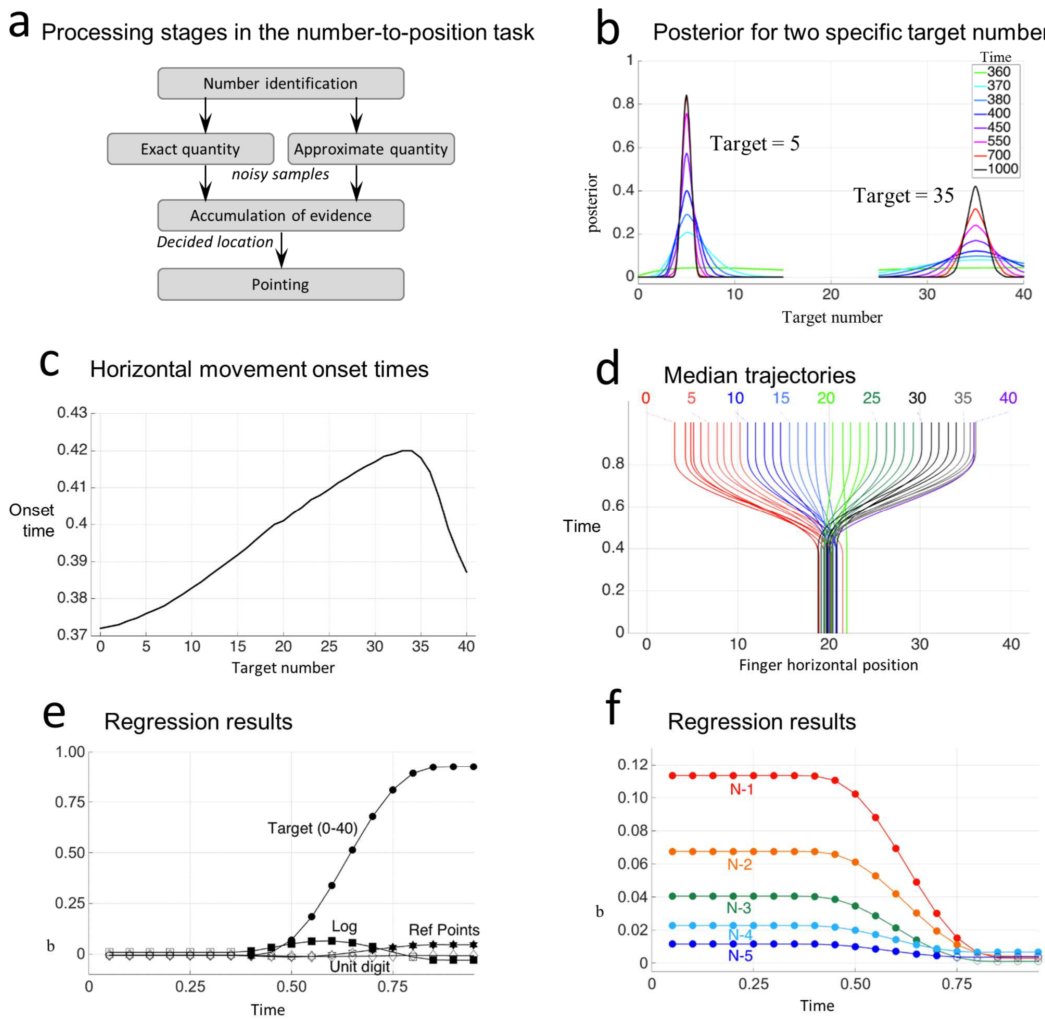

Fig. 3.12. Model and simulations of the processing stages in the number-to-position task. (a) Proposed stages. Incoming digits are identified and the corresponding quantity is separately encoded in approximate and exact systems. Next, evidence accumulation is used to infer the posterior distribution of target locations given the incoming noisy samples. Finally, a pointing stage brings the finger to the location that minimizes pointing errors. (b-f) Simulation results. (b) The posterior probability function in different time points, for two specific target numbers. As the trial progresses the posterior curve becomes steeper. Crucially, the curve converges more quickly for small target numbers such as 5 than for symmetric large target numbers such as 35. (c) Small-number advantage: the horizontal movement onset times are earlier for small target numbers than for larger targets. The onset times were calculated using the onset detection algorithm described above (Section 3.2.2.4.1). (d) Median trajectories. (e-f) The regression b values (dependent variable = x coordinate, predictors = $N_{0\text{-}40}$, log'($N_{0\text{-}40}$), unit digit, SRP, and the last 5 targets). The regression captures several effects of the real data – strong linear factor, transient logarithmic factor, and an effect of several prior trials in early trajectory parts, which decays exponentially for older trials.

Under this assumption of a single movement, given that the posterior distribution is sharper for small numbers than for large numbers, the movement onset time should always be slower for large compared to small numbers. However, the choice of the threshold θ has a crucial impact on the shape of the response function. If the participants use a low threshold θ, the finger deviates towards the decided location early on, at a time when the posterior distribution is sharp for small target numbers but not for large ones. This results in a greater separation between small numbers than between large numbers, leading to an approximately logarithmic response pattern (as observed in children and in patient ZN). If the participants use a higher threshold θ, the finger's deviation towards the decided location happens later, at a time when the posterior distributions for both small and large numbers are already sharply centered on the appropriate target value, so the responses become arrayed in a linear manner.

Effect of prior targets. In our experiments, the prior p(n) was flat over all target numbers. The participants, however, were not told this, and may (explicitly or implicitly) believe that some targets are more likely than others. In agreement with this idea, in all experiments, we observed an effect of the recent target numbers on the early part of the trajectory. Such a prior-trial effect cannot be explained merely as a perseveration of the motor response on the immediately previous trial, because that response was influenced solely by the target of that particular trial and not of the previous trials. As we shall now see, the exponentially decreasing influence of previous targets can be explained as a constantly updated Bayesian prior over the possible targets.

Formally, we capture this idea using a second-order optimal observer model. The assumption is that subjects use the distribution of recent targets to estimate the probability distribution of a new target *T*. For simplicity, we assume that participants only consider linear distributions over the range of target numbers, i.e., a set of distributions of the form $p(n|\lambda) = \frac{1}{n_{targets}}\left(1 + \lambda\frac{n-mean}{max-mean}\right)$ with $mean = \frac{max+min}{2}$, where min and max are respectively the minimum and the maximum of the range of target numbers. This equation describes a linear probability distribution over the numerical interval [*min, max*]. $\lambda \in [-1,1]$ is a hyperparameter that governs the relative emphasis of small numbers over large numbers: $\lambda = -1$ indicates that participants expect a majority of small numbers, $\lambda = 0$ a flat distribution, and $\lambda = +1$ a majority of large numbers.

We assume that the participants' expectations about the target numbers changes as a function of the recent target numbers they received. This is achieved by constantly maintaining an internal distribution of the possible values of λ. At the beginning of the experiment, this distribution is flat over the interval [-1, 1]: all values of λ are equiprobable. At the end of each trial, based on the target they just received, subjects revise their posterior distribution of λ. We denote this revised distribution by $p(\lambda_i)$ (this is the estimate at the end of trial *i*, after taking into account the target T_i, and therefore serving as a prior for trial *i+1*). At this time, we assume that the participants have precisely identified the trial's target number T_i, so they can use it to revise their previous distribution $p(\lambda_{i-1})$. According to Bayes' rule, this update should be:

$$p(\lambda_i|T_{1:i}) \propto p(\lambda_{i-1}|T_{1:i-1})\, p(T_i|\lambda_i) \qquad [12]$$

This optimal equation, however, would simply imply that subjects accumulate perfect evidence about the distribution of targets, without any forgetting, in which case they would quickly converge to a distribution centered on the correct value λ=0 (unbiased distribution of target numbers). The evidence, however, indicates a strong effect of recent trials, which suggests the existence of local expectations (e.g., after a streak of large numbers, subjects expect to see more large numbers). We model this as forgetting in the updating process. Formally, as in previous work (Behrens, Woolrich, Walton, & Rushworth, 2007; Meyniel, Schlunegger, & Dehaene, 2015), we assume that there is a probability k_3 that the participants carry the current posterior estimates $p(\lambda_{i-1})$ onto the next trial, and a probability of $1-k_3$ that they revert to a flat prior. In other words, k_3 controls the relative weight of the prior expectation relative to the incoming evidence at a given trial: $k_3 = 1$ means no forgetting (optimal Bayesian integration), and $0 \leq k_3 < 1$ mean underweighting of the prior information and, correspondingly, a stronger effect of the last target on the estimation of λ.

The value of λ_i can now be calculated by applying Bayes rule:

$$p(\lambda_i|T_{1:i},k_3) \propto p(\lambda_i,T_{1:i-1},n_i|k_3)p(T_{1:i}) \qquad [13]$$

$$\propto \int_{-1}^{+1} p(\lambda_{i-1},\lambda_i,T_{1:i-1},T_i|k_3)d\lambda_{i-1} \qquad [14]$$

$$\propto \int_{-1}^{+1} p(T_{1:i-1}|k_3)p(\lambda_{i-1}|T_{1:i-1},k_3)p(\lambda_i|\lambda_{i-1},T_{1:i-1},k_3)p(T_i|\lambda_{i-1},\lambda_i,T_{1:i-1},k_3)d\lambda_{i-1} \qquad [15]$$

$$\propto \int_{-1}^{+1} p(\lambda_{i-1}|T_{1:i-1},k_3)p(\lambda_i|\lambda_{i-1},k_3)p(T_i|\lambda_i)d\lambda_{i-1} \qquad [16]$$

In [14], we removed the constant term $p(T_{1:i})$ and marginalized over λ_{i-1}. In [15], we applied the chain rule. In [16], we removed the constant term $p(T_{1:i-1}|k_3)$ and simplified the other probabilities by considering that some terms are independent of each other. In the resulting expression [16], the term $p(\lambda_{i-1}|T_{1:i-1},k_3)$ reflects the prior; the term $p(\lambda_i|\lambda_{i-1},k_3)$ – the forgetting factor; and the term $p(T_i|\lambda_i)$ – the probability that the present trial target would indeed be T_i given a certain λ value.

Once we know the distribution $p(\lambda_i)$, we can marginalize over λ_i to obtain the prior probabilities for target number of the next trial:

$$p(n_{i+1}|n_{1:i}) = \int_{-1}^{+1} p(n_{i+1}|\lambda_i)p(\lambda_i)\ d\lambda_i \qquad [17]$$

Intuitively, the effect of those equations is that after receiving, say, a large number such as 40, participants infer that the estimated likelihood of being in an experiment with a large λ is high, and therefore they expect to receive other large target numbers on subsequent trials. As a consequence, even in an unbiased experiment where all targets are presented equally frequently, participants will be biased to point towards recently presented targets.

Simulations. Fig. 3.12c-f shows simulations of movement time, movement trajectory and regressor estimates. It can be seen that the model provides a reasonable qualitative fit for most of the experimentally observed effects (here and in Chapter 2). The horizontal movement onset is an asymmetrical function of target size, with faster responses for small numbers than for large numbers (Fig. 3.12c). As a result, simulated finger trajectories depart from the center faster for smaller numbers than for larger number (Fig. 3.12d). Consequently, regression analyses exhibit a transient log effect followed by a sustained linear effect (Fig. 3.12e). This effect disappears when regression is locked on the horizontal movement onset. Finally, an effect of previous targets is observed on the initial part of the movement, with approximately exponential decay over the past trials (Fig. 3.12f).

The model may also account for two additional subtle features of the data: the influence of the spatial reference points (SRP) equation, and the fact that the regression weight of the log function becomes negative late in the trial. Both effects arise because the model only considers hypotheses in the range [0,40], thus truncating the posterior distribution to this range and shifting the responses away from the endpoints 0 and 40 and towards the center of the number line (a regression to the mean typical of Bayesian models, see e.g. Fischer & Whitney, 2014; Jazayeri & Shadlen, 2010). The reference point effect captures this small displacement, while the negative log captures a slight asymmetry of this effect due to differential variability for small and large numbers. In actual data, the reference point effect is larger, seemingly because of an additional repulsion of responses away from the line midpoint 20, which is not captured by the current model (but might be if one assumed an additional process of comparing the target to the midpoint).

The simulations in Fig. 3.12 were obtained with $k_1 = 0.7,\ k_2 = 20.0,\ k_3 = 0.7,$ $\theta = 0.15$, with a delay of $\tau_{approx} = \tau_{exact} = 350\ ms$ for the onset of samples arising from the exact and approximate representations, and with the assumption of calculation iteration every 1 ms. Because the model remains coarse and unspecified, especially as concerns movement programming, we did not attempt a quantitative fit of the data, but we did observe that the above effects are generic across a larger range of parameters. Scalar and compressive representations of approximate number give virtually identical results. Importantly, having only an exact linear representation cannot account for the results: simulating it leads to a disappearance of the transient log effect. Conversely, however, it is possible to account for the results with a single approximate representation – there is a range of parameters (e.g., $k_1 = 0.7,$ $k_3 = 0.7,\ \theta = 0.12,\ \tau_{approx} = 350\ ms$) for which the movement onset is delayed for large numbers, resulting in a transient log, and yet the internal distribution at the time of movement is precise enough to yield near-linear pointing. The only quantitative inadequacy of this approximate-only model is that the weight of the linear regressor never converges to 1, i.e. the final pointing remains sublinear. The fact that the linear weight does converge to 1 in adult data (Fig. 3.2, 3.6, 3.9b) thus confirms that adults are supplementing their approximate representation with a linear understanding of exact number.

Both the single (approximate) and the dual-representation models can also account for the children's data by lowering the posterior threshold θ required for making a decision. Lower threshold leads to an earlier decision to move. In this earlier time point, less evidence was accumulated, so the decision about a target location is based on a more approximate representation, thus magnifying the difference between small and large numbers. This results in a more logarithmic mapping (Fig. 3.13, created by lowering the threshold θ from 0.15 to 0.07), which bears much similarity to the children data in Fig. 3.10a-b. Finally, the effect of dual-task interference may be simulated in several ways, either by differing the onset of the exact representation relative to the approximate representation, or by assuming that, during dual-task interference, both representations suffer from additional noise, such that the rate of evidence accumulation is lower. Further research will be needed to disentangle these possibilities.

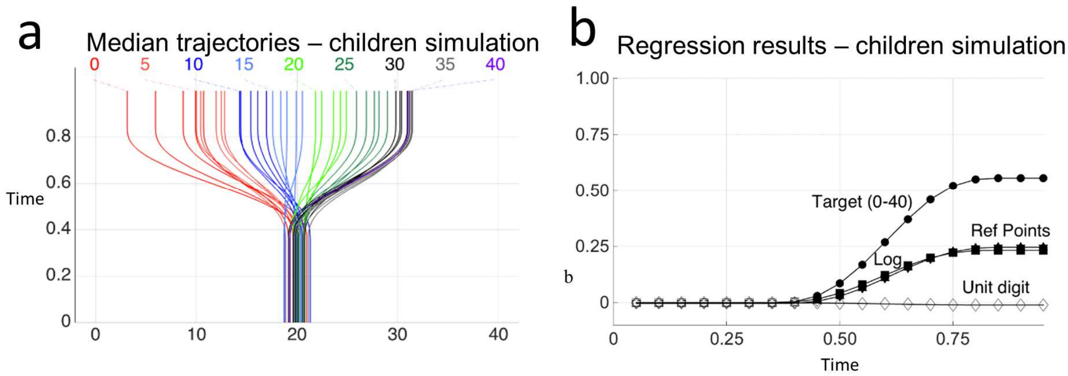

Fig. 3.13. Simulation of children data. (a) Median trajectories. (b) Regression b values.

One aspect that is not captured very accurately by this model is the shape of the distance effect for the movement onset time: the real data show a clear dependency on distance from the mid-point (Fig. 3.4, Fig. 3.11d), which is absent in the simulated data (Fig. 3.12c). This finding suggests that the model's simple decision mechanism (a fixed threshold on posterior probability, inspired by Gold & Shadlen, 2001) may have to be replaced by a more complex mechanism of comparison between the new aim (point to the target) and the initial aim (point to the midpoint), as indeed suggested by recent studies of motor programming (Fishbach et al., 2007) and error correction (Charles et al., 2014). Such refinements, however, add much complexity to the model and are therefore better left for future research.

3.7.3. Conclusion of Chapter 3

Performance of the number-to-position task, as studied in the present experiments with adult participants, is entirely compatible with a strictly sequential processing model that combines a quantification stage (using both exact and approximate representations), an optimal decision-making stage, and a movement stage that minimizes pointing errors. Our main empirical finding is that in adults, these stages appear to be separable: once the variable duration of the decision stage is controlled for (by aligning trajectories on the horizontal movement onset times), the finger trajectories show virtually no logarithmic effect, but only linear pointing. Many other details are captured by the optimal decision making model.

While this model nicely accounts for the performance of healthy adult participants, an examination of the performance of the aphasic patient ZN and of 4th grade children indicated that this model cannot be the whole story. The logarithmic effect in these experiments cannot be solely explained by differential durations of a decision stage, as a logarithmic effect continued to be found long after the horizontal movement onset. We saw that two classes of explanations can be proposed: either those subjects genuinely fail at the conceptual level, i.e., they simply do not understand that the task calls for linear pointing (Booth & Siegler, 2006; Dehaene et al., 2008; Siegler & Booth, 2004; Siegler & Opfer, 2003); or they attempt to point linearly (as our model does), but their decision-to-move is based on partial evidence which is coarser for large than for small numbers. More research will be needed to separate those possibilities.

4. Parallel and serial processes in number-to-quantity conversion

Abstract. Converting a multi-digit number to quantity requires processing not only the digits but also the number's decimal structure. We investigated this structural processing: first, we asked whether the digits are processed serially or in parallel. Second, given that the same digit represents different quantities when in different decimal roles (e.g., "2" can mean 2, 20, etc.), we asked how digits are assigned decimal roles. To answer these questions, we used the number-to-position task with finger trajectory tracking. Crucially, the decade and unit digits could appear with a lag. When the decade digit was delayed, the decade effect on finger movement was delayed by the same amount. However, a lag in the unit digit delayed the unit effect by 35 ms less than the lag duration, a pattern reminiscent of the psychological refractory period, indicating an idle time window of 35 ms in the units processing pathway. When a lag transiently caused a display of just one digit on screen, the unit effect increased and the decade effect decreased, suggesting errors in binding digits to decimal roles. We propose that a serial bottleneck is imposed by the creation of a syntactic frame for the multidigit number, a process launched by the leftmost digit. All other stages, including the binding of digits to decimal roles, quantification, and merging them into a whole-number quantity, appear to operate in parallel across digits, suggesting a remarkable degree of parallelism in expert readers.

4.1. Introduction

How do we combine the digits of a multi-digit number into a single quantity? In the visual system, the digits in a number such as "22" appear at distinct retinotopic locations and must therefore be processed independently; yet at some point they must be weighted according to their position in the overall number. The very same symbol, say 2, changes its meaning depending on its decimal role – e.g., it may mean two units (e.g. in "32") or twenty units (in "23"). The number recognition process must therefore be broken into several operations: (1) Visual parsing and identification of each digit and their relative positions. (2) Binding each digit to a decimal role – units, decades, etc. (3) Quantifying each digit, i.e., multiplying the digit value by the weight implied by its decimal role, and merging these quantities. (4) Using the resulting quantity in whichever task is required. In the number-to-position task, the quantity is converted into a manual movement towards the appropriate location on the number line.

The study described in this chapter focuses on the 2nd and 3rd operations in this list: we aimed to understand how the digits of a multi-digit number are bound to decimal roles and quantified. We examined these issues using the number-to-position mapping task. In the previous chapters, experiments with two-digit numbers found that the effects of the decade and unit digits on finger movement were in a 10:1 ratio and that their buildup was nearly simultaneous. Consequently, we suggested that the decade and unit digits were quantified simultaneously. In fact, however, several cognitive architectures may underlie this finding. One possibility (termed the lexical model) is that the entire two-digit string is recognized holistically, similar to the lexical route for the visual recognition of known words (Coltheart, Rastle, Perry, Langdon, & Ziegler, 2001; Ellis & Young, 1996; Friedmann & Coltheart, in press). This would require that all numbers and their corresponding quantities be lexically stored (see Cohen, Dehaene, & Verstichel, 1994 for counter-evidence). Alternatively, each digit may be quantified independently and simultaneously, and the per-digit quantities would affect the finger movement in parallel (parallel-decomposed model). Last, any serial processing of the digits may be undetectable by the finger-tracking paradigm if the finger deviates towards a specific target location only after the decade and unit quantities were fully processed and merged (max model). According to this model, the finger deviation time would reflect the maximum of the processing times of the individual digits, irrespective of the order in which they are processed.

To evaluate these possibilities, we asked participants to perform the trajectory-tracked number-to-position task with two-digit numbers. Crucially, we systematically varied the temporal order and delay separating the onsets of the decade and unit digits: on some trials the decades appeared shortly before the units, or vice-versa. This method allows examining the temporal dependencies that link the two digits. If all four operations described above (parsing, binding to decimal role, quantification, movement) unfold independently and in parallel for each digit, delaying a digit by a delay Δt should delay the effect of this digit on finger movement by the same Δt, but it should not delay the effect of the other digit on finger movement. Conversely, if there is full dependency between the digits – either because all digits are needed to recognize the whole number as a word, or because the decision to move the finger depends on the availability of all digits – then delaying either digit by Δt would delay the effects of both digits on the finger movement by Δt. Note that the between-digit dependency does not have to be symmetric: for example, it is possible that the processing of the unit digit depends on the decade digit but not vice versa (such would be the case if we assume purely sequential processing – first the decade digit, then the unit digit).

Delaying the onset of a digit might also perturb the binding of digits to decimal roles. If the decade digit transiently appears alone on screen, it might be incorrectly quantified as a single-digit number. Conversely, a stand-alone unit digit might be interpreted as the decade of a two-digit number. To reduce the likelihood that visual confusions would cause such binding error, the digits always appeared in predictable locations on screen, and when delaying the onset of a digit, its position was temporarily occupied by a placeholder character (i.e., only two-character strings were always displayed).

4.2. General method

4.2.1. Participants

All participants were right-handed adults with no reported cognitive disorders, and were compensated for participation. Their mother tongue was Hebrew, in which words are written from right to left but numbers are written like in English. Hebrew and left-to-right readers were found to exhibit similar patterns of results in our paradigm (see Chapter 2).

4.2.2. Procedure

The general procedure followed the one described in Chapter 2. The crucial difference was that here, the digits of a two-digit number did not always appear simultaneously. The fixation stimulus was changed accordingly, as detailed below.

4.2.3. Statistical analysis

4.2.3.1. Factors affecting the finger movement

We analyzed finger trajectories using the method introduced in Chapter 2. One regression was run per participant and per time point in 50 ms intervals. Implied endpoints were regressed against the decade (0, 10, 20, …, denoted D), the unit digit (U), the target of the previous trial (N-1), and a bias function (SRP, equation [1]). SRP may reflect a spatial aiming strategy that relies on the middle and ends of the number line (Barth & Paladino, 2011; Rouder & Geary, 2014; Slusser, Santiago, & Barth, 2013; Section 2.3.2.6).

To examine whether a given predictor had a significant group-level effect per time point, the participants' regression b values (significant and non-significant) were compared with zero using t-test (one-tailed p is reported).

In the previous chapters, we used a logarithmic predictor to account for potential logarithmic quantity representation. The log predictor was not used here, because in Chapter 3 we showed that it captures a temporal bias rather than logarithmic representation, but including it yielded essentially the same results.

4.2.3.2. Comparing the decade and unit effects between conditions

To assess how delaying the decade or unit digits modulates their effect on finger movement, we compared the decade and unit regression coefficients (b[U] and b[D]) between conditions with different onset times of the digits. To estimate the difference in the timing of an effect between two conditions, we calculated delay between the regression curves of that effect – i.e., the Δt that minimized the sum of squared differences between the curves: $\min_{\Delta t} \int (b_{cond1}(t) - b_{cond2}(t - \Delta t))^2 dt$. This integral was computed on the average b values over participants, and restricted to the time window in which the regression curve was on rise (which was always in the range 350-650 ms), because these time windows were the most informative. To approximate the integral, the regressions were run in 10-ms intervals and the b values were interpolated to 1-ms granularity with cubic spline interpolation.

To assess the significance of the difference between conditions, we compared the per-participant b values between the two conditions on each time point using paired t-test (one-tailed p). This analysis too was restricted to the time window in which the regression curve was on the rise.

4.2.3.3. Assessment of binding errors using changes of mind

Binding errors cause the decade digit to be sometimes interpreted as the unit digit or vice versa. We assumed that binding errors are often corrected later in the trial, resulting in a change in the finger direction. To measure such changes of mind, we calculated the finger horizontal acceleration by deriving the x coordinates twice (with Gaussian smoothing before each derivation, σ = 20 ms). We then counted, per trial, the number of "x acceleration bursts" – trajectory sections with acceleration ≥ 2.38 mm/s during at least 70 ms (Dotan, Meyniel, & Dehaene, 2017). Each acceleration burst potentially reflects a change of mind.

4.2.3.4. ANOVA

The speed of performing the number-to-position task varied a lot between participants. Our goal in the present study was not to explain these inter-individual differences, but to focus on the within-participant factors that affect people's behavior in the number-to-position task. For this reason, in all ANOVA's in this study – most of which concern reaction times – we use repeated measures design and report effect sizes as partial η^2, a measure independent of the between-participant variance. For standardization we also report η^2 for one-way ANOVAs, and generalized η^2 (Bakeman, 2005; Olejnik & Algina, 2003), denoted η_G^2, for ANOVA with several factors.

4.3. Experiment 4.1: Delaying the unit digit

In this experiment, the decade digit always appeared at t = 0 and the unit digit appeared either simultaneously or after a short delay. If the unit digit is processed independently of the decade digit, any delay in the unit digit onset should be fully reflected in its effect on the finger movement.

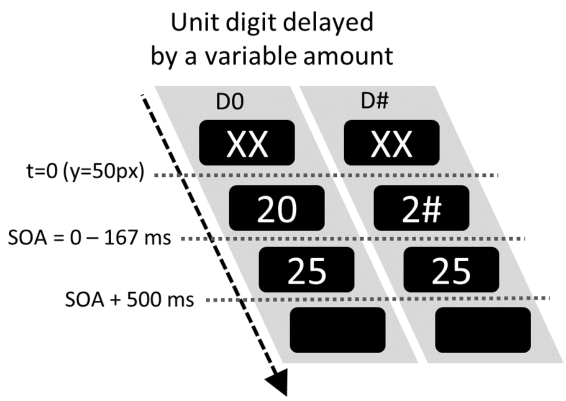

Fig. 4.1. Experiment 4.1 task design: the fixation was XX. At t=0, the decade digit appeared and the unit position was occupied by a '0' or '#' character (in two separate blocks). The placeholder character was replaced by the unit digit after 0, 33, 67, 100, 133, or 167 ms. The '0' placeholder makes the transient stimulus a valid two-digit number, and was aimed to reduce errors in binding of digits to their decimal roles.

To discourage the binding of digits to incorrect decimal roles, the position of the delayed unit digit was temporarily occupied by a '0' placeholder character (Fig. 4.1, "D0" block) – e.g., the target number 25 appeared as 20 and then changed to 25. With a '0' placeholder, the transient stimulus (20) is a valid two-digit number; we hoped this would facilitate the processing of the displayed decade digit as part of a two-digit number (rather than as a single-digit number), and thus discourage binding to incorrect decimal roles. However, a possible disadvantage of the '0' placeholder is that, being an acceptable digit, the '0' may undergo the full processing pathway on top of the target unit digit or instead of it. As a result, finger movement would reflect a mixture of the target unit digit and the '0' placeholder. Because '0' is always smaller than or equals to the target digit, the average unit effect on finger movement would be reduced. To control for this possible confound, we added a second block with a non-digit placeholder character (D# block). The '#' character should not undergo the digit processing pathway, so we predicted that it should not cause the unit under-representation artifact, perhaps at the cost of more errors in binding digits to decimal roles. By comparing the D0 and D# blocks we could examine the notion of binding errors.

4.3.1. Method

28 right-handed adults, aged 25;10 ± 2;7, performed two blocks of the number-to-position task. The transient unit placeholder character was ‘0’ in one block and ‘#’ in the other. We used a 0-60 number line. In both blocks, the decade digit always appeared at t = 0, and the unit digit appeared at t = 0, 33 ms, 67 ms, 100 ms, 133 ms, or 167 ms (mixed design; Fig. 4.1). Each target number between 10 and 50 was presented twice per block and SOA (492 trials per block).

The fixation indicator was uppercase XX. In the SOA=0 condition, it changed into the target number at t=0. In longer SOAs, the decade digit appeared at t=0, and the unit position changed at t=0 into ‘#’ or ‘0’, and after the SOA duration – to the target digit. Both digits disappeared 500 ms after the unit digit onset. Note that at t=0, a visual change occurred in both decimal positions, in order to minimize the between-digits differences in attentional or spatial bias.

The finger trajectory data of each SOA condition was separately submitted to the two-stage analysis and analyzed for between-condition delays as described in General Method. As we shall see below, in the D0 block b[U] did not converge to the same value in all SOA conditions, so the delay estimations were not based on the raw b values but on the ratio between b[U] and its endpoint value.

4.3.2. Results

4.3.2.1. General analyses

The trial-level measures and the trajectory regressions showed the pattern typical to our paradigm (Table 4.1). In both blocks, in the SOA=0 condition (Fig. 4.2a,b) the decade and unit effects almost overlapped, with a small over-weighting of b[U] relative to b[D] during a transient time window, similarly to the findings in Chapter 2.

Table 4.1. Trial-level measures and regression patterns. In all experiments, the regression effects showed the pattern typical to this paradigm (as described in the previous chapters): dominant effect of the target number (both digits), significant effect of the previous trial in the early trajectory parts, and a spatial-reference-points bias (SRP) effect in the late trajectory parts.

Experiment	Exp. 4.1, D#	Exp. 4.1, D0	Exp. 4.2	Exp. 4.3
Failed trials (%)	5.4 (4.5)	4.6 (3.5)	2.8 (2.6)	4.3 (3.5)
Movement time (ms)	1012 (161)	980 (137)	1036 (177)	1272 (186)
Endpoint bias [a]	-0.73 (0.88)	-0.75 (0.8)	-1.36 (0.87)	-5.81 (4.71)
Endpoint error [a]	2.89 (0.84)	2.83 (0.81)	4.68 (1.17)	23.8 (7.58)
b[N-1]				
Peak value	0.20-0.22	0.23-0.26	0.20-0.21	.19
Peak time point	350-400	350-400	350-400	500
b > 0.05 until (ms)	600-700	650-750	600-700	850
b[SRP]				
Significant from (ms)	450-600	450-550	450-500	500
Endpoint value	0.29-0.32	0.27-0.31	0.29-0.30	.39
Endpoint b[U]/b[D]	1.01-1.06	0.87-1.03	0.85-0.91	0.92 [b]

[a] Endpoint bias and error use each Experiment’s number line scale (0-60, 0-100, or 0-400)
[b] The ratio shown for Experiment 4.3 is b[D]/b[H]

4.3.2.2. Partial processing of the 0 placeholder

In the D0 block, the regression analyses showed that the unit effect b[U] converged to gradually lower endpoint values as the SOA increased (Fig. 4.2c). This suggests that, as we predicted, the ‘0’ placeholder character was partially processed as the target unit digit, and that the degree of this partial processing depended on the duration of presenting the ‘0’ placeholder. This effect was confirmed by a repeated measures ANOVA on b[U](endpoint) with SOA as a within-subject numeric factor ($F(1,27) = 13.6$, $p < .001$, $\eta^2_P = .34$, $\eta^2_G = .09$). No such linear trend was found in the D# control block ($F(1,27) < 0.2$). The difference between the blocks was demonstrated using a two-way repeated measures ANOVA on b[U](endpoint), with SOA as a within-subject numeric factor and the block (D# or D0) as a between-subject factor: the SOA x Block interaction was significant ($F(1, 27) = 7.83$, $p = .01$, $\eta^2_P = .22$. $\eta^2_G = .02$).

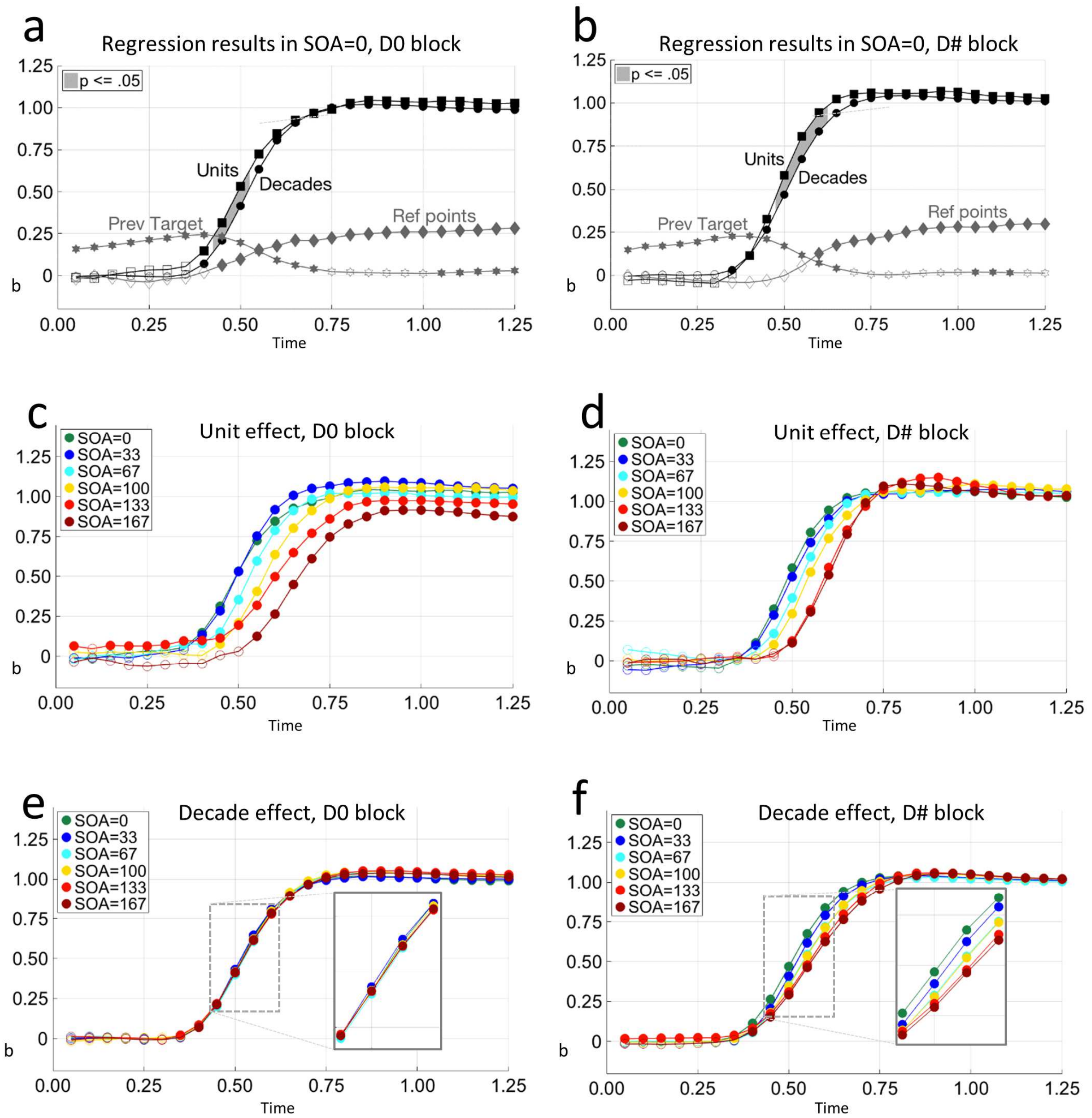

Fig. 4.2. Time course of the effects in Experiment 4.1. This and subsequent plots show the b values of the regressions (one regression per time point, participant, and condition) on implied endpoints. The b values were averaged over participants, plotted as a function of time, and compared to zero with t-test (full dot = significant). (a,b) Time course of the effects in the SOA=0 condition (simultaneous presentation of the two digits). The decade and unit effects rise together in almost 10:1 ratio, with a small over-weighting of the units. (c,d) Time course of b[U], the unit regression effect, in all SOA conditions. (c) b[U] showed an IDLE pattern in the D0 block: delaying the unit digit by 33 ms had little effect on b[U], whereas longer delays caused a linearly-increasing delay of b[U]. (d) In the D# block, b[U] did not show the IDLE pattern. (e,f) Increasing the SOA created a small decrease/delay in the decade effect in the D# block but not in the D0 block.

4.3.2.3. D0 block: the effect of delaying the unit digit

If the unit digit is processed independently of the decade digit, delaying the unit digit onset (extending the SOA) should delay the unit effect on finger movement (b[U]) by the same amount. Fig. 4.2c clearly shows that this was not the case: the shortest SOA (33 ms) had no impact on b[U]; only extending the decade-unit SOA beyond 33 ms created an increasingly larger delay in b[U]. The b[U] delay, calculated as described in Methods, showed this pattern very clearly (red line in Fig. 4.3). To quantify the increase in the delay, we regressed the b[U] delay by SOA (only SOA > 0 were included). The regression slope (dashed line in Fig. 4.3) was 1.11 – i.e., beyond SOA = 33 ms, extending the SOA by a certain amount delayed b[U] by a similar amount.

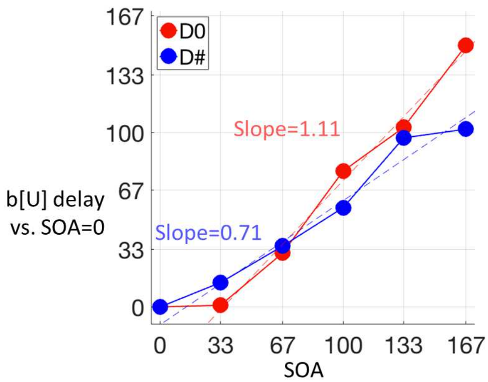

Fig. 4.3. Experiment 4.1: the delay in the unit effect on finger movement between SOA=0 and each other SOA. The lowest SOA (33 ms) had no effect on b[U], and extending the SOA beyond that created a linearly increasing delay (IDLE pattern). The dashed lines are regressions of the b[U] delay against SOA (SOA=0 excluded). In the D0 block, this regression has a slope that approached 1.0 and it crosses the x axis at SOA=35 ms, i.e., extending the SOA beyond 35 ms delayed the unit effect by SOA - 35 ms.

One explanation for this pattern is that the unit quantity was not processed as soon as it appears, but only after a short interval. This interval is an idle time window in the units processing pathway. Small delays in the onset of the unit digit would be fully absorbed in this idle time window and have no impact on finger movement, and larger delays would be partially absorbed. The duration of the idle time window in the units processing pathway can be estimated as the smallest SOA that would create a delay in the unit quantification and consequently in b[U]. We estimated this value as 35 ms, the SOA for which the delay-per-SOA regression predicts delay = 0 (Fig. 4.3). This pattern of results – b[U] delay that increases linearly with SOA, starting from a certain SOA – is hereby referred to as the "Idle Digit Latency Effect" pattern (or IDLE pattern in short).

4.3.2.4. D# block: No IDLE pattern in the unit effect

The results in the control block did not show an IDLE pattern: the b[U] delay increased more or less linearly with SOA except a slight deviation on SOA = 133 ms (Fig. 4.2d and the blue line in Fig. 4.3). The slope of the b[U] delay per SOA was 0.71 (Fig. 4.3), i.e., the SOA was not fully reflected in the finger movement. Alternatively, it is possible that the SOA was fully reflected in finger movement, but another factor partially canceled this effect – either by quickening the b[U] effect or by increasing it (in our regression analysis method, earlier/higher effects are virtually indistinguishable). We propose that this other factor is errors in binding the digits to their decimal roles, as explained next.

4.3.2.5. Errors in binding digits to decimal roles

If a digit is bound to an incorrect decimal role, this may appear in our task as a bias in the digit effect on finger movement: if a unit digit is bound to the decade role, it would be processed with 10 times its real quantity. Conversely, if a decade is bound to the unit role, it would be processed with ⅒ times its quantity. Extending the period during which the decade digit appeared alone on screen may increase the likelihood that the decade digit would be incorrectly bound to the units decimal role. This should be the case when increasing the SOA in the D# block (but not in the D0 block). We used two methods to examine this prediction.

The first method was based on the regressions. If longer SOA means more binding errors, and binding errors result in a lower decade effect b[D], then b[D] should continuously decrease with SOA in the D# block. Assuming that many binding errors are transient and are eventually corrected, this SOA-b[D] effect should be observed in a relatively early time window. This was indeed the case: we analyzed b[D] by SOA in each time point using repeated measures ANOVA, with b[D] as the dependent variable, SOA as a numeric within-subject factor, and the participant as the random factor. A continuous decrease of b[D] by SOA in an early time window was observed in the D# block, during b[D] buildup (400-750 ms, $F(1,27) > 5.64$, $p < .03$, $.17 < \eta^2_P < .65$, $.02 < \eta^2_G \leq .10$; Fig. 4.2f). This pattern was not found in the D0 block in any time point (Fig. 4.2e, $F(1,27) < 2.3$, $p > .14$; Fig. 4.2e).

Another method to assess binding errors is based on identifying potential changes of mind. We reasoned that the finger direction is dominated by the decade digit, so a transient binding error may appear as a change in finger direction, because the finger would first aim according to the unit digit, erroneously perceived as the decade, and then correct its direction to the decade digit. An experimental condition with more binding errors would therefore show more within-trial direction changes. We indexed these direction changes as the number of left-right accelerations per trial (see Methods). To maximize sensitivity, we analyzed only trials with large gap between the two digits ($|D\text{-}U| \geq 5$). We predicted that larger SOA's would yield more within-trial accelerations. This prediction was confirmed: the number of acceleration bursts (#Acc) increased linearly with SOA (repeated measures ANOVA with SOA as a within-participant numeric factor, $F(1,27) = 7.3$, $p = .01$, $\eta^2_p = 0.21$, $\eta^2_G = 0.01$). Moreover, in each SOA except 100 ms, #Acc was larger than in SOA=0: paired t-test, $t(27) \geq 2.4$, one-tailed $p \leq .01$, $0.45 <$ Cohen's $d < 0.65$ (in SOA=100: $t(27) = 1.25$, one-tailed $p = .11$). Neither these patterns was observed in the D0 block, $F < 1$ and $t(27) \leq 0.8$.

These results indicate that when the placeholder character was '#', the decade digit was sometimes bound to the unit role, and that longer SOAs increased the probability of such an erroneous binding. Using the '0' character as placeholder largely eliminated these binding errors.

4.3.3. Discussion of Experiment 4.1

Experiment 4.1 presented two-digit numbers in which the unit digit was delayed by a variable amount. In the critical block (D0), where the transient stimulus was always a valid two-digit number, a clear IDLE pattern was observed: a short delay in the unit digit onset (33 ms) did not delay the unit effect on finger movement, whereas longer delays caused a linearly-increasing delay in the unit effect. This pattern suggests the existence of an idle time period in the units processing pathway, during which the unit digit is not yet needed, so its absence has no impact on the number-to-quantity conversion process. Small delays in the onset of the unit digit ($SOA \leq T_{idle}$, where T_{idle} denotes the idle time window duration) are fully absorbed in this idle time window, so they have no effect on the finger movement. Larger delays ($SOA > T_{idle}$) are partially absorbed, so the unit effect on finger movement is delayed by $SOA - T_{idle}$. The results of Experiment 4.1 suggest that $T_{idle} \approx 35$ ms.

This idle time window resembles the classical psychological refactory period effect (PRP) (Pashler, 1984, 1994; Sigman & Dehaene, 2005), in which the processing of task A is delayed until the processing of a simultaneous task B is completed. In our case, the unit processing may wait for the completion of a process triggered by the decade digit: the process is a bottleneck, and until completed, the unit processing cannot continue and must wait, hence the idle time window. While PRP effects typically suggest serial processing, note that our findings are also quite unlike the classical PRP effects in a specific respect: here, in the baseline condition

(SOA=0) where all information appears simultaneously, the decade and unit effects did not have a sequential impact on finger movement, but a parallel impact. We revisit this issue in Experiment 4.3.

Another finding was that the decade effect b[D] decreased with SOA in the D# block. This pattern suggests that the decades and units were sometimes confused, so that decades were processed as units (and perhaps also vice versa). One reason for this to occur could be that the digits were sometimes bound to an incorrect decimal role. The likelihood of these binding errors was increased by displaying the decade digit alone on screen for increasing durations, and they were nearly eliminated when only valid two-digit numbers were presented (as in the D0 block).

4.4. Experiment 4.2: Delaying decades or units

A possible concern about Experiment 4.1 is that it does not reflect the normal functioning of the cognitive system: the IDLE pattern may result from the experimental design, in which the unit digit was often delayed but the decade digit was never delayed. This design could have encouraged serial processing of the decade digit followed by the unit digit. Thus, the observed idle time window in the units processing pathway might be an artifact of the design. To control for such artifacts, Experiment 4.2 delayed either the decades or units with equal probabilities. This symmetric design also allowed examining whether delaying the decade digit onset would delay its effect on finger movement, similarly to the corresponding effect we observed for units.

4.4.1. Method

The participants were 20 adults, aged 26;9 ± 4;5. The design was as in Experiment 4.1, but with different onset times of the digits: each digit could appear either at t=0 or at t=100 ms (2x2 mixed design: a no-delay condition, where both digits appeared at t=0, delay decade, delay unit, and delay both digits). Tach target number between 10 and 90 appeared twice per condition (648 trials). Like in Experiment 4.1, both X characters changed when the finger started moving (t = 0). In the no-delay condition, they changed immediately to the target (Fig. 4.4a). In the delay-decade and delay-unit conditions, at t=0 only the decade or the unit digits appeared; the other digit changed at t=0 into a lowercase ‘x’ placeholder, and after 100 ms to the target. In the delay-both condition, the fixation changed to ‘xx’ at t=0, and after 100 ms to the target. The ‘0’ placeholder was not used here because, unlike Experiment 4.1, it would have created an invalid two-digit number when the unit digit was displayed first.

The trajectory data was analyzed per condition using the two-stage regression analysis described in methods, with the SRP predictor modified to match the 0-100 number line length.

4.4.2. Results

Trial-level measures and the general pattern of trajectory effects are listed in Table 4.1. In the no-delay condition (Fig. 4.4b), the decade and unit effects were similar but there was a small but significant over-weighting of b[U] relative to b[D] in a transient time window – again imputable to erroneous binding of digits to decimal roles.

To examine whether the decade quantification depends on unit quantification and vice versa, the regression coefficients of each digit were compared between the four experimental conditions (Fig. 4.4c-d). We hereby describe the results with respect to each digit.

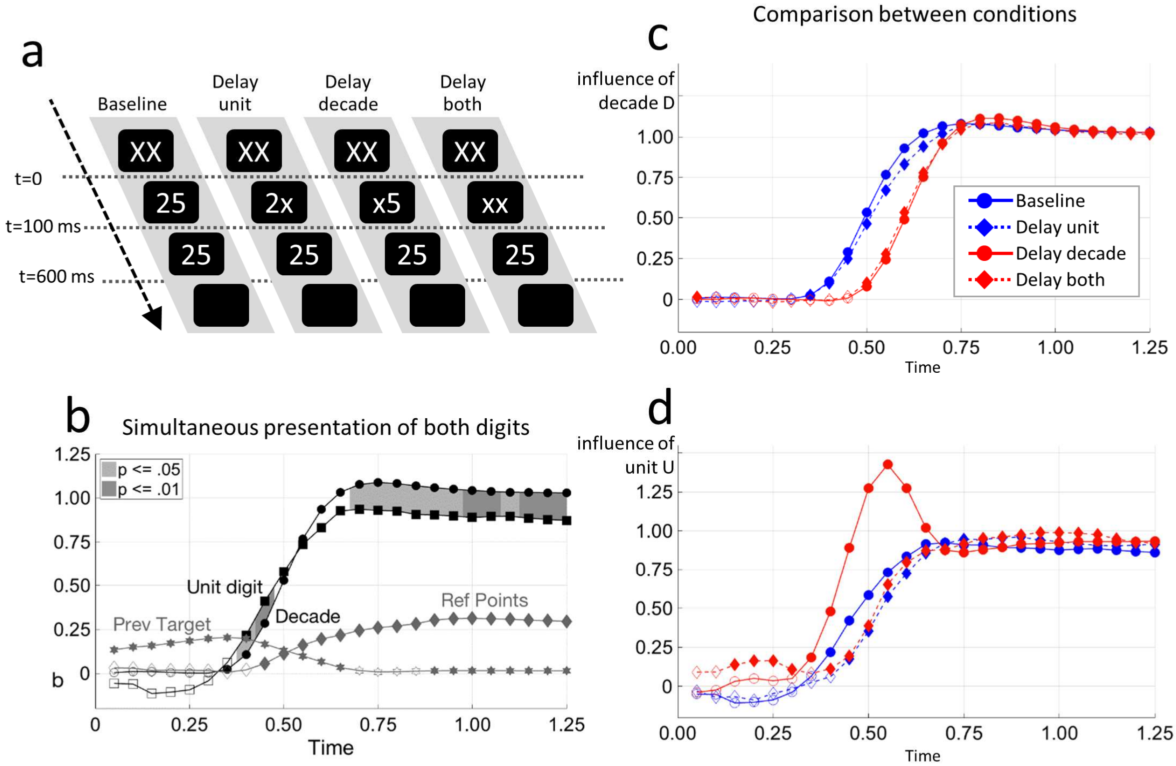

Fig. 4.4. Experiment 4.2. (a) Task design: 4 mixed conditions. The fixation was 'XX'. At t=0, each 'X' changed either immediately to the target digit, or to a lowercase 'x' and after 100 ms to the target digit. (b) When neither the decade digit nor the unit digit are delayed, their effects build up together and in almost 10:1 ratio, with slight over-weighting of the unit digit. (c) The decade effect depended solely on the decade digit onset time, and was almost unaffected by delaying the unit digit. (d) Delaying the unit onset by 100 ms (no-delay vs. delay-unit) delayed its effect by only ~65 ms (IDLE pattern). Presenting the unit digit before the decade digit (delay-decade) resulted in an exaggerated unit effect.

4.4.2.1. Decade quantification

If the decade quantity is processed independently of the unit quantity, its effect on finger movement should depend solely on the decade onset time. This was indeed the case (Fig. 4.4c). Delaying the decade digit onset by 100 ms caused a significant delay of about 100 ms in b[D] (no-delay vs. delay-decade and delay-unit vs. delay-both in Table 4.2). Conversely, delaying the unit digit onset by 100 ms only had a minor effect on the decade effect (no-delay vs. delay-unit and delay-decade vs. delay-both in Table 4.2). Thus, the decade digit was quantified and caused a corresponding finger movement as soon as it appeared, almost independently of the unit onset time.

Table 4.2. Comparison of regressions effects between condition pairs in Experiment 4.2. The decade effect was coupled with the decade digit onset time, and was hardly affected by the unit onset. The unit effect showed the IDLE pattern (effect delayed by less than the lag in the digit onset).

Comparison[a]	Delay between conditions (ms)	Significant differences between regression lines			
		When (ms)	t(19)	1-tail *p*	Cohen's d
b[D]					
none vs. decade	104	350	1.96	.03	.02
		400-700	> 2.9	< .004	0.67 - 3.06
unit vs. both	80	350-700	≥ 2.5	≤ .01	0.57 - 2.46
none vs. unit	18	450-700	≥ 2.1	< .03	0.47 - 1.48
decade vs. both	-8				
none vs. both	95	350-700	> 2.6	< .01	0.01 - 0.33
b[U]					
none vs. unit	65	400-550	≥ 1.9	< .04	0.43 - 0.96
none vs. both	50	450-500	≥ 2.3	< .02	0.04 - 0.21

[a] Condition names: "none" = both digits appeared at t=0; "decade", "unit", and "both" denote delay of the corresponding digit/s.

4.4.2.2. Unit quantification

Experiment 4.1 showed an IDLE pattern in the unit effect following a delay in the unit digit onset: 35 ms of the delay were absorbed in a putative idle time window, and only additional delay was observed in finger movement. This pattern was replicated here: comparing no-delay and delay-unit (blue lines in Fig. 4.4d) showed that delaying the unit onset by 100 ms delayed its effect b[U] on finger movement, and the size of this delay (65 ms, Table 4.2) was smaller than the visual stimulus delay by 35 ms – an exact replication of the two equivalent conditions in Experiment 4.1 (SOA = 0, 100) and of the estimated idle time window duration (35 ms).

Presenting the unit digit before the decade digit resulted in an unexpected pattern of results: extremely high b[U] values. The b[U] difference between no-delay and delay-decade cannot be explained as a mere delay – the peak b[U] value was higher in delay-decade (peak b[U] = 1.37 at 550 ms) than in no-delay (peak b[U] = 0.93 at 700 ms, paired $t(19) = 2.35$, two-tailed $p = .03$, Cohen's $d = 1.79$). Namely, what we see in the delay-decade condition is not a *delay* of the unit effect but an *exaggerated increase* in the unit effect. A similar but smaller effect was observed when both digits were delayed by 100 ms: the unit effect in the delay-both condition was not delayed by 100 ms compared to no-delay as expected, but only by 50 ms. This lower-than-expected delay can be interpreted as an exaggerated b[U] in the delay-both condition, which appears in the delay analysis as an earlier b[U] effect (because an earlier and a higher regression effect are indistinguishable in our regression analyses). The lower-than-expected b[U] delay cannot be interpreted as faster-than-expected processing of the full two-digit number in the delay-both condition, because the b[D] delay was ~100 ms, as expected.

We explain the exaggerated b[U] effect in the delay-decade and delay-both conditions is in terms of digit binding errors: presumably, delaying the decade digit induced errors in binding digit to decimal roles, thereby amplifying the weight of the unit digit. The effect was the strongest when only the decade digit was delayed, i.e., when the unit digit transiently appeared alone on screen, which may have facilitated processing it as a decade digit.

4.4.3. Discussion of Experiment 4.2

The main findings of Experiment 4.1 were replicated: first, when the two digits of a two-digit number were presented simultaneously, they influenced finger motion in a near-synchronous manner, with some overweighting of the unit digit relative to the expected 10:1 ratio. Second, the effect of the decade digit depended only on its presentation time, irrespectively of when the unit digit was displayed. Third, delaying the unit onset by 100 ms delayed its effect on finger movement by only 65 ms, presumably because the unit delay was partially absorbed in a 35 ms idle time window in the units processing pathway.

Presenting the unit digit before the decade digit resulted in a large increase in the unit effect on finger movement. We interpret this b[U] increase as resulting from errors in binding the digits to decimal roles: presumably, when the unit digit appeared first, it was more often processed as if it was a decade digit and quantified 10 times its real value, causing the exaggerated unit effect on finger movement. A similar b[U] increase effect, but smaller, was observed when both digits were delayed, suggesting that this condition too induced some binding errors. The decade-unit confusions may have been symmetric, i.e., it is possible that the decade digit was sometimes processed as a unit digit (indeed, in Experiment 4.1 we observed binding errors via the decade effect). However, the effect of binding errors on the decade digit is harder to measure, because it is 10 times smaller than their effect on the unit digit.

4.5. Experiment 4.3: Three-digit numbers

The IDLE pattern indicates the existence of a bottleneck process that causes an idle time window in the units processing pathway. What could this process be? The apparently simplest explanation seems to be sequential processing of the digits: if the units can only be processed after the decades, this would create an idle time window in the units processing pathway. If correct, this explanation would extend previous findings on sequential processing of digits in a multi-digit number: sequential effects were previously reported in number comparison and number reading, but only for numbers with 4 digits or more (Friedmann, Dotan, & Rahamim, 2010; Meyerhoff et al., 2012), and in one case also for 3-digit numbers (in a number comparison task, Bahnmueller, Huber, Nuerk, Göbel, & Moeller, 2015).

However, the sequential-processing view has a major flaw: it predicts that when decades and units are presented simultaneously, the decade effect on finger movement should precede the unit effect. This was not the case: in simultaneous presentation, the decade and unit effects consistently build up in parallel (Fig. 4.2a,b, Fig. 4.4b, and Chapters 2, 3). To explain this parallel pattern, the sequential-processing view could assume the existence of another factor, which masked the decade-unit delay: binding errors, which increase the unit effect and decrease the decade effect, could cause a virtual negative delay between the decade and unit effects. By pure coincidence, this virtual negative delay could happen to be of the same size as the decade-unit delay, in which case the two effects would exactly cancel each other.

Such a coincidence, replicated in so many experiments, seems unlikely. Nevertheless, to examine this further, we ran the task with 3-digit numbers, with simultaneous presentation of all digits. The coincidence interpretation, which seemed unlikely for 2-digit numbers, seems even more unlikely for 3-digit numbers. Thus, if the parallel effects of decades and units in the simultaneous-presentation condition were coincidental, we can expect the coincidence to cease here: the regression effects of the hundreds, decades, and units should not overlap each other.

4.5.1. Method

The participants were 20 right-handed adults, aged 25;10 ± 4;1. The task design was like in the previous experiments, but here all digits always appeared simultaneously at t=0. The number line ranged from 0 to 400, and each number between 0 and 400 was presented once.

We applied the same regression analysis method as above, this time with 5 predictors: the unit digit U, the decades D (0, 10, 20, …), the hundreds H (0, 100, 200, 300 or 400), the previous target N-1, and the spatial-reference-point-based bias function SRP, modified to match the 0-400 number line length.

4.5.2. Results and Discussion

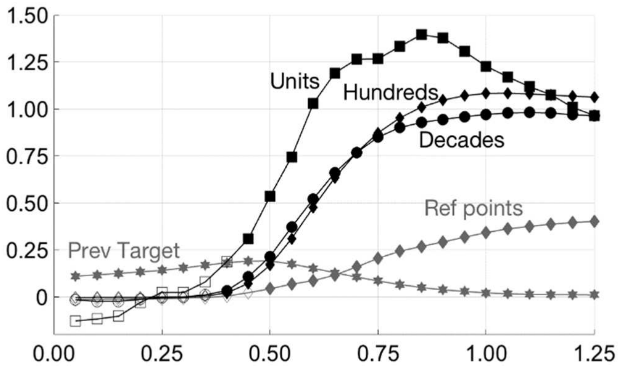

Fig. 4.5. Regression results of Experiment 4.3 (3-digit numbers, simultaneous presentation of all digits). The hundred and decade effects build up in almost exact parallel, replicating the findings of two-digit numbers. The 3-digit design implies that unit-hundred binding errors would have a strong effect on the unit digit (1:100), and indeed the unit effect is over-weighted here more than in the two-digit case.

The trial-level measures and the general pattern of trajectory effects (Table 4.1) resembled the previous experiments, but the finger was slower here by about 20% than in previous experiments, suggesting that the task is more difficult for 3-digit numbers than for 2-digit numbers. Crucially, the effects of the two leftmost digits (hundred and decade) built up in almost exact parallel, with a transient small overweighting of the decade digit (Fig. 4.5), replicating the decade-unit parallel buildup observed in previous experiments. The unit effect was higher than the decade and hundred effects during a transient time window, suggesting again a certain degree of binding errors. The effect of binding errors on the unit effect was unsurprisingly much larger here than in experiments with two-digit numbers, because each unit-hundred binding error results in processing the unit digit with 100 times its real weight.

The almost perfect alignment of the hundred and decade effects is hard to reconcile with the sequential-processing view. This view must now assume that the virtual delay caused by binding errors was exactly identical not only with the real decade-unit delay in the previous experiments but also with the delay between the hundred and decade digits in the present experiment. Such coincidence seems hardly plausible. The results therefore strongly suggest that a purely sequential processing of the digits is not the best explanation. Rather, the quantities provided by the different digits appear to feed finger movement in parallel, yet – as shown in the previous experiments – with a small idle time during which the units can be delayed without any consequence on behavior. In the General Discussion below, we present a model that formalizes those hypotheses and can account for the results.

4.6. Discussion of Chapter 4

4.6.1. The properties of number-to-quantity conversion processes

This chapter examined how the decade and unit digits of a two-digit number interact when transforming the number to quantity. We used a number-to-position mapping task, which forces the participants to convert the digit string to quantity, and we monitored the finger trajectories to get an insight into the temporal dynamics of this quantification process.

The main findings, which any theory of number comprehension should account for, can be summarized as follows. First, *when decades and units were simultaneously presented, their effects on finger movement built up in parallel*, with a small over-weighting of the unit digit relative to the expected decade-unit ratio of 10:1. Second, *the timing of the decade effect depended only on the decade digit*: delaying the presentation of the decade digit by Δt induced an identical delay Δt in the effect of the decade quantity on finger movement. Delaying the presentation of the unit digit hardly modified the decade effect on finger movement. Third, unlike the decade effect, *the timing of the unit effect depended on both digits*. Delaying the unit digit by up to about 35 ms had almost no effect on finger movement. Delaying the unit digit by $\Delta t > 35$ ms induced a delay in the effect of the unit quantity on finger movement, and the size of this delay was smaller than Δt by about 35 ms. Last, *the relative weighting of decades and units was sometimes biased*. Imposing a decade-unit onset discrepancy resulted in an amplification of the amplitude of the unit digit and, correspondingly, a reduction in the amplitude of the decade digit. These weighting errors were largest in Experiment 4.2 and in the D# block in Experiment 4.1, where the two-digit stimulus was preceded by a transient single-digit number.

In turn, these findings afford several conclusions:

1. **Immediate processing of the decade digit.** The finding that the decade effect depended solely on the decade digit onset time indicates that the decade digit is processed as soon as it appears, without having to wait for the unit digits. This means that the quantity system, and the decision stage following it, operate as a continuous integrator of the information carried by the various digits. In particular, when the decade and unit information become available at different times, the finger aiming can be initially guided by just the decade digit and be corrected later, when the units information arrives. The trajectory-tracking paradigm is therefore sensitive enough to track the time course of the processing of the two digits with a high temporal resolution. This continuous nature of movement programming, and the ability to capture it with finger tracking, is supported by several other studies, both with continuous spatial response like here (Pinheiro-Chagas et al., 2017; Chapter 5) and with binary responses (Dotan et al., 2017; Finkbeiner et al., 2014, 2008; Finkbeiner & Friedman, 2011; Freeman, Dale, & Farmer, 2011; Santens et al., 2011; Song & Nakayama, 2008a, 2008b, 2009).

   This finding clearly refutes any model that assumes a single decision point for movement – in particular the lexical model, which postulates that the entire two-digit string is recognized and mapped to a whole-number quantity, and the max model, which postulates that the finger deviates only after both digits were quantified and merged. Such models cannot account for the finding that in some conditions, the finger first moved according to the decade quantity whereas the unit effect kicked in later. This discrepancy between the decade and unit effects implies either a continuous updating or, at least, two movement decisions, one based on the decade and another based on the full two-digit number.

2. **Idle time window in the units processing pathway.** The unit effect showed a completely different pattern: any lag of Δt in the unit digit onset time resulted in a delay of Δt - 35 ms in the unit effect, and a lag smaller than 35 ms caused no delay. This indicates that the pathway for processing the unit digit contains an idle time window of approximately 35 ms, during which the unit digit appears to be waiting for the end of a bottleneck process initiated by the decade digit. Below, we propose a model that specifies what this process might be.

   The pattern of the unit effect clearly shows that the digits are not processed fully independently and in parallel, as proposed by the parallel-decomposed model presented in the Introduction. If this were the case, the unit digit should have been processed immediately as it appeared, with no idle time window, and any delay in the unit onset would be fully reflected in its effect on finger movement.

3. **Errors in binding digits to decimal roles.** In our experiments, all stimuli were two-digit numbers. When the transient stimulus was a single-digit number, we observed a bias in the weights of the digits – decades were underweighted and units were overweighed – and this bias increased as a function of the duration of displaying the single-digit transient stimulus. Our interpretation of this pattern is that the decade and unit digits are sometimes bound to incorrect decimal roles. Thus, the decade digit is sometimes processed as units, with $1/10$ its weight, and correspondingly the unit digit is sometimes processed as decades, with 10 times its weight. These binding errors are facilitated by displaying a digit alone on screen rather than as part of a two-digit number.

4. **Parallel decade and unit processing in simultaneous presentation.** When the digits were presented simultaneously, they contributed to the quantity in parallel and in 10:1 ratio. This finding is extremely robust – it was observed in several experiments with two-digit numbers (here and in the previous chapters), and even between the hundred and decade digits of three-digit numbers (Experiment 4.3). The unit digit was typically slightly overweighed relatively to the decade digit, suggesting perhaps that some degree of binding errors existed even when the digits were displayed simultaneously.

   The existence of binding errors could have conceivably supported the possibility that decades and units are actually processed serially. Serial processing should result in serial effects on finger movement – first the decade effect, then the unit effect – but this serial pattern could be masked by binding errors: such errors increase the unit effect relatively to the decade effect, making the unit effect appear earlier (because in our regression analyses, a larger effect is almost indistinguishable from an earlier effect). However, our data is not in good agreement with this serial-processing model. Complete masking of the serial pattern would require that the virtual delay, induced by binding errors, would have exactly the same duration as the real delay, induced by serial processing. To accommodate the robust finding of parallel effects of the different digits, a serial-processing model would have to explain why the real and virtual delays consistently have identical durations, in two-digit and three-digit numbers. At present, we see no such explanation.

This remarkable degree of coordination between decades and units is a robust finding in expert readers, yet apparently it is not a trivial ability for the quantification system. Children, even not very young ones, show a completely different pattern. In the previous chapter, we described the performance of 4^{th} grade children in the number-to-position task with simultaneous presentation of the two digits (Section 3.5.1). We observed a discrepancy of 50-100 ms between the decade and unit effects on finger movement, suggesting that the children were processing the digits serially, first the decades, then the units. Thus, even after learning to read multi-digit numbers, the cognitive system may require several more years to develop its full ability to process the digits in parallel and weight them correctly relative to each other. Furthermore, this ability can be impaired following a brain damage, making an adult exhibit the serial pattern typical to children (Chapter 9).

Methodologically, the present data joins several studies in showing the ability of finger tracking to dissect the temporal aspects of cognitive processes, in particular when using manipulations that change the stimulus during a trial (Dotan et al., 2017). Finger tracking therefore has the potential of becoming a useful and simple behavioral tool to dissect cognitive processes temporally, and may even provide sufficient accuracy to serve as a diagnostic tool for single individuals (Chapter 9). At the same time, the present study also demonstrates a limitation of this paradigm: it confounds time and space, such that an earlier effect on the finger movement is almost indistinguishable from a larger effect on the finger movement. In our task, this limitation confounded the effects of delaying a digit and of binding errors. In the future, newer analysis methods may perhaps be able to address this limitation (e.g., single-trial analyses, Dotan et al., 2017).

4.6.2. A revised model of number-to-quantity conversion

Our findings refute all the number-to-quantity conversion models presented in the Introduction: the lexical model, the parallel-decomposed model, and the max model. To account for the data, we propose the following hypothetical model of number processing (Fig. 4.6). This is an enhancement of the model we proposed in Chapter 3, which referred to several stages in the number-to-position task (symbol identification of the digits, quantification, decision, and pointing), and proposed a detailed Bayesian model of the decision stage. The present data suggest a more refined model of the first two stages, identification and quantification, during which subjects must convert the digit shapes on screen into a representation of the corresponding quantity.

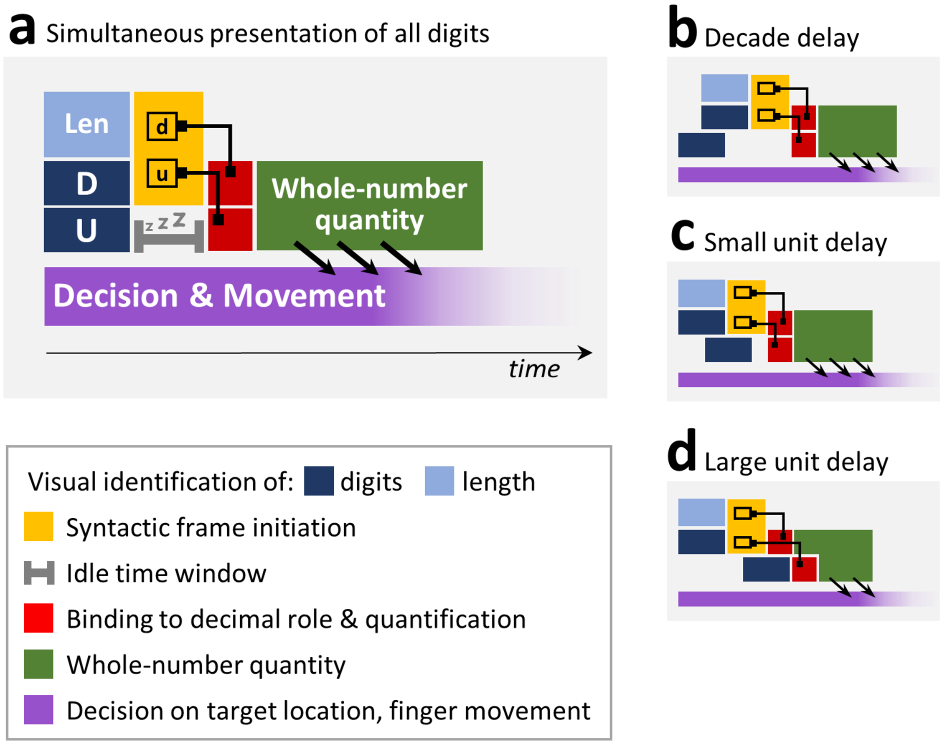

Fig. 4.6. Proposed model for two-digit number comprehension. The appearance of the leftmost digit triggers the identification of number length (light blue) and the subsequent formation of a *syntactic frame* (yellow). In parallel, the two digits are visually identified (dark blue). The formation of the syntactic frame imposes a bottleneck: while the frame is being created, the processing of digits is idle (grey). This idle time window lasts about 35 ms. Once the frame is ready, each digit can be assigned to a decimal role, quantified accordingly (red), and merged into a whole-number quantity representation (green), which continuously feeds the decision process that drives the finger movement (purple). Each panel illustrates how the predicted delays arise in each stimulus condition: (a) When the digits are presented simultaneously, they simultaneously integrate into the whole-number representation and affect the finger movement. (b) Delaying the decade digit onset by Δt delays the syntactic frame initiation, and consequently should delay the quantification of both digits by the same Δt. In our experiments, the unit delay in this case was masked by a large overarching effect of unit amplification, which we attribute to an erroneous assignment of digits to their decimal roles. (c) Short delays (Δt < 35 ms) in the onset of the unit digit of a two-digit number do not affect the syntactic frame creation, so they are fully absorbed by the idle time window and have no effect on finger movement. (d) Longer delays in the unit onset are partially absorbed by the idle time window, so the unit effect is delayed but by less than Δt.

A two-digit number must be initially processed as two independent shapes, because the two digits are projected to distinct retinotopic locations. At some point, however, the system must stop processing the digits independently and bind them to distinct roles, one being the leftmost decade digit (worth 10) and the other being the rightmost unit digit (worth only 1). This process is not trivial because the absolute location of the digits may vary relative to the fixation point, and indeed the order of digits is encoded by a dedicated process (Friedmann et al., 2010; Chapter 7; for an analogous distinction in word reading, between spatial location of letters and their within-word positions, see Dehaene et al., 2004). The digit identities, their relative order, and the number of digits are sufficient information to assign each digit its decimal role as decades or units. We propose that this is done by binding the digits to a *syntactic frame* of the multi-digit number. The term "syntactic frame" is a concept borrowed from models of digit-to-verbal number transcoding, where it was found indispensable to account for the errors made by various patients with brain lesions (Cohen & Dehaene, 1991; McCloskey, 1992; McCloskey et al., 1986). Here, it refers to a mental representation of the structure of the multi-digit number that takes into account its overall length, but not yet the specific digit values. The syntactic frame of a single-digit number has one placeholder 'u', for the unit quantity; the syntactic frame of two-digit numbers has two placeholders, 'd' for the decade quantity and 'u' for the unit quantity; and so on. After the syntactic frame was created (Fig. 4.6a, yellow), the digits, which were visually identified (dark blue), are assigned decimal roles by binding each digit to the corresponding placeholder in the syntactic frame, and quantifying the digit according to this decimal role (red): the value of the 'd' digit is interpreted as a decade, i.e., multiplied by 10 compared to the value of the 'u' digit. The per-digit quantities are combined into a whole-number quantity (green) and fed to the decision process that determines the target location and eventually drives the finger movement (purple).

The selection of an appropriate syntactic frame requires knowing how many digits the number has. The model postulates that this information is extracted by a dedicated number-length encoder process (Fig. 4.6a, light blue color), which is a part of the visual analyzer of numbers. The number-length detector is thought to be one of several processes that encode the number's decimal structure even before specific digits were identified. When reading numbers aloud, this quick identification of the number structure allows the verbal system to prepare itself for saying a verbal number with this structure (Cohen & Dehaene, 1991; Chapter 7). The same idea may apply here: quick identification of the number structure (its length) is needed to initiate the appropriate quantity syntactic frame as fast as possible so not to delay the subsequent processing stages.

Our model assumes that decades and units can be processed asynchronously and independently at all stages, except a single bottleneck point – the creation of the syntactic frame.

Crucially, the identification of number length, and consequently the initiation of the syntactic frame, is triggered by the decade digit in the two-digit number (or, more generally, the leftmost digit in numbers of arbitrary length). This explains why any delay in the decade digit impacts on the syntactic frame initiation and consequently on the binding and quantification of both digits (Fig. 4.6b). Assuming that the syntactic frame takes about 35 ms to form, the model can also account for the IDLE pattern in two-digit numbers: a short delay (< 35 ms) in the visual presentation of the unit digit has no effect (Fig. 4.6c) because this delay is entirely absorbed by the contemporaneous formation of the syntactic frame. As a result, the timing of the unit quantification process remains unchanged. Only a delay of $\Delta t > 35$ ms in the unit digit onset is large enough to make the visual identification process end after the syntactic frame initiation, thereby delaying the unit quantification by $\Delta t - 35$ ms (Fig. 4.6d). Importantly, the number length detector does not rely on the unit digit. However, it may rely on identifying *any* digit in the unit position, so the process may break down, at least occasionally, if the unit position is occupied by a non-digit placeholder. This may explain why the IDLE pattern was not observed in the D# condition in Experiment 4.1.

Our model may settle a long-lasting debate about whether the quantity representation of two-digit numbers is holistic or decomposed. Some studies argued that a holistic quantity of the two-digit number is created (Dehaene et al., 1990; Fitousi & Algom, 2006; Reynvoet & Brysbaert, 1999; and Chapter 2), whereas others argued for decomposed single-digit quantities (Meyerhoff et al., 2012; Moeller, Fischer, et al., 2009; Nuerk & Willmes, 2005). Our model reconciles these views: each digit is first processed independently and quantified separately, but the per-digit quantities are immediately merged to a whole-number quantity representation.

4.6.3. Conclusion

Our data suggests that converting multi-digit numbers to quantity involves a mixture of serial and parallel processing. On one hand, there was a bottleneck process that imposes serial processing. On the other hand, all other stages seemed to process the digits in a parallel and asynchronous manner, both before and after the bottleneck – i.e., not only in the visual recognition of digits, but even in the deeper processing stages of conversion to quantity. This remarkable degree of parallel processing is not trivial, and indeed it apparently takes years to develop (Section 3.5.1). Future studies may investigate its stages of development, as well as whether certain kinds of input are needed to facilitate this development.

5. Tracking the mental updating of Bayesian priors°

Abstract. This chapter focuses on the decision-making processes involved in a number-to-position mapping task. Bayesian theories of decision making predict that optimal decisions are reached by starting from a prior probability distribution, acquired from previous trials, and then updating it according to the specific evidence received on the current trial. Here, using our trajectory-tracked number-to-position task, we provide direct behavioral evidence that human decisions unfold in this order. We manipulated either the prior, via the distribution of target numbers in an experimental block, or the initial finger direction. In both cases, the finger first pointed in the instructed direction, then to the inferred prior, and finally to the trial-specific target. During the intermediate stage, pointing to the prior was observed even when it implied transiently deviating away from the target. This pattern fits a Bayesian model where decisions are initially driven by prior knowledge and only then by the trial-specific evidence.

5.1. Introduction

Hundreds of times a day we need to choose one action among several possible. The last two decades have seen major advances in our understanding of the cognitive and neural processes involved in simple decision making, and in our ability to account for simple decisions with formal mathematical models (Glimcher, 2003; Gold & Shadlen, 2007; Kiani, Corthell, & Shadlen, 2014; Ratcliff & McKoon, 2007). Such models consider that humans decisions are close to the normative optimum provided by Bayesian reasoning. They assume that the subject maintains a likelihood for each possible decision outcome (Barthelmé & Mamassian, 2010; Knill & Pouget, 2004; Smith & Ratcliff, 2004), and continuously updates these likelihoods by accumulating evidence arising from the stimulus, until a decision criterion is met (Gold & Shadlen, 2002; Kiani, Hanks, & Shadlen, 2008; Lo & Wang, 2006; Ratcliff & Rouder, 1998; Roitman & Shadlen, 2002). Neurophysiological recordings strongly support this model of decision making (Gold & Shadlen, 2001; Kopp, 2006; Zylberberg, Fetsch, & Shadlen, 2016). The activity of neurons in prefrontal parietal cortex appears to build over time, with a slope monotonically related to the available evidence, up to a threshold level, as expected under the hypothesis that they encode the likelihood or likelihood ratios of the decision outcomes (Kiani & Shadlen, 2009; O'Connell, Dockree, & Kelly, 2012; Roitman & Shadlen, 2002). Furthermore, stronger activity, presumably reflecting sharper distribution of likelihoods, results in higher confidence in the decision (Kiani & Shadlen, 2009).

° This chapter has supplementary material in Appendix B.

One of the pillars of the Bayesian framework, and the topic of the present study, is the prior. Even before any evidence is obtained, Bayesian decision theory dictates that we should entertain a prior distribution of likelihoods, based on our past experience and expectations. Evidence accumulated about the specific situation at hand gradually modifies the prior, and it becomes a posterior distribution likelihoods. This formulation predicts that in a task with continuous response, the behavior within a trial should be initially governed by the prior and only later by the posterior.

In line with the Bayesian notion of a prior that is based on past experience and expectations, pointing studies showed that the location where the participant pointed at was affected by the distribution of stimuli in the experiment (Cicchini et al., 2014; Kording & Wolpert, 2004). Decisions are also affected by the recently-presented stimulus (Abrahamyan, Silva, Dakin, Carandini, & Gardner, 2016; Cicchini et al., 2014). However, the use of end-of-trial measures limited the ability of these experiments to resolve the inner dynamics of a trial. The present study specifically explored this point: it examined whether, within a trial, the effect of prior occurs before the effect of the trial-specific target (the posterior). Discovering such temporal dynamics would provide further support to the Bayesian interpretation of decision tasks.

We examined this prediction with the number-to-position mapping task, which is a simple decision task. As in the previous chapters, we recorded the finger pointing throughout the trial and examined the factors that affected the pointing in different temporal stages.

In Chapter 3, we proposed a full Bayesian account of the number-to-position task. This model assumes that the participants maintain the likelihood for each possible target location. In each trial, the likelihoods are initialized to a prior inferred from the distribution of previous target numbers in the experiment. Then, these likelihoods are gradually updated according to the present target, with the finger movement reflecting this gradual update (Section 3.7.2; see also Cicchini et al., 2014). In the experiments described in the previous chapters, where the distribution of stimuli was flat, we observed a pattern consistent with this model: the finger initially moved towards the middle of the number line, which is the expected target location given a flat prior. During this stage, the finger was additionally affected by the numbers presented on previous trials, with exponentially decreasing influence of trials N-1, N-2, N-3, etc. Mathematical modeling showed that these findings were in prefect agreement with the notion of a Bayesian prior that gets updated by recent information and gradually forgets older information (Abrahamyan et al., 2016; Chapter 3).

Nevertheless, pointing towards the middle cannot be taken as an unambiguous marker of prior-driven behavior. It could be simply attributed to the task's motor instructions, which prohibited pointing sideways at the beginning of a trial. To address this confound, we designed two experiments that dissociate between the finger direction implied by the prior distribution of numbers, and the initial pointing direction given by the instructions. In Experiment 5.1, the instruction required initial pointing to the middle of the line, but the distribution of target numbers was biased towards small, average, or large numbers in different blocks, therefore inducing a prior biased to the left, the middle, or the right of the number line. Conversely, in Experiment 5.2 the prior was constant (a fixed, flat distribution of target numbers), but the initial pointing direction was manipulated by explicitly instructing participants to start each trial by pointing left, middle or right. Bayesian theory predicts that in both experiments, finger movement would be governed first by the instructed pointing direction, then by the prior, and finally by the trial's target number. Counterintuitively, this model also predicts that on trials where the instructed direction pointed straight at the target, but the prior differed, subjects would transiently deviate away from the target in order to accommodate the prior.

5.2. Method

5.2.1. Participants

Participants were right-handed adults with no reported cognitive disorders, with Hebrew as their native tongue, and were compensated for participation. There were 18 participants in Experiment 1 (mean age = 25;1, SD = 2;4) and 24 participants in Experiment 2 (mean age = 26;3, SD = 4;1).

5.2.2. Procedure

We used the number-to-position task described in Chapter 2, with a 0-100 number line. Each experiment included 3 blocks, administered in random order, with 202 trials per block. In Experiment 5.1, unknown to the participants, each condition (block) had different target distribution (Fig. 5.1a): flat (equal probability for all targets); biased to large numbers (each target appeared $\left\lceil 3\frac{target}{100}\right\rceil$ times, and 0 appears once), or biased to small numbers (a mirror symmetry of the large-number condition distribution). Each condition was preceded by 21 calibration trials, in which the target numbers were selected (with flat distribution) from a biased range – 0-42 (in the small-bias block), 58-100 (in the large-bias block), or from the whole range

0-100 (in the flat block). Calibration trials were aimed to reset the prior bias according to the experimental condition and to override any potential prior from previous blocks. They were administered as part of the block but not analyzed.

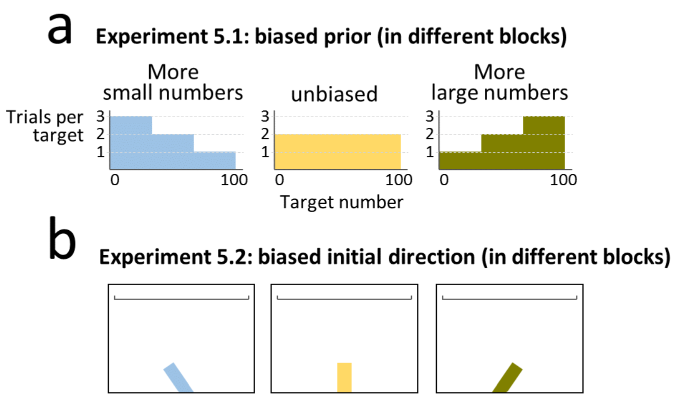

Fig. 5.1. Logic of the experiments dissociating Bayesian prior from initial finger direction. (a) Experiment 5.1 manipulated the Bayesian prior by setting different distribution of target numbers per block. The initial finger direction was straight up in all blocks. (b) Experiment 5.2 used a fixed prior (flat distribution of target numbers) and manipulated the finger initial direction across blocks.

In Experiment 5.2, all conditions had a flat distribution of targets, and they differed only in the finger initial direction (Fig. 5.1b): the participants were explicitly instructed to start each trial with movement towards the left end of the line (corresponding to target 0; the start rectangle was tilted 30º to the left), to the middle of the line (corresponding to 50: straight start rectangle), or its right end (corresponding to 100; start rectangle tilted 30º rightwards).

5.2.3. Data preprocessing and trajectory analysis

The data was preprocessed as described in Section 2.2. To analyze trajectories we used the two-stage regression method introduced in Chapter 2. Within each experiment, all three conditions were analyzed together. One regression was run per participant and time point in 50 ms intervals on the implied endpoints. The predictors were the target number (denoted N), the target number of the previous trial (denoted “N-1”), and the Condition. In Experiment 5.1, the Condition predictor was the average target number per block (42.5, 50, or 57.5). In Experiment 5.2, Condition was the initial direction (0, 50, or 100). In Experiment 5.1, we also included the targets of preceding trials (N-2 to N-10) as additional predictors, to account for possible correlation between them and the Condition predictor. Furthermore, in Experiment 5.1, to avoid statistical biases due to between-condition differences in the number of trials with each target number, only one trial per target was included in the regression analysis (the last occurrence of each target number).

5.3. Results

5.3.1. Experiment 5.1: Manipulating the prior

Visual inspection of the trajectories suggested that participants were sensitive to the distribution of target numbers: in the two biased-prior conditions, the trajectories were transiently biased leftwards or rightwards (Fig. 5.2a), a pattern that was observed even on single trials for some participants (Fig. 5.2b). To quantify this pattern, the trajectory data was submitted to the two-stage regression analysis described in Methods: on each time point, the implied endpoints were regressed against the Condition (defined as the average target per block), the target number, and the target of the 10 previous trials (N-1 to N-1). This analysis revealed a succession of three effects (Fig. 5.2c): in the beginning of a trial, only the constant factor had a strong effect (blue curve), indicating that the finger aimed towards a more or less constant direction – the middle of the number line. This constant factor immediately started declining and completely disappeared by 500 ms post stimulus onset. In parallel, the finger movement became governed by the Condition factor (red curve). The Condition factor peaked at 400 ms, and started declining as the finger direction became dominated by the trial-specific target number (green).

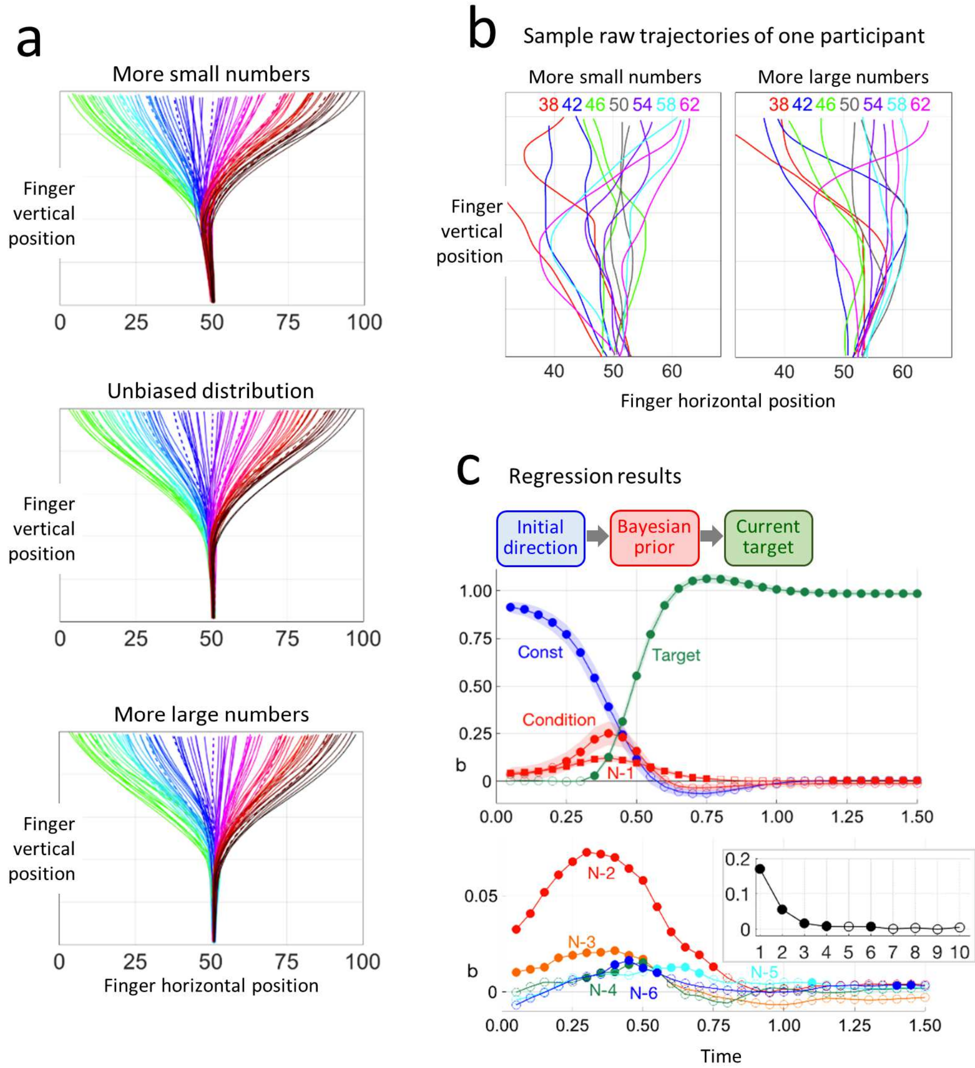

Fig. 5.2. Results of Experiment 5.1, which manipulated the distribution of target numbers across blocks. (a) Average trajectory per target number in each condition. The condition-induced bias is clearly visible in the early trajectory parts. (b) For some participants, the condition-induced bias can be observed even in single trials. (c) Results of regressing implied endpoints against the present and 10 previous target numbers and the per-condition average target. The per-participant mean b values are plotted as a function of time (full dot = t-test significantly higher than 0). Shaded areas show one standard error below/above mean. b[const] is divided by 50, so b=1.0 corresponds with the middle of the line. The colors denote how the effects can be grouped to three sequential stages: instruction-driven (blue) → distribution-driven (red) → trial-driven (green). The inset in the bottom panel shows the mean b value over the interval 0-600 ms for the predictors of the 10 recent targets.

Importantly, although the Condition predictor correlated with the target numbers (r = .32), its effect could not be reduced to an effect of the recent target numbers, because it was significant although the regression model included the 10 recent target numbers. The Condition effect therefore reflects a genuine effect of the block-specific distribution of targets – namely, an effect of prior. Parallel to the Condition effect, the finger was additionally affected by the target of the previous trial (N-1) and, to a smaller extent, by the targets of several trials farther back (bottom panel in Fig. 5.2c), in accord with the findings in Chapter 3. The Condition and recent-trials effects rose and declined in similar time windows, suggesting that they resulted from a single cognitive process – the Bayesian prior. Thus, the prior was not solely affected by the long-term distribution of targets, but also leaned towards the targets presented recently. The recent-targets effect decreased exponentially for trials farther back (Fig. 5.2c inset) – the exact pattern predicted under the hypothesis that the prior is learned from accumulation of evidence over trials, with some forgetting of older information (see Section 3.7.2).

The pattern of results agrees with the three-stage Bayesian process described in the Introduction: pointing to a default initial direction (blue); pointing according to the per-condition distribution of target numbers, i.e., according to a Bayesian prior (red); and pointing to the estimated location of the target number, i.e., according to a Bayesian posterior (green). These three stages affected the finger sequentially: the effect of each stage started declining around the same time as the next effect started ascending.

5.3.2. Experiment 5.2: manipulating the finger initial direction

In this experiment, participants were instructed for a different initial finger direction in each block. Visual inspection of the average trajectories suggested that the participants complied with this instruction (Fig. 5.3a, Fig. B.1). Strikingly, single-trial trajectories suggested that, as predicted, in an intermediate stage, participants quickly returned to pointing towards the middle of the number line, which is the direction implied by the prior in this experiment, even when this movement transiently carried the finger *away* from the target (Fig. 5.3b). For instance, when the target number was close to zero, and the instruction was to initially point towards 0, the trajectory was not an optimal straight line, but often deviated transiently towards the middle of the line.

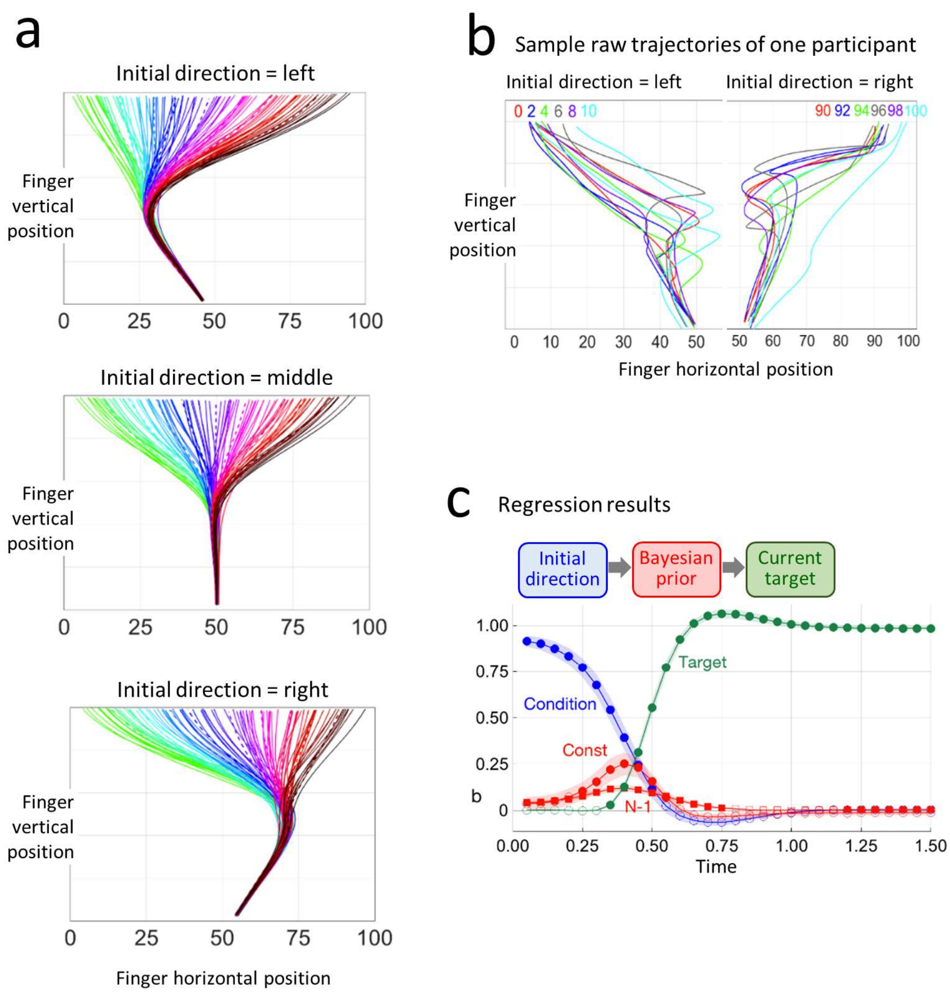

Fig. 5.3. Results of Experiment 5.2, which manipulated the finger initial direction per block. (a) Average trajectory per target number (over all participants) in each condition. Clearly, the participants complied with the instructed initial direction. (b) Single trials clearly show that some participants transiently point towards the middle of the line, even when this drove the finger away from the target number. (c) Time course of the effects (same plot type as Fig. 1c). Here the predictors were the target number of the present and previous trials (N-x) and the per-condition initial direction (0, 50, or 100). The order of effects is the same as in Experiment 5.1 (instruction → prior → target), even if here they are reflected by different predictors.

To evaluate the significance of this pattern, trajectories were submitted to a two-stage regression analysis where the implied endpoints were regressed against the Condition (the initial direction), the target number, and the target of the previous trial N-1 (see Methods). As in Experiment 5.1, three successive effects were observed (Fig. 5.3b). In the early trajectory parts (blue), the finger direction was dominated by the instructed initial direction (left/right/middle). This effect quickly declined, and completely disappeared by 550 ms. In parallel, starting from ~200-250 ms, the finger started moving towards a fixed direction (Constant effect, red color) –

presumably the middle of the line – and was additionally affected by the previous trial. As in Experiment 5.1, the two predictors of this middle stage unfolded with a similar time course, suggesting that they originated in a single cognitive process. Their peak was at 400 ms, and then they declined as the finger direction became dominated by the target number (green).

This temporal organization again suggests three consecutive stages of pointing, first according to the initial direction (blue); then according to a Bayesian prior, which in this experiment is a constant direction with an additional effect of the previous trial (red); and finally to the estimated location of the target number (green).

5.4. Discussion of Chapter 5

We start by summarizing the results. In two different experiments, the factors affecting finger movement could be separated into three successive stages, which both imply the same succession of cognitive stages and similar dynamics: (a) Initially, the finger pointed to a default direction (peak = 0 ms; end = 500 ms), whether this direction was fixed (in Experiment 5.1) or varied according to instructions (in Experiment 5.2). (b) Very quickly, the finger deviated towards the mean of the distribution of target numbers in the experimental block, as predicted by Bayesian decision making models. In Experiment 5.1, the distribution was condition-dependent, so we observed a Condition effect in the regression. In Experiment 5.2, the distribution was flat in all conditions, so the regressions showed a contribution of the constant term, corresponding to a strategy of pointing towards the midline. At this stage the finger was also influenced by targets presented in the recent past. (c) Finally, from 350 ms on, the finger started deviating towards the target number of the present trial.

Importantly, each of our experimental manipulations affected just one of these processing stages. In Experiment 5.1, manipulating the distribution of target numbers affected stage (b) but not stage (a), whereas in Experiment 5.2, manipulating the default direction affected stage (a) but not stage (b). Neither manipulation affected stage (c). This disordinal interaction indicates that the three stages of finger movement reflect three distinct stages of cognitive processing.

A striking result of Experiment 5.2 is that participants reverted to the prior even on trials when this behavior transiently led their finger away from the target number (Fig. 3b). While such behavior may seem suboptimal, Bayesian theory fully account for its occurrence and explains why pointing towards the prior is an efficient strategy *on average*: as long as information about the target is still absent, this strategy minimizes the average error or and therefore reduces the average distance to future target location. More precisely, pointing towards the mean minimizes the average quadratic error, while pointing towards the median minimizes the average absolute distance. The present data cannot distinguish between these possibilities, but subtle manipulations of the shapes of the prior distributions might be able to address this point in the future.

Interestingly, during stage (b), the finger direction was affected by both the average distribution of targets (long-term prior) and by targets presented in the recent past. Those findings indicate that the prior is partially based on a long-term mean, and partially based on an exponentially decreasing update based on the history of recent targets. During the intermediate stage (pointing by target distribution), the finger was additionally affected by several recent target numbers. The size of this recent-trials effect decreased exponentially for trials farther back, replicating our previous findings (Fig. 3.3). Its time course unfolded in almost perfect temporal alignment with the time course of the long-term prior (Condition in Experiment 5.1, Const in Experiment 5.2), thus strongly suggesting that both effects originate from the same processing stage – pointing to a Bayesian prior. Indeed, previous mathematical modeling has demonstrated how this exponential pattern can arise in a mathematical model where the prior gets partially updated by new target numbers, with older targets gradually being forgotten (Abrahamyan et al., 2016; Section 3.7.2).

Several alternative interpretations of the results can be ruled out. First, the findings cannot be explained by a model that assumes a single decision point per trial, because we clearly identified several successive direction changes, which were sometimes observed even in single trials (Fig. 5.2b, 5.3b). By the same argument, finger trajectory does not result from a single decision-to-move, but seems to be continuously fed by the progressive accumulation of evidence arising, first from the prior, then from the target. Indeed, in studies where the decision was based on sequentially presented stimuli, the finger changed its direction and speed during the trial, indicating progressive accumulation of the sequential evidence (Buc Calderon, Verguts, & Gevers, 2015; Dotan et al., 2017).

Second, the effects that precede the target number (Const, Condition, N-1) cannot be ascribed to a default behavior or a general aiming preference. Such default behavior should be observed right from the beginning of the trial, whereas in both experiments we observed some pre-target effects building up only after the trial started, and peaking hundreds of milliseconds later. Furthermore, those pre-target effects did not correspond to a fixed default behavior, but adapted flexibly to changes in long-term prior and the history of recent targets.

Third, as noted above, the perfect alignment of the middle stage effects – the target distribution effect and the recent-targets effect (red curves) – suggests that they have a single cognitive origin. Models that attributes these effects to different mechanisms – e.g., explaining the distribution effect as an overt strategy, or the recent-trial effect as a motor-level perseveration, cannot explain this parallelism.

Methodologically, the present study confirms that finger tracking can illuminate the temporal dynamics of cognitive tasks (Finkbeiner et al., 2014; Freeman et al., 2011; Friedman et al., 2013; Pinheiro-Chagas et al., 2017; Song & Nakayama, 2008a). The present work also shows how the number-to-position task, which was typically used to study numerical cognition, can also serve as a platform to study decision making. Compared to other common decision-making tasks, the number-to-position task offers two major benefits: first, it has multiple possible responses, whereas most tasks have two possible responses (Abrahamyan et al., 2016; Dotan et al., 2017; O'Connell et al., 2012; Pahl, Si, & Zhang, 2013; Resulaj et al., 2009). Second, it involves semantic access (to the quantity represented by the number), whereas most other tasks involve perceptual decision making. At the same time, the number-to-position task is still simple and well-defined enough to allow for accurate mathematical modeling (Cicchini et al., 2014; Chapter 3).

The present findings clearly demonstrate that the brain encodes a Bayesian prior distribution, based on the experience from a specific experimental block. The exact organization of this distribution, however, is yet unknown. An interesting question may be whether the brain stores a prior distribution in a detailed manner, e.g., a likelihood for each of the possible responses, or in a summarized manner, e.g., mean and standard deviation (Meyniel, Sigman, & Mainen, 2015). The use of multiple-response paradigms such as number-to-position may be useful to investigate this question, and open the door to examine more accurately how the brain represents distributions of response-likelihoods, and to what extent it operates as a Bayesian machine.

6. From multiple digits to quantity: Discussion

In a series of experiments, we investigated how multi-digit Arabic numbers are encoded as quantities. We used the number-to-position mapping task, which forces participants to convert a symbolic number into a quantity. The task was performed on an iPad while tracking the participants' finger trajectories, and the nearly-continuous measure of finger position allowed analyzing the subsequent processing stages in a trial. To examine specific processes, different experiments manipulated the participant's attentional state, the number of digits presented, the order and timing of their onset, the length of the number line, the participants' prior expectations for the number to appear, and the finger's initial direction.

To account for the findings in these experiments, we propose the following cognitive model of the number-to-position mapping task (Fig. 6.1). The model describes four main processing stages: visual identification (blue); quantification, which includes several sub-stages, detailed below (yellow-red-green); decision (purple), also with several sub-stages; and pointing (brown).

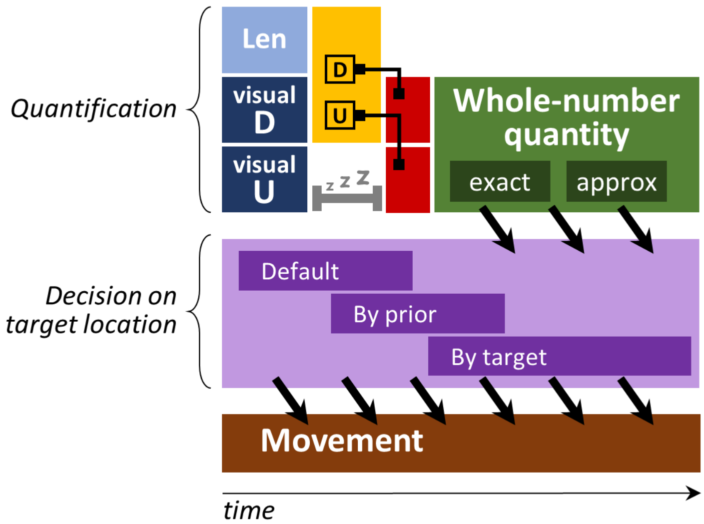

Fig. 6.1. A model for the processes involved in the number-to-position task. The quantification processes converts the multidigit string to quantity representations. The decision process is a Bayesian mechanism that uses this quantity, as well as other information about the task, context, and distribution of target numbers, to determine a target location. Movement is the motor processes that drive the finger to the decided location.

Visual identification includes two processes. One process identifies how many digits the number has (Light blue). Another process identifies the digit symbols, presumably processing each digit independently of the other, i.e., if one of the digits appeared first – it can be identified without waiting for the remaining digit/s. Both processes of visual identification – the length detector and the digit identity encoder – are presumably a part of the visual analyzer of numbers, and presumably also serve other tasks such as number reading (see Chapter 7).

Quantification. At some stage, we create a syntactic frame (yellow) – a quantity-oriented mental representation of the number's structure, which takes into account the number of digits in the multi-digit number but not yet their specific values. Concretely, the syntactic frame is a set of placeholders, one per decimal position. Thus, it can be created even before the specific digit symbols were identified; it only requires knowing how many digits the number has, and therefore depends only on the number-length detector. In two-digit numbers, the initiation of the syntactic frame seems to be triggered by the processing of the leftmost (decade) digit, suggesting perhaps that the number length detector relies on the leftmost digit.

Once the frame was created and the digit symbols were identified, each digit is bound to a placeholder in the syntactic frame, and quantified according to this role (red). This is the stage where decades and units are assigned different weights – the decade digit weight will be 10 times the unit digit weight. Next, the per-digit weighted quantities are combined to a single representation of the multi-digit number's quantity (green).

According to the model, almost all the mechanisms described so far can process the different digits simultaneously – in two-digit numbers, and perhaps even in three-digit numbers (Chapter 4). The only exception is the initiation of the syntactic frame, which is a bottleneck in the quantification pathway: until a syntactic frame was created, the process of binding-to-role and quantification is pending and cannot start.

Decision (purple). The next stage determines the intended location to which the participant will aim the finger. This decision starts from the beginning of the trial, even before the quantity information is available. Initially the finger points to a default location – typically the middle of the number line, but explicit instructions can modify the default direction (as was the case in Experiment 5.1). This default direction is quickly overridden: first, a tentative target location is determined by the distribution of target numbers in the experiment. When quantity information becomes available, the decision is changed and the finger deviates towards the target location, which is determined using a nearly-linear scale. This transition occurs faster for smaller numbers (Chapter 3), presumably because their quantity representation is clearer than that of larger numbers (due to scalar variability or compressed quantity representation).

Pointing (purple). The motor system continuously moves the finger towards the number line, and also deviates it towards the target location determined by the decision stage. The number and frequency of finger deviations is an open issue. The mathematical model described in Chapter 3 took the simple assumption of a single deviation per trial, yet tablet-based pointing tasks clearly allow for more than one deviation per trial (Chapter 4; Dotan et al., 2017). One possibility is that the finger direction is changed only when the new target location, as determined by the decision stage, is distant enough from the finger's current direction (Charles et al., 2014; Fishbach et al., 2007). Our findings agree with this notion, however, further research would be needed to investigate it in detail.

Several questions remain unanswered by this model. In the quantification stage, the exact nature of the syntactic frame is yet unclear. Chapter 4, which examined this matter, used mostly two-digit numbers. Does a similar process occur for longer numbers, or are longer numbers quantified in a hierarchical manner? Does the syntactic frame handle the digit 0 in a special manner, as is apparently the case in syntactic processing in transcoding symbolic numbers (Cohen & Dehaene, 1991; Chapter 7)?

Another issue is the existence of holistic two-digit quantity representation. Although initially we argued (in Chapter 2) that the logarithmic effect in the number-to-position task indicates holistic quantities, this conclusion may perhaps be questioned based on the newer interpretation of the logarithmic effect as resulting from an artifact of processing speed. Another open question concerns the mechanism that merges the decade and unit quantities into a single quantity: is this the same process that is also responsible for summing two quantities when performing additions (Chapter 4; Pinheiro-Chagas et al., 2017)?

The subsequent processing stages also raise several questions. The output of the decision stage may be a continuously-updated decision, or alternatively a discrete set of decisions. We are also not certain how to explain the "spatial reference points" pointing bias (Section 2.3.2.6): it may originate in the stage of decision about the target location, as we suggested in Chapter 3, but it could also originate in other processing stages, e.g. pointing. Finally, understanding the pointing stage would require answering several questions, listed in the discussion of Chapter 3.

Methodologically, the experiments described in this section join several studies in demonstrating the power of the finger-tracking and its ability to tap subsequent stages of decision making, in number processing and in other domains (Faulkenberry, Cruise, Lavro, & Shaki, 2016; Finkbeiner et al., 2014, 2008; Finkbeiner & Friedman, 2011; Marghetis et al., 2014; Santens et al., 2011; Song & Nakayama, 2008a, 2008b, 2009). In our view, the present research makes two main contributions to the increasingly-used technique of finger tracking. The first contribution is the use of a task with continuous response (whereas previous studies used finger tracking mostly with dual decision tasks). The second is the set of fine-grained analysis method that we introduced here – e.g., the per-time-point multiple regression analysis and the analysis of movement onsets.

The cognitive architecture we proposed here, a syntactic bottleneck process (the creation of a syntactic frame) surrounded by several parallel processes of visual recognition and quantification, suggests that the quantification process is not driven by single-digit processing but by the syntactic processes that handle the number's decimal structure. This central role of structural processing is not unique to number comprehension: in number reading too, several processes explicitly represent the number's decimal and verbal structure, and these structural mechanisms apparently drive the reading process (see Chapter 7). Structural processing prevails also in word reading: the morphological structure of words is explicitly represented in several processing stages, from visual analysis processes to phonological production processes (Beyersmann, Castles, & Coltheart, 2011; Dotan & Friedmann, 2015; Kohn & Melvold, 2000; Rastle, Davis, & New, 2004; Reznick & Friedmann, 2015). It may be the case that when faced with the need to convert compound stimuli from one representation to another, the cognitive system consistently tends to mediate the conversion process via deep representations of the compound stimulus structure.

Section B

From digits to number words

Section B: From digits to number words

Section A examined how we understand the meaning of a number, but very often it is not the number's meaning that we are concerned with. Consider daily situations such as writing down on a piece of paper a phone number that someone told you; reading that number to someone; writing a cheque; reading aloud your laptop's serial number to a customer support representative. In all these situations, what you would be concerned with is merely the number symbols, not the quantity they represent. And in many of these situations, you would have to transcode numbers from one symbolic code to another – either from digits to number words or vice versa.

Section B of this dissertation examines the pathway of number reading – converting a sequence of digits into a corresponding sequence of oral number words. Our focus in this section is again syntax: we wish to characterize the syntactic mechanisms of number reading, i.e., the mechanisms that specifically handle the structural complexity of multi-digit numbers. This is done in Chapter 7: the study described in this chapter examined in detail the number processing abilities of seven individuals with number reading disorders. Based on the selective impairment of these individuals, we identified several specific processes in the number reading pathway, and we propose a detailed cognitive model of number reading.

The two remaining chapters in section B examine the relation between the number reading pathway and other, potentially-similar processing pathways. Chapter 8 examined whether number reading and word reading make use of shared cognitive mechanisms. It describes the processes involved in reading words and numbers, proposes possible similarities between them, reviews associations and dissociations between impairments of word and number reading, and presents two case studies with previously-unreported word-number dissociations. We conclude that word reading and number reading are implemented by separate mechanisms. Chapter 9 examines the relation between number reading (converting multidigit strings to number words) and number comprehension (converting multidigit strings to quantity). We report an aphasic patient who can comprehend multidigit number but cannot read them aloud, and conclude from this dissociation that at least some syntactic mechanisms involved in number reading do not serve number comprehension.

7. A cognitive model for multi-digit number reading: Inferences from individuals with selective impairments°

Abstract. Reading multi-digit numbers aloud involves visual analysis of the digit string and oral production of the verbal number. To examine these processes in detail, we investigated the number processing abilities of seven individuals with different selective deficits in number reading. In particular, some participants were impaired in visual analysis of digit strings – in encoding the digit order, encoding the number length, or parsing the digit string to triplets (e.g., 314987 → 314 and 987). Other participants were impaired in verbal production, making errors in the number structure (shifts of digits to another decimal position, e.g., 3,040 → 30,004). These selective deficits yielded several dissociations: first, a double dissociation between visual analysis deficits and verbal production deficits. Second, several dissociations within visual analysis: a double dissociation between errors in digit order and errors in the number length; a dissociation between order/length errors and errors in parsing the digit string into triplets; and a dissociation between the processing of different digits – impaired order encoding of the digits 2-9, without errors in 0 position. Third, within verbal production, a dissociation between digit shifts and substitutions of number words. On the basis of these selective impairments and previous findings, we propose a detailed cognitive model of number reading. The model postulates that within visual analysis, separate sub-processes encode the digit identities and the digit order, and additional sub-processes encode the number's decimal structure: its length, its triplet structure, and the positions of 0. Verbal production consists of one process that generates the verbal structure of the number, and another process that retrieves the phonological forms of each number word. We propose that the verbal number structure is first encoded in a tree-like structure and then linearized to a sequence of number-word specifiers, similarly to syntactic trees of sentences.

7.1. Introduction

Number reading is a complex cognitive operation involving several different sub-processes, each of which can be impaired and cause a different number reading malfunction (Basso & Beschin, 2000; Cappelletti et al., 2005; Cipolotti & Butterworth, 1995; Cipolotti et al., 1995; Cohen & Dehaene, 1991; Cohen et al., 1997; Dehaene et al., 2003; Delazer & Bartha, 2001; Deloche & Willmes, 2000; Dotan & Friedmann, 2015; Friedmann, Dotan, et al., 2010; McCloskey et al., 1985, 1990, 1986; Moura et al., 2013; Noël & Seron, 1993; Starrfelt & Behrmann, 2011; Starrfelt, Habekost, & Gerlach, 2010; Temple, 1989). How do these cognitive mechanisms operate? In the present study, we propose a detailed model of number reading. In doing so, we draw inspiration from word reading, another complex and potentially-similar

° This chapter is a manuscript submitted to the journal *Cortex*. The text here is identical with the submitted manuscript, except reformatting and removing some parts that would, if remained, repeat other sections of this dissertation.

cognitive function. Similarly to number reading, word reading also involves a variety of processes: visually analyzing the sequence of letters, accessing the appropriate entries in orthographic, phonological, and semantic mental lexicons, generating the phonological output, and articulation. After several decades of research, we now have a cognitive model with detailed specification of the processes involved in word reading and of the flow of information among these processes (Coltheart et al., 2001; Ellis, 1993; Ellis & Young, 1996; Friedmann & Coltheart, in press; Friedmann & Gvion, 2001; Humphreys, Evett, & Quinlan, 1990; Marshall & Newcombe, 1973; Patterson & Shewell, 1987; Shallice, 1988). This model turned out to be invaluable in several ways. From a theoretical point of view, an accurate model of word reading enables us to better understand the reading mechanisms, and supports detailed investigation of other language process such as morphology and lexical retrieval (Biran & Friedmann, 2012; Dotan & Friedmann, 2015; Friedmann, Biran, & Dotan, 2013; Funnell, 1983; Gvion & Friedmann, 2016; Job & Sartori, 1984; Reznick & Friedmann, 2009, 2015). From a clinical point of view, such a detailed model improves our ability to identify specific impairments in number reading and to treat individuals with such impairments (Castles & Friedmann, 2014; Colenbrander, Nickels, & Kohnen, 2011; Coltheart & Kohnen, 2012; Friedmann et al., 2013; Friedmann & Coltheart, in press; Friedmann & Gvion, 2001; Marshall & Newcombe, 1973; Nickels, 1997; Nickels, Rapp, & Kohnen, 2015; Rapp, 2005; Temple, 2006). The cognitive model of reading could not have been as useful had it not been very explicit in terms of information processing: the model accurately describes the function of each cognitive sub-process involved in reading, and the kind of information transferred between these processes, in a manner detailed enough to allow for computational implementation (Coltheart et al., 2001). This high level of granularity is what allows characterizing the interaction between reading and other language processes, and makes it possible to identify specific cognitive disorders in specific processing stages.

The reading of numbers (such as “256”) is implemented, at least in part, by separate mechanisms than the word reading mechanisms (Abboud, Maidenbaum, Dehaene, & Amedi, 2015; Friedmann, Dotan, et al., 2010; Hannagan, Amedi, Cohen, Dehaene-lambertz, & Dehaene, 2015; Shum et al., 2013; for a review, see Chapter 8). However, number reading was not investigated as much as word reading, and less is known about it. During the 1990’s, there was much debate about the representation of symbolic numbers and the transcoding processes that convert between these representations (Cipolotti & Butterworth, 1995; Cipolotti et al., 1995;

Cohen & Dehaene, 2000; Dehaene & Cohen, 1997; McCloskey, 1992; McCloskey et al., 1990, 1986; Sokol, McCloskey, Cohen, & Aliminosa, 1991). At present, a widely accepted model is the triple-code model of number processing (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene et al., 2003), which holds that separate cognitive and neural circuits represent numbers as sequences of digits, as verbal number words, and as quantities. With respect to number reading, the triple-code model postulates that the visual parsing of digital numbers and the verbal production of number words are handled by separate processes, connected by a direct digit-to-verbal transcoding pathway that is at least partially separate from the access to number semantics. Indeed, several studies showed that the visual analysis of numbers can be selectively impaired (Cohen & Dehaene, 1995; Friedmann, Dotan, & Rahamim, 2010; McCloskey et al., 1986; Noël & Seron, 1993), and so can the verbal production of numbers (Benson & Denckla, 1969; Cohen et al., 1997; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Marangolo et al., 2004, 2005; Chapter 9).

The triple-code model, as well as many of the above studies, characterized the different number representations and transcoding pathways. Other studies, though fewer, were specifically concerned with offering a detailed cognitive model of number reading. Michael McCloskey and his colleagues (McCloskey, 1992; McCloskey et al., 1986) proposed a model where number reading – transcoding a digit string into number words – is mediated by a central semantic representation, which essentially reflects the number's decimal structure (e.g., 2,031 = $2 \times 10^3 + 0 \times 10^2 + 3 \times 10^1 + 1 \times 10^0$). Their model postulates that converting this representation to number words begins by creating a *syntactic frame*, which reflects the verbal structure of a number with a given number of digits – e.g., for a 4-digit numbers the syntactic frame is *[_:ones] [thousand:multiplier] [_:ones] [hundred:multiplier] [_:tens] [_:ones]*. The notations [_:ones] and [_:tens] represent placeholders for a number word of the corresponding lexical class[4]. The syntactic frame is created and then "filled" with the specific digit identities (resulting, for the example above, in *[2:ones] [thousand:multiplier] [_:ones] [hundred:multiplier] [3:tens] [1:ones]*). In the filled frame, each slot uniquely identifies a single word. Some numbers have irregular structure – e.g., in English, the digit 0 is never verbalized –

[4] McCloskey and his colleagues experimented in English and mentioned ones, teens and tens as lexical classes for words. However, the specific lexical classes may depend on the characteristics of verbal numbers in a specific language. Our study was conducted in Hebrew, in which the number words for hundreds and thousands often introduce some verbal irregularity and may therefore be lexicalized. This would result in hundreds and thousands as two additional lexical classes. However, this question – whether hundreds and thousands are indeed lexical classes in Hebrew – was not in the scope of the present study.

which results, as in the above example, in an unfilled slot. This unfilled slot would be discarded from the frame after it has been filled. Another example for irregularity in English is that a digit 1 in the decades position results in a teen word. This too will result in modifying the filled frame, e.g., *[1:tens] [3:ones]* would be changed into *[3:teens]*. The filled frame is a plan for phonological retrieval: each combination of lexical class and digit value, or the specification of a multiplier word, is used to retrieve the corresponding phonological form of a single word. McCloskey suggested that this form is retrieved from the phonological output lexicon, but in Dotan and Friedmann (2015) we showed that number words are actually retrieved from a dedicated phonological store that is separate from the phonological output lexicon of words.

Cohen and Dehaene (1991) proposed a modified version of this reading model: they proposed that the visual analysis of the digit string is immediately followed by verbal production, without mediation of a central semantic representation. The challenge for any number-reading model is how to handle the number's decimal and verbal structure; Cohen and Dehaene proposed that this is implemented by two separate processes, one visual and one verbal. The visual process, which is a part of the visual analysis of numbers, is responsible for parsing the number's decimal structure, which concretely consists of the number length (how many digits it has) and the positions of 0 and 1. The remaining digits (2-9) are identified by a separate process. Within the verbal mechanisms, Cohen and Dehaene accepted McCloskey's notion of a syntactic frame, but proposed that it is quickly converted into what they termed a *number word frame*. Conceptually, the number word frame is the number's verbal structure. Concretely, it is a sequence of lexical classes (ones, teens, tens) of the number words to be produced (the frame for 24,013 is *[_:tens] [_:ones] [thousand] [and] [_:teens]*). The number word frame is generated based on the number's decimal structure: the number length determines how many words will be produced, the positions of 0 indicate number words to skip, and the existence of 1 in the decade position separates teens from tens. The word frame is then filled with specific digit values and goes on to phonological retrieval and articulation.

In terms of information flow, the concept of number word frame may seem like a small difference from McCloskey's model: instead of filling the syntactic frame and only then modifying it according to 0's and 1's, as McCloskey proposed, Cohen and Dehaene propose that the syntactic frame is first modified by 0-1, resulting in a number word frame, and only then filled. Theoretically, however, the difference is important: Cohen and Dehaene propose a concrete representation of the number's verbal structure that is independent of specific digits or number words.

Here we propose another model, which is a mixture of the above models with few modifications and additions. Similarly to previous models of number reading, it accounts only for the reading of positive, base-10 integers, and admittedly ignores very long numbers, whose reading may involve different processes (in this study we considered only numbers up to 6 digits). The model also ignores the issue of "lexicalized" numbers such as "1984", which may be identified as a whole and be processed in different pathways (Cohen et al., 1994). A general illustration of this model appears in Fig. 7.1, and we revisit it with more detail in the Discussion of this chapter. The model postulates that within visual analysis, one process extracts the number's decimal structure, which consists of the number length, the positions of 0 (but not of 1), and the way the number is parsed into triplets (e.g., 24013 is parsed as 24 and 013). Two other processes encode the digit identities and their relative order, and can provide the 1-9 value of each digit in the correct order. Within the verbal system, the number's decimal structure is used to generate a number word frame, defined as in Cohen and Dehaene's model. The number word frame is a sequence of word specifiers, each of which can be a number word lexical classes, a multiplier words (e.g., thousand, hundred; we hereby refer to them as "decimal words"[5]), and the function word "and". These are used, in conjunction with the 1-9 digit value, to from the dedicated phonological store the corresponding sequence of words, which are then sent to articulation.

The main components of this model are similar to those proposed by Cohen and Dehaene (1991): no semantic representation is mediating the digit-to-verbal transcoding; and we followed Cohen and Dehaene's assumption of separate processes for visual parsing and verbal production, each of which is further divided into a "structural" component (decimal or verbal) and a "lexical" component (digits or words). From McCloskey's model, we borrowed the notion that number words are retrieved according to lexical class and digit value. However, our model

[5] In English, number words such as "hundred" and "thousand" are special in two respects. First, semantically, they impact the quantity in a predictable manner – they are multiplied by the preceding word, such that the quantity of "three hundred" is three times hundred, hence the term "multiplier". Second, lexically: each multiplier is a single lexical item, separate from the units word (they are the "building blocks" of multidigit verbal numbers, Cohen et al., 1997; Dotan & Friedmann, 2015). We wish to keep the term "multiplier" to refer to the semantic notion, and use the term "decimal word" to refer to the lexical notion. Indeed, in some languages such as Hebrew, not all multipliers are decimal words. For example, "hundred" is a multiplier in the semantic sense, yet it is not an independent word: apparently, it is not a separate lexical entry in the phonological storage of number words, and for some numbers it is also not a separate orthographic entry (e.g., 200 is a single word – מאתיים, MATAYIM).

also proposes some modifications and enhancements to the existing models. First, we propose a different internal organization of the decimal structure extraction. In our model, the decimal structure does not consist of number length and 0,1 positions, but of number length, the positions of 0 (not 1), and the number's triplet structure. Below, we bring evidence from number reading impairments in support of these claims. Second, the process of digit identification was broken here in two – a digit-identity encoder and a digit order encoder (Friedmann, Dotan, & Rahamim, 2010). Third, we accept Cohen and Dehaene's definition of the number word frame, however, how this frame is obtained is different in our model: we discarded the notion of a syntactic frame, and in the Discussion of this chapter we describe several specific processes involved in generating the number word frame.

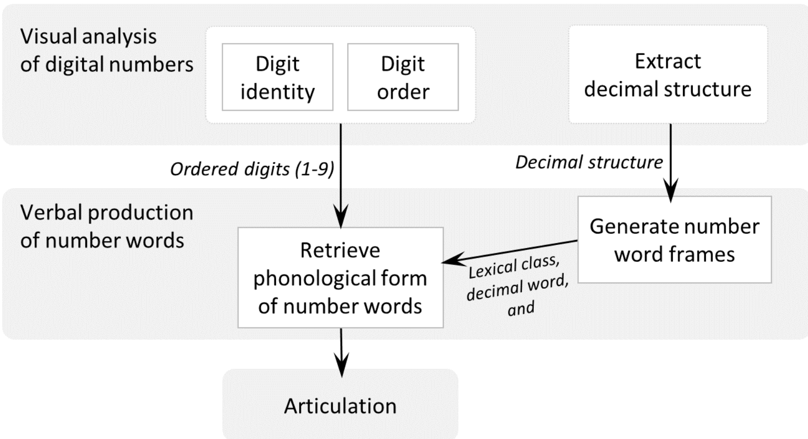

Fig. 7.1. A proposed cognitive model of number reading. Separate processes handle the visual analysis of the digit string and the verbal production of the number words. The visual analyzer has several distinct sub-processes: the digit identity encoder and digit order encoder provide the identity of each digit (1-9) in their respective order. Another set of sub-processes extract the number's decimal structure. This decimal structure is used to generate a number word frame – the number's verbal structure. The word frame is a sequence of one lexical class per number word (ones, teens, tens), and further specifies where decimal words ("thousand", "hundred") and the word "and" should be embedded in the number. Each entry in this sequence, in conjunction with the corresponding digit value, is used to retrieve the phonological form of one number word at a time.

The present study reports seven neuropsychological case studies whose performance led us to propose the above model. We report individuals with selective impairments in three of the components depicted in Fig. 7.1: the encoding of digit order, the extraction of decimal structure, and the generation of number word frames. Previous studies showed that selective impairments are possible also in the three remaining components – the digit identity encoding (Cohen &

Dehaene, 1991), the phonological retrieval of number words (Cohen et al., 1997; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Girelli & Delazer, 1999; Marangolo et al., 2004, 2005), and the articulation of number words (Shalev, Ophir, Gvion, Gil, & Friedmann, 2014; Chapter 9). Furthermore, we report specific dissociations that support the separation of decimal structure extraction into three distinct sub-processes – encoding the number length, identifying the positions of zeros, and splitting the number into triplets.

7.2. Method

7.2.1. Participants

Seven individuals with various number processing impairments participated in this study: HZ and OZ were undergraduate students. EY was a PhD candidate whose performance was reported in Friedmann, Dotan, and Rahamim (2010). MA was a self-employed woman with undergraduate degree. ED and NL were sisters: ED had an undergraduate degree and worked in an administrative job, and NL was a BA student. Finally, UN was a retired lawyer who was recovering from a stroke that he had 3 months prior to our meeting. All participants were native speakers of Hebrew, with normal or corrected-to-normal vision. Table 7.1 shows their background information.

Table 7.1. The participants' background information

	HZ	EY	MA	ED	NL	OZ	UN
Gender	F	F	F	F	F	M	M
Age	24	34	26	31	24	20	79
Dominant hand	R	R	R	R	R	R	R
Education years	13	20	15	15	13	14	20
Acquired/Developmental deficit	D	D	D	D	D	D	A

All control participants were native speakers of Hebrew with at least 12 years of education and no reported cognitive disorders (other than the difficulties with numbers). They were compensated for participation. All participants and control participants gave informed consent to joining the study. The Tel-Aviv University ethics committee approved the experimental protocol.

7.2.2. General procedure

The participants were tested in a series of 1- to 2-hour sessions in a quiet room. All tests were conducted in Hebrew. Unless specified otherwise, EY read stimuli from the computer screen, where each stimulus was presented for 400 ms, and the other participants read the stimuli from paper, where they were printed as vertical lists. Each task is described in the text below, additional methodological comments appear in the supplemental online material, and Table C.1 lists the tasks used in this study and the processes that each task can tap. When the participants made both a correct and an erroneous response, the response was classified as an error. Error percentages were calculated out of the total amount of target numbers.

Control participants with outlier error rates were excluded (see Table 7.2 for demographic details of the control participants in all experiments). Statistical comparisons of individual performance between conditions were done using chi-square test or Fisher's exact test. Individual participants were compared to control groups using Crawford and Garthwaite's (2002) one-tailed t-test. In cases of a control group ceiling effect (mean error rate ≤ 2%), the low variance does not allow for a reliable statistical comparison. We arbitrarily decided that in such cases, 7% errors or more would be considered as impaired performance.

Table 7.2. Control participants per experiment

Experiment		No. of participants	Outliers	Age		
				Range	Mean	SD
7.1	Number reading	20	3	20;7 - 30;4	25;5	2;7
	Number reading (EY)	10	-	21;3 - 42;4	27;0	5;10
7.2	Sequence identification	20	1	20;9 - 42;0	31;7	8;4
	Sequence identification (EY)	10	-	23;0 - 35;5	29;2	5;6
7.3	Same-different decision	24	1	20;9 - 42;0	30;0	6;11
	Same-different decision (EY)	10	-	27;8 - 35;0	28;8	4;10
7.4	Number matching	20	1	21;3 - 42;6	26;1	4;4
7.5	Number repetition	20	2	20;7 - 42;4	26;1	4;8
	Number repetition (EY)	10	-	22;10 - 28;8	25;2	2;0
	Number repetition (UN)	15	-	21;9 - 30;1	25;0	2;4
7.8	Same-different (length)	20	1	20;10 - 43;4	29;6	7;3
7.10	Multiply/divide by 10	20	2	21;4 - 44;4	27;10	5;4
7.11	Read numbers as triplets	20	-	24;8 - 49;4	35;1	7;7

The "outliers" columns indicates the number of control participants who were excluded as outliers – i.e., their error rate exceeded the 75th percentile of error rates by more than 150% the inter-quartile distance. Some control groups were run for experiment versions used for a specific participant. These cases are indicated in the "Experiment" column by parentheses with the participant's initials.

7.3. Experimental investigation

7.3.1. Background: language assessment

The participants' cognitive and language abilities were examined using several tasks (Table 7.3): digit and word spans (FriGvi, Friedmann & Gvion, 2002; Gvion & Friedmann, 2012; comparison to control data was done using Crawford & Garthwhite (2002) t-test); picture naming (SHEMESH, Biran & Friedmann, 2004); reading single words, nonwords, and word pairs (TILTAN, Friedmann & Gvion, 2003); lexical decision (TILTAN, Friedmann & Gvion, 2003), a task in which they classified letter strings as words or nonwords without reading them aloud (the task included both migratable nonwords, i.e., nonwords in which letter migration can yield an existing word, and non-migratable nonwords); nonword reading (TILTAN, Friedmann & Gvion, 2003); nonword repetition (BLIP, Friedmann, 2003); and writing single words and nonwords (TILTAN, Friedmann, Gvion, & Yachini, 2007).

Table 7.3. Memory spans, and error percentages in language tasks

	No. of items	HZ	EY	MA	ED	NL	OZ	UN
Memory spans								
Digit (free recall)		6	5^{*}	5^{*}	5^{*}	5^{*}	6	3^{**}
Digit (matching)		7		7	7	7	7	4
Word (free recall)		6		4^{*}	4½	5	6	3^{*}
Word (matching)		7		5	7	7	7	2^{**}
Picture naming	**100**	1		0	2	3	2	26^{***}
Word reading	**136**							
All errors		14^{***}	16^{***}	2	1	3	1	12^{***}
Migration errors [a]		24^{***}	30^{***}	0	0	0	0	0
Lexical decision	**60**							
Migratable nonwords	**15**	60^{+++}	65^{+++}	0	0	0	7^{+++}	0
Non-migratable nonwords	**15**	20^{+++}	5	0	0	13^{+++}	7^{+++}	7^{+++}
Nonword reading	**40**							
All errors		43^{***}	17^{**}	8	3	8	5	57^{***}
Migration errors		35^{***}	17^{***}	8	0	3	3	3
Nonword repetition	**48**	2		4^{+}	2	2	4^{+}	46^{***}
Word writing	**50**	14^{***}		4	2	2	4	28^{***}

Comparison vs. control group: $^{+}$ $p < .1$ * $p < .05$ ** $p < .01$ *** $p < .001$

$^{+++}$ Errors ≥ 7%, control group ≤ 2% errors

[a] Migration errors out of all words with lexical potential for interior migration

These tasks showed that HZ and EY had letter position dyslexia, a selective deficit in letter position encoding by the visual analyzer (Friedmann, Dotan, & Rahamim, 2010; Friedmann & Gvion, 2001; Friedmann & Rahamim, 2007; Kezilas, Kohnen, McKague, & Castles, 2014; Kohnen, Nickels, Castles, Friedmann, & McArthur, 2012). Both of them had high rate of letter migration errors in word reading and in lexical decision – two tasks that have orthographic input so they involve the orthographic-visual analyzer. Conversely, they did not have migration errors in tasks that did not involve the orthographic-visual analyzer (i.e., tasks without orthographic input): neither had errors in spontaneous speech, and HZ did not have many migrations also in formal tasks without orthographic input – picture naming, nonword repetition, and sentence elicitation. HZ also had a mild surface dysgraphia.

MA, ED, NL, and OZ had intact word reading, writing, and naming (for a detailed comparison of number reading with word reading, see Chapter 8). UN, the participant with acquired aphasia, had impairments in writing and naming and a low digit span (lower than that of the other participants). He also had some difficulty in reading. Most of his reading errors were surface errors and vowel letter errors, which typically originate in processing stages later than visual analysis (Friedmann & Lukov, 2008; Gvion & Friedmann, 2016; Khentov-Krauss & Friedmann, 2011).

EY, MA, ED, and NL had slightly low scores on memory span tasks that involved production, suggesting a slightly low capacity of phonological working memory. For MA, ED, and NL, we tested and found normal scores in phonological working memory tasks not involving verbal production, indicating that this capacity limit was specifically in the production-related phonological memory. This mild phonological working memory impairment did not seem to impact their speech: they performed well in naming and in nonword repetition, tasks that are typically sensitive to phonological working memory deficits (Friedmann et al., 2013). A deficit in production-related phonological working memory (the phonological output buffer) sometimes causes substitutions of number words (Dotan & Friedmann, 2015). However, as we will see below, here this was not the case for any participant except UN.

7.3.2. Experiment 7.1: Assessment of number reading

The participants' ability to process symbolic numbers was first assessed with a number reading task, which involves digit input and verbal output.

7.3.2.1. Method

The participants read aloud a list of 120 Arabic numbers with 3, 4, 5, or 6 digits (30, 38, 47, and 5 items, respectively), in random order. The digit 0 appeared in 63 of the numbers, and the other numbers contained only the digits 2-9. EY read a different list of 316 numbers with 3, 4, or 5 digits (100, 84, and 132 items, respectively). The digit 0 appeared in 134 of the numbers, and the others contained only the digits 2-9.

7.3.2.2. Results

All participants had many errors in the number reading task (Table 7.4). The errors were classified as follows: *transposition*, or a *digit order error*, is a change in the relative order of digits (e.g., 1234 → 1324). In word reading, transposition errors are the hallmark of letter position dyslexia, a deficit in letter position encoding by the visual analyzer (Friedmann & Gvion, 2001; Friedmann & Haddad-Hanna, 2012, 2014; Friedmann & Rahamim, 2007; Kezilas et al., 2014; Kohnen et al., 2012). A similar deficit also exists in the visual analyzer of numbers (Friedmann, Dotan, & Rahamim, 2010; Chapter 8).

A *decimal shift* is the production of a number word as if the corresponding digit was in a different decimal position (e.g., 2345 → "two thousand and thirty… sorry, three hundred and forty five" – in this example, the error was spontaneously corrected).

It is important to point here to a crucial difference between decimal shift errors and digit order errors: Whereas both reflect situations where one or more digits appear in an incorrect decimal position, these are two different error types. Digit order errors are digit displacements that result in erroneous relative order of digits (which, in turn, causes erroneous order of the corresponding number words – e.g., 2345 → 2354). In contrast, decimal shifts are digit displacements that keep the relative order of digits (and hence do not result in erroneous order of number words).[6]

The distinction between decimal shifts and digit order errors was demonstrated in our data by the finding of a double dissociation between the two error types: EY had only digit order errors, whereas MA, ED, NL, OZ, and UN had only decimal shift errors. This double dissociation indicates that the digit order errors and decimal shift errors have different cognitive

[6] Some errors can arguably be classified both as an order error and as a decimal shift: this is the case when a non-leftmost digit was transposed with zero (e.g., 3,405 → 3,045). These errors were rare (only 9 errors for all participants pooled together), and were classified as decimal shifts.

origins. In the following sections, we confirm and clarify this dissociation and its implication for the number reading model.

Decimal shift errors are especially interesting when they occur in the leftmost digits of the number (e.g., reading 234 as 2034, 2304, or 2340) – hereby, *first-digit shifts*. Such errors may indicate that the participant was processing the number structure incorrectly. For example, the above example may originate in the 3-digit number 234 being processed as if it has 4 digits. Decimal shifts were therefore analyzed by their position in the target number.

Table 7.4. Percentage of errors in number reading (Experiment 7.1). EY had many transpositions, without decimal shifts. MA, ED, NL, OZ, and UN had many decimal shift errors, with only few transpositions. HZ had many errors of both types.

	Order	Decimal shift	Substitutions	Thousand [a]	All errors [b]
HZ	18 $^{+++}$	30 $^{+++}$	2	0	46 ***
EY	17 ***	0	2	0	27 ***
MA	4	18 $^{+++}$	0	0	20 ***
ED	3	17 $^{+++}$	1	6	23 ***
NL	5	14 $^{+++}$	2	1	23 ***
OZ	2	22 $^{+++}$	3	13 $^{+++}$	32 ***
UN	1	24 $^{+++}$	17 $^{+++}$	15 $^{+++}$	44 ***
Controls (SD)	0.5 (0.7)	1.1 (1.1)	0.7 (0.8)	0.9 (1.3)	2.8 (1.3)
EY Controls (SD) [c]	2.1 (1.5)	0.03 (0.1)	2.2 (2.3)	0	6.6 (4.3)

Comparison to the control group: *** $p \leq .001$ $^{+++}$ Errors ≥ 7%, control group ≤ 2% errors

[a] The rate of errors related with the decimal word "thousand" was counted out of the 52 numbers that contained the word “thousand” (5 or 6 digits).

[b] The percentage of items with any error.

[c] EY read a different list of numbers. Her control group had more errors than the other control group, perhaps because they saw each number for 400 ms (like EY) whereas the other control group had unlimited exposure.

There were also errors related with the decimal word "thousand" (in 5- and 6-digit numbers): omission of the word "thousand" or addition of an excessive "thousand". Last, there were *substitutions* of a digit by another digit (e.g., 234 → 294).

The participants showed different error patterns: HZ and EY had a high rate of digit order errors; all participants but EY had many decimal shift errors; UN and OZ, and to a lesser extent ED, had errors in the decimal word "thousand"; and UN had many substitution errors. Importantly, even when making mistakes, participants rarely produced invalid number names (e.g., 2030 → "two thousand and three thousand" or "two, thirty"): none of them produced more than 2 invalid number names in this task.

7.3.2.2.1. Digit order errors

Digit order errors may result from impaired encoding of digit order by the visual analyzer. Note that according to the model in Fig. 7.1, different visual analyzer sub-processes encode the order information of different digits: the order of 1-9 are encoded by the digit order encoder, whereas the presence of 0 and its positions are encoded by another, dedicated process, as part of extracting the number's decimal structure. A spared 0-detector could potentially compensate for an impaired digit order encoder if the number has 0. To examine this, we analyzed the order errors of HZ and EY (the two participants who had order errors) in numbers with or without 0. EY had 25.8% order errors in numbers that included only the digits 2-9, but only 6.0% order errors in numbers that included 0 (χ^2 = 19.58, one-tailed p < .001). This suggests that she had a selective impairment in digit order encoding, yet this impairment spared the encoding of the positions of 0. In contrast to EY, HZ' order errors in numbers with 0 (35%) were as frequent as in numbers that included only the digits 2-9 (46%; χ^2 = 1.49, one-tailed p = .11), suggesting that her impairment was not as selective as EY's: she was impaired both in the digit order encoder and in the 0 detector.

7.3.2.2.2. Decimal shift errors

All participants except EY had decimal shift errors. Some decimal shifts involved omissions of digits and number words, and in other cases a zero was omitted (so no word was omitted). Participants usually self-corrected their decimal shift errors (e.g., 2345 → "two thousand and thirty… sorry, three hundred and forty five": 85% self-corrections for HZ, 100% for the other participants, but UN self-corrected only 32% of these errors. We assume that the spared digit identity encoding of all participants (except UN) served them as a cue to detect their mistake.

Table 7.5 shows decimal shift errors according to their position in the target number: shifts of the leftmost digit or digits (e.g., 4,320 → 40,320 or 432), shifts of the first digits of the second triplet (e.g., 4,320 → 4,032), or shifts of other digits (e.g., 4,320 → 4,302). The table clearly shows that decimal shift errors were most frequent in the leftmost digits. We examined whether the erroneously produced number had more or fewer digits than the target number (rightmost columns in Table 7.5). No clear tendency was found – "longer" and "shorter" errors did not significantly differ for any of the participants (Binomial test, $z \leq 1.25$, two-tailed $p \geq .21$).

Table 7.5. Decimal shift errors in number reading, classified by the decimal position of the target digit. For each participant, most of the errors occurred in the first (leftmost) digits. The table shows raw number of errors in reading 120 numbers in Experiment 7.1, and percentages out of the total number of errors.

	Position of decimal shift			First-digit-shift made the number… [a]	
Sample error in	Leftmost	First digit of 2nd triplet	Other		
target=12345 →	120,345	12,034	12,304	Longer	Shorter
HZ	33 (92%)	3 (8%)	1 (3%)	13	14
MA	18 (82%)	4 (18%)	0	9	7
ED	20 (100%)	0	0	11	5
NL	14 (82%)	4 (24%)	0	5	7
OZ	17 (65%)	10 (38%)	1 (4%)	6	8
UN	22 (76%)	6 (21%)	1 (3%)	6	11

[a] In the two right columns, the numbers sum to less than the total number of first-digit-shift errors because the longer/shorter direction of some errors was ambiguous.

7.3.2.3. Discussion of Experiment 7.1

All the participants showed impaired oral reading of numbers, yet they showed different types of errors in number reading. Two participants – HZ and EY – had high rates of digit order errors. These errors may originate either in the visual analyzer, which encodes the digit order, or in verbal production processes. In Section 7.3.3 we assess the exact locus of deficit underlying the order errors.

EY's order errors were almost absent from numbers that included the digit 0. Our explanation to this pattern is that EY's impairment selectively disrupted the processing the digit order, but the positions of 0 are processed by another mechanism, which was not impaired for EY. This issue is systematically examined in Section 7.3.4.

All participants except EY had many decimal shift errors. These errors were not uniformly distributed across all decimal positions – most of them occurred in the leftmost digits. This pattern may have two explanations. One possibility is that the participants processed the number length incorrectly, i.e., they processed numbers (e.g., 4,320) as if they had more digits (reading it as 43,200) or fewer digits (432). Alternatively, the participants may have grouped the digits incorrectly to triplets (e.g., as 43-20 rather than 4-320). Under both interpretations, these first-digit shift errors indicate a deficit in a dedicated mechanism that handles the number structure.

The reading task, however, cannot indicate whether this deficit was in the visual analysis or in the production stage. We further investigate the origin of these errors in Section 7.3.5. Note that a tendency to err in the leftmost digits is unlikely to result from a plain memory difficulty: serial recall tasks typically show *better* recall of the first items in the list (Baddeley, 1968; Gvion & Friedmann, 2012; Hanten & Martin, 2000; Jahnke, 1965).

OZ and UN, and marginally ED too, had errors related with the decimal word "thousand", e.g., reading "12345" as "twelve, three hundred and forty five". In the Discussion of this chapter, we propose a possible explanation of these errors.

Last, UN had many digit substitution errors. We will show in the next sections that these substitutions resulted from a deficit in the verbal output, and in the Discussion of this chapter we discuss his locus of deficit in more detail. No other participant had many substitution errors, indicating that they had no deficit in processing digit identities – neither in visual analysis nor in verbal production.

7.3.3. Impaired encoding of digit order in the visual analyzer

HZ and EY had many digit order errors in number reading, indicating a digit order processing deficit. To identify the functional locus of this deficit, we administered several tasks sensitive to digit order information in different processing stages. To tap the encoding of digit order by the visual analyzer, we used tasks with visual digit input and without verbal output. To tap the use of digit order information by the verbal production system, we used tasks with verbal number production and without visual digit input. A digit-order encoding deficit in the visual analyzer should cause order errors in the visual input tasks but not in the verbal production tasks.

The results of each task are reported here in full, including decimal shift errors, but these errors will be discussed only below, in Section 7.3.5.

7.3.3.1. Input tasks

We administered three tasks that tap digit-order encoding within the visual analyzer: sequence identification, same-different decision, and number matching.

7.3.3.1.1. Experiment 7.2: Sequence identification

7.3.3.1.1.1. Method

The participants saw 4-digit strings printed on paper, and were asked to circle strings that consisted of only consecutive digits (e.g., 3456). In these consecutive strings, digits always appeared in ascending order. The non-consecutive strings were derived from a consecutive string either by transposing two adjacent digits (e.g., 3546) or by substituting a digit (e.g., 3496). A digit-order encoding deficit in the visual analyzer should cause a difficulty in the digit-transposition stimuli but not in the digit-substitution stimuli.

The task included 100 consecutive sequences and 100 non-sequence digit strings: 53 digit-transposition strings and 47 digit-substitution strings. No number included 0 or 1. EY performed a computerized version of this task, with 150 consecutive-digit strings, 75 transposition strings, and 75 substitution strings: each stimulus was presented centered on the computer screen for 400 ms, and she clicked on one of two buttons with the mouse.

7.3.3.1.1.2. Results

HZ and EY had significantly more errors than the control group in the transposition stimuli (Table 7.6), and had more errors in the transposition stimuli than in the substitution stimuli ($\chi^2 = 62.21$ for HZ, 24.0 for EY; two-tailed $p < .001$ for both). In the other stimulus types – substitution stimuli and sequence stimuli – they performed like the control group. Because the task involved the visual analyzer but not verbal production of numbers, these results reaffirm that HZ and EY had a digit-order encoding deficit in the visual analyzer.

OZ showed a similar pattern of errors – more errors in transposition stimuli than the control group, and more than his own errors in substitution stimuli ($\chi^2 = 6.11$, two-tailed $p = .01$). However, his transposition error rate was significantly lower than HZ's and EY's ($\chi^2 > 5.55$, two-tailed $p < .02$), and he had no transposition errors in the number reading task. Thus, it seems that he did not have a digit-order encoding deficit, or at most – had a mild one. The other participants (MA, ED, NL, UN) performed well in all stimulus types, confirming that their digit order encoding in the visual analyzer, as well as their digit identity encoding, were intact.

7.3.3.1.2. Experiment 7.3: Same-different decision

To further assess the visual analyzer without verbal production, participants were shown pairs of numbers and judged whether the numbers in each pair were identical or not. A digit-order encoding deficit in the visual analyzer should create a difficulty in this task if the two numbers in a pair differ only in the order of digits.

7.3.3.1.2.1. Method

The participants saw 144 pairs of 4-digit numbers printed on paper, and were asked to circle pairs with two identical numbers. These were 50% of the pairs. In the remaining pairs, the second number was derived from the first number by transposing two adjacent digits (36 transposition pairs, e.g., 1234-1324) or by substituting a digit (36 substitution pairs, e.g., 1234-1237). Transpositions and substitutions were evenly distributed across all decimal positions. No number included 0 or 1. EY performed a computerized version of this task – each pair was presented on screen for 1300 ms, and she responded by clicking one of two buttons with the mouse. Her task included 120 identical pairs, 75 transposition pairs (50 unit-decade, 20 decade-hundred, and 10 hundred-thousand), and 75 substitution pairs (25, 25, 15, and 10 items with a substitution in the unit, decade, hundred or thousand digit, respectively). UN did not perform this task.

7.3.3.1.2.2. Results

Table 7.6 shows the results in this task. Notably, the transposition pairs were significantly harder than the substitution pairs even for the control group (paired $t(23) = 3.43$, one-tailed $p = .001$, Cohen's $d = 1.43$). Namely, even unimpaired individuals make some transposition errors in this task.

Except MA, all participants had significantly more transposition than substitution errors ($\chi^2 > 5.06$, one-tailed $p \leq .01$), and significantly more transposition errors than the control group. However, the transposition error rate was the highest for HZ and EY – each of them had significantly more transpositions than MA, ED, NL, and OZ ($\chi^2 > 15.91$, two-tailed $p < .001$), whose error rates were similar (no pairwise differences between MA, ED, and OZ, $\chi^2 < .84$, two-tailed $p > .36$; and NL had fewer errors). HZ and EY were also the only participants whose error rates exceeded those of the worst-performing control participant.

HZ also had many substitution errors, but her predominant error type was still transpositions: they were more frequent than her substitutions ($\chi^2 = 31.7$, one-tailed $p < .001$), and they all went undetected by her, whereas she self-corrected all but 5 substitution errors.

The same-different decision task does not require verbal production of numbers. Thus, HZ's and EY's high transposition error rates clearly indicate that they have a digit-order encoding deficit in the visual analyzer. The other participants had a more-moderate (even if significant) transposition error rate in this task. One possibility is that they had a milder digit-order encoding deficit. Another possibility, however, is that their transposition errors reflect the normal difference of difficulties between transposition pairs and other pairs, which was observed even in the control group, and was amplified for the participants due to a general difficulty in number reading or in memory.

Table 7.6. Error percentages in tasks that tap the visual analyzer (tasks that involve visual digit input but do not involve production of verbal numbers). HZ and EY had high rates of transposition errors. MA, ED, NL, and UN had lower transposition error rates. OZ had many transposition errors only in one task.

Experiment 7.2 *– sequence identification*

	Sequence	Transposition	Substitution
HZ	1	83 ***	4 *
EY	1	36 +++	4
MA	1	0	0
ED	3	4	0
NL	4	2	0
OZ	7	17 ***	2
UN	6	4	0
Controls (SD)	3.3 (3.9)	2.7 (3.4)	0
EY Controls (SD)	0.9 (1.0)	1.3 (1.3)	0.1 (0.4)

Experiment 7.3 *– same-different decision*

	Equal	Transposition	Substitution
HZ	1	100 ***	39 +++
EY	0	63 +++	0
MA	0	14 +	3
ED	1	22 **	0
NL	0	6	0
OZ	0	19 **	3
Controls (SD)	1.2 (2.1)	4.7 (5.5)	0.6 (2.3)
EY Controls (SD)	0.1 (0.4)	1.9 (2.5)	0.4 (0.6)

Experiment 7.4 *– number matching*

	Equal	Transposition	Substitution	Number length
HZ	6	19 +++	4	29 +++
MA	0	3	0	5
ED	2	5	0	6
NL	7	0	0	0
OZ	6	2	0	2
Controls (SD)	3.1 (2.7)	0.3 (0.6)	0.2 (0.4)	0.5 (0.8)

Comparison with control group: + $p \leq .1$ * $p \leq .05$ ** $p \leq .01$ *** $p \leq .001$
+++ Errors ≥ 7%, control group ≤ 2%

7.3.3.1.3. Experiment 7.4: Number matching

In this variation of same-different decision, the participant was presented with a list of numbers and compared each number in this list to a fixed sample number. This task too involves visual digit input with no verbal output, so it taps the visual analyzer. The exact task design and stimulus selection were mainly motivated by considerations of diagnosing number length encoding impairments. These considerations will be explained in detail in Section 7.3.5.1.

7.3.3.1.3.1. Method

The task was designed as 10 blocks. In each block, the participants saw a sample number and 49 target numbers printed underneath. They were instructed to circle all targets that were identical with the sample number, working as accurately and quickly as possible. The sample numbers consisted of a digit that repeated 4 or 5 times, and one different digit in an interior position (e.g., 22322, 777747, etc.). Of the 490 target numbers, 191 were identical with the sample, 100 were derived from the sample by transposing two digits (777747-777477), 100 were derived by adding/deleting a repeated digit (number-length difference, 228222-22822), and 99 were derived by substituting the non-repeated digit (33533-33933). The numbers were printed on A4 paper, two blocks per sheet. EY and UN did not perform this task.

7.3.3.1.3.2. Results

Only HZ had significantly more errors than the control group in the transposition targets (Table 7.6). This further indicates that she had a digit-order encoding deficit in the visual analyzer, whereas MA, ED, NL, and OZ did not.

7.3.3.1.4. Interim summary: Digit order errors in the visual input tasks

The tasks described here, all of which specifically tap digit order encoding in the visual analyzer, showed a consistent pattern: EY and HZ had many digit order errors, whereas MA, ED, NL, and UN did not (except the same-different task, where they had transposition errors, but still significantly fewer than HZ and EY). This indicates that EY and HZ, but not the other participants, have a digit-order encoding deficit in the visual analyzer. The only inconsistent finding was OZ’s high rate of transposition errors in the sequence identification task (which was still much lower than EY’s and HZ’s). It is therefore possible that OZ too had a mild digit-order encoding deficit.

7.3.3.2. Output task: Number repetition – Experiment 7.5

The participants performed a number repetition task, which involves verbal production without visual digit input. This task taps the phonological retrieval mechanisms of number words (Dotan & Friedmann, 2015; McCloskey et al., 1986). If the transposition errors result from a deficit in phonological retrireval, they should appear in this task too. The task may also expose a verbal production difficulty in other stages (e.g., the generation of number word frame), but not necessarily: in another study we observed a patient with a deficit in verbal production of numbers, who nevertheless managed to repeat numbers correctly, apparently by using various strategies (Chapter 9).

7.3.3.2.1.1. Method

The experimenter said aloud each number and the participant repeated it. HZ, MA, and ED repeated the 120 numbers from Experiment 7.1. UN's digit span was very low, so he repeated 120 numbers in which only 2 or 3 digits were non-zero. The numbers had 3, 4, 5, or 6 digits (22, 39, 37, and 22 items per length, respectively). To allow for direct comparison of his number repetition with his number reading, in a separate session he also read the same numbers from paper. EY repeated 82 numbers – one block of 40 four-digit numbers, and another block of 42 five-digit numbers.

7.3.3.2.1.2. Results

All participants had almost no digit order errors in the repetition task (Table 7.7). This suggests that HZ's and EY's digit order errors in number reading (which we saw in Experiments 7.1 - 7.4) did not originate in an impaired production process, and certainly not in impaired phonological retrieval.

7.3.3.3. Interim summary: the assessment of digit order errors

The results of the experiments above are clear: HZ and EY had "digit order dyslexia" – a digit-order encoding deficit in the visual analysis of Arabic numbers. They had high rates of digit order errors in all tasks that involved visual digit input – reading aloud, same-different decision, sequence identification, and number matching, but only few order errors in number repetition, a task that involved verbal output without visual digit input. HZ's deficit was more severe. Indeed, this disturbed her in real life situations – e.g., she had a real difficulty when waiting in a bus station where both line 28 and 82 were stopping.

Table 7.7. Percentage of errors in the number repetition task (Experiment 7.5), which involved production of verbal numbers but did not involve visual digit input. The rate of digit order errors was low for all participants.

Task		Digit order	1st digit shifts	Decimal shifts	Substi-tutions	Thousand errors[a]	All errors[b]
Number repetition	HZ	1	0	0	3	0	4
	EY[c]	0	0	0	9	0	12
	MA	3	0	0	4	0	9
	ED	1	0	0	8	0	11 $^{+}$
	NL	1	1	1	11 *	0	16 **
	OZ	1	0	0	7	0	8
	UN[c]	0	13 $^{+++}$	29 $^{+++}$	27 $^{+++}$	2	48 $^{+++}$
	Controls (SD)	1.1 (1.2)	0.5 (1.2)	1.1 (1.0)	3.6 (3.5)	0	4.6 (3.6)
	EY controls (SD)	1.5 (1.1)	0	0	5.4 (5.4)	0	6.6 (5.8)
	UN controls (SD)	0.1 (0.2)	0.1 (0.3)	0.1 (0.3)	0.4 (0.4)	0	0.6 (0.7)
UN's reading of the same stimuli		0	48	48	7	0	52

Comparison to the control group: $^{+}$ $p < .1$ * $p \le .05$ ** $p \le .01$
$^{+++}$ Errors ≥ 7%, control group ≤ 2%

[a] The rate of errors related with the decimal word "thousand" was counted out of the 52 numbers that had a sufficient number of digits (5 or 6).

[b] The percentage of items with any error.

[c] The stimuli lists of EY and UN were different from those of the other participants.

MA, ED, NL, and UN had relatively few digit order errors in all tasks, indicating that they had intact digit order encoding in all stages.

OZ had no digit order errors in the output-only tasks and in the number reading task, and relatively few digit order errors in two of the three input-only tasks, but he had many transpositions in third input-only task (sequence identification). Thus, he may have had a mild impairment in digit order encoding in the visual analyzer.

7.3.4. Impaired encoding of 0 positions in the visual analyzer

In Experiment 7.1, EY showed an interesting performance pattern: she had many digit order errors when reading numbers that included only the digits 2-9, but virtually no errors when reading numbers that included also the digit 0. This is an important finding, as it suggests the existence of another mechanism, separate from the digit order encoder, which selectively encodes the position of 0. Presumably, this mechanism was spared for EY, and this is what allowed her to avoid order errors when the number included the digit 0. We further tested this dissociation using two experiments in which the presence of 0 in the number was carefully controlled. These experiments were administered to the two participants with digit-order encoding deficit, EY and HZ.

According to Cohen and Dehaene's (1991) model, a dedicated process encodes not only the positions of 0, but also of 1. To test this possibility, we also controlled for the presence of 1 in the number. Moreover, Cohen and Dehaene suggested that the importance of 1 is the verbal irregularity it creates when it appears in the decades position (it cues that the number should include an x-teen word). This may imply that 1 would have an effect only when appearing in the decades position. We therefore compared the participants' performance in numbers where the digit 1 appeared in different positions.

7.3.4.1. Experiment 7.6: reading numbers with 0, 1, or neither

7.3.4.1.1. Method

EY and HZ read 350 four-digit numbers: 100 numbers included the digit 0 in the hundreds or decades positions (x0xx and xx0x, 50 items per type), and 150 numbers included the digit 1 (xxx1, xx1x, and x1xx, 50 items per type). Additional 100 control numbers included neither 0 nor 1 and were derived from the xxx1 and xx1x numbers by substituting the digit 1 with a digit that was neither 0 nor 1 (xxx6 and xx3x). The 350 numbers were administered in random order in four blocks.

In Experiment 7.1, transpositions with 0 (e.g., 2304 $\rightarrow$ 2034) were classified as decimal shifts. Here, to avoid any bias that may artificially reduce order errors in numbers with 0, we classified transpositions with 0 as order errors.

7.3.4.1.2. Results

Both participants had many digit order errors (Table 7.8). Importantly, EY had merely a single error in numbers with 0, more order errors in numbers with 1 (χ^2 = 16.6, one-tailed p < .001), and even more order errors in numbers with neither 0 nor 1 (χ^2 = 25.5, one-tailed p < .001), replicating the dissociation she showed in Experiment 7.1 between numbers with and without 0. Within numbers with 1, she made more order errors involving the digit 1 than order errors not involving 1 (χ^2 = 4.62, one-tailed p = .02). Her performance was unaffected by the position in which the digit 1 appeared: she had similar digit-order error rates in xxx1 (14%), xx1x (20%), and x1xx (18%, $\chi^2(2)$ = .54, two-tailed p = .76), and for each decimal position of 1, the error rate in numbers with 1 was lower than in numbers with 2-9 ($\chi^2 > 10.31$, one-tailed $p < .001$).

Table 7.8. Error percentages in Experiment 7.6 – reading aloud numbers with 0, with 1, or with only the digits 2-9. EY had fewer order errors in numbers with 0/1 than in numbers without these digits. HZ showed no such sensitivity to 0/1.

		Numbers with 0	Numbers with 1	Numbers with only 2-9
EY	Order errors	1	17	47
	Transpositions with 0/1	0	6	
	Only in the digits 2-9	1	13	
	All errors [a]	2	20	48
HZ	Order errors	59	35	42
	Transpositions with 0/1	50	18	
	Only in the digits 2-9	10	19	
	All errors [a]	62	42	45

[a] The percentage of items with any error.

HZ did not show this kind of sensitivity to 0 and 1. In fact, she showed the opposite pattern: *more* digit order errors in numbers with 0 than in numbers without 0, 1 ($\chi^2 = 5.78$, two-tailed $p = .02$). Table 7.8 shows that this pattern resulted from her high rate of transpositions of 0 with another digit (e.g., 4302 → 4032), suggesting that at least some of these errors were in fact decimal shifts rather than order errors. This interpretation is supported by two findings: first, when excluding transpositions of 0 with another digit (and correspondingly excluding from the control numbers transpositions of 3 or 6 with another digit), HZ showed similar order error rates in numbers with 0 (10%) and without 0,1 (12%, $\chi^2 = 0.2$, one-tailed $p = .32$). Second, when we compared HZ's transpositions of 0 with another digit against her transpositions in the same decimal positions in the numbers without 0-1, we observed more transpositions with 0 (50% versus 21%, $\chi^2 = 18.36$, $p < .001$).

The different patterns exhibited by HZ and EY cannot be explained by the slightly different methods of stimulus presentation (EY read the numbers on a computer screen with limited exposure, HZ read them from paper): HZ's error pattern did not change when she re-read the Experiment 7.6 stimuli under EY's conditions (from a computer screen with 400 ms exposure). Crucially, HZ's and EY's different stimulus presentation methods cannot explain the main finding in the present experiment – the effect of 0 and 1 on EY's reading.

The results can also not be attributed to visual differences between 0 and 1 and the other digits. According to such a visual account, what helped EY was visual parameters such as the unique shape of 0 (circle) and 1 (line). To rule out this explanation, we administered EY a control experiment in which she saw a circle-shaped character that was not zero. We capitalized on EY's letter position dyslexia in word reading, and on the fact that the Hebrew letter Samekh (ס, pronounced /s/) has a circle-like shape, similarly to the English letter O. EY read a list of 51 words with the letter ס as a middle letter (because letter position dyslexia affects only middle letters), mixed with 51 words without ס. The words were presented on the computer screen for 400 ms in Guttman-Yad font (ס). The visual account predicts that EY would have fewer transposition errors in words with ס than in words without ס, but this was not the case: she had 24% migration errors in words with ס and 25% in words without ס ($\chi^2 = 0.50$, 1-tailed $p = .41$).

7.3.4.2. Experiment 7.7: same-different decision in numbers with 0, 1, or neither

Experiments 7.1 and 7.6 showed that EY can read numbers without digit order errors if the number includes 0. As we saw in Section 7.3.3, EY's digit order errors originate in a visual analyzer deficit. We therefore hypothesized that her ability to avoid digit order errors also originates in the visual analyzer. To examine this hypothesis, we administered her a same-different decision task and manipulated the presence of 0 in the numbers. This task involves visual input but no verbal output, and as demonstrated in Experiment 7.3, EY's impaired digit order encoder fails in distinguishing between numbers that differ in the order of digits. If her visual analyzer can avoid digit order errors in numbers with 0, EY should be able to tell apart transposed pairs that contain 0. HZ performed the task too, as control.

7.3.4.2.1. Method

HZ and EY saw 300 pairs of 4-digit numbers and decided, for each pair, whether the two numbers were identical (143 pairs) or differed in the order of two adjacent digits (157 pairs). Of the transposition pairs, 53 pairs contained the digit zero, 51 pairs contained the digit 1, and 53 pairs contained only the digits 2-9. Both 0 and 1 could appear in the units (19%), decades (15%) or hundreds (66%) position. The numbers in each pair appeared next to each other on the computer screen. The rest of the methodological details were like in the same-different experiment described above (Experiment 7.3).

7.3.4.2.2. Results

Similarly to number reading (Experiments 7.1, 7.6), EY had significantly fewer errors in detecting transpositions when the numbers included the digit 0 than when they included only the digits 2-9 (Table 7.9, $\chi^2 = 16.5$, one-tailed $p < .001$). However, unlike Experiment 7.6, the existence of 1 in the number did not improve EY's performance ($\chi^2 = 0.27$, one-tailed $p = .30$). The specific position of the digit 0 or 1 had no significant effect on EY's error rate (Fisher's $p = .44$ for 0, Fisher's $p = .34$ for 1).

HZ did not show a facilitating effect of 0 or 1 compared to the 2-9 pairs ($\chi^2 \leq 0.66$, one-tailed $p \geq .21$), replicating her performance pattern in the reading aloud task.

Table 7.9. Percentage of errors in same-different decision (Experiment 7.7). EY had fewer errors in numbers with 0 than in other numbers, whereas HZ showed no sensitivity to 0.

	Pairs differing in digit order			
	Only 2-9	With 1	With 0	Identical pairs
HZ	68	73	60	20
EY	47	41	11	3

7.3.4.3. Interim summary: the assessment of 0-position encoding

The two number reading Experiments (7.1, 7.6) clearly show that EY had a highly selective deficit in number reading: she had difficulty in digit order encoding, but this difficulty had almost no impact on numbers that included the digit 0. A similar facilitating effect of 0 was observed in a task with visual input and no verbal output (same-different, Experiment 7.7). Our best explanation to this pattern is that the visual analyzer has a dedicated sub-process that detects the presence of 0 in the number and encodes its positions, as part of the decimal structure extraction (Fig. 7.1). EY had selective impairment in the digit-order encoding mechanism, but her 0-detector was still intact. This allowed her, for numbers with 0, not only to identify the position of 0 but also to use it as pivot for ordering the remaining digits. HZ was impaired in both processes, so the presence of 0 in the number did not help her.

The findings were slightly different with respect to the digit 1. The presence of 1 in the number helped EY to avoid digit order errors in reading aloud but not in the input-only task (same-different). This suggests that 1 has a special status in the speech production stage but not in the visual analysis stage. We further elaborate on the implications of this finding in the Discussion of this chapter.

7.3.5. Impaired processing of the number's structural information

In Experiment 7.1, all participants except EY had many decimal shift errors. These errors occurred mainly in the leftmost digits, a pattern that can potentially result from impairments in several possible sub-processes, all of which handle the number's decimal or verbal structure. In the present section we identify, per participant, the locus of deficit underlying these first-digit shift errors. One possibility is that the errors result from erroneous encoding of the number length in the visual analyzer, which would make participants produce a number as if it had fewer digits or more digits (e.g., 4,320 → 43,200). Section 7.3.5.1 examines this possibility. A second possibility, examined in Section 7.3.5.2, is that the errors result from impaired triplet parsing in the visual analyzer (e.g., 4320 → 43,20 → "forty three, twenty"). A third possibility, assessed in Section 7.3.5.3, is that first-digit shift errors result from impaired detection of 0's and their positions: ignoring a 0 or encoding an excessive 0 would change the perceived number of digits in the number (e.g., 4,320 → 43,200), and transposing a 0 would shift the decimal position of the transposed non-0 digit (e.g., 4,320 → 4,302). Finally, in Section 7.3.5.4 we examine the possibility that the decimal shift errors result from impaired generation of number word frames in the verbal production stage. Such impairment could potentially distort the number length, the positions of 0's, or the number's triplet structure.

7.3.5.1. Can decimal shift errors result from impaired number-length encoding in the visual analyzer?

The participants performed two visual tasks without verbal production, which were sensitive to number length: same-different decision and number matching. If the participants have impaired number-length detection in the visual analyzer, they should have difficulties in these tasks.

Both tasks required the participants to judge whether visually presented numbers were identical or not. Pilot experiments suggested a major methodological challenge in designing this kind of tasks: participants often rely on alternative strategies rather than on number length information. For example, if we ask whether 1234 and 12345 were identical, the participant could detect the difference by relying on the digit identities (only the second number has "5"). In the pair "1234 =? 12343", they could rely on the order between 3 and other digits. Thus, pairs such as 1234-12345 and 1234-12343 could yield good performance even if number length encoding is impaired. To prevent these alternative strategies, we used numbers in which all digits but one were identical (e.g., 99949). Number length was manipulated by changing the number of instances of the repeated digit, e.g., 99949-9949: both numbers contain only 4’s and 9’s and in the same relative order, so they are indistinguishable by digit identity and digit order. In the supplemental online material, we discuss more fine-grained methodological aspects of these tasks.

7.3.5.1.1. Experiment 7.8: same-different decision

7.3.5.1.1.1. Method

The participants saw 240 pairs of numbers with 3-6 digits, and decided whether the two numbers in each pair were identical or not. In all numbers, one digit was non-9 and the other digits were 9. There were 120 identical pairs and 120 different pairs. In the different pairs, the second number was derived from the first by adding or removing a single 9 digit (e.g., 99949-9949, or 99949-999499, 60 pairs), or by substituting the non-9 digit (e.g., 99949-99979, 60 pairs). The two numbers appeared in the center of the screen one after another for 1000 ms each, with a 500 ms delay between them. The participants answered using two keyboard keys. EY did not perform this task. HZ had very high error rates in all stimulus types, suggesting impulsivity, so she later performed the task again while answering verbally rather than with the keyboard. We report her performance in both response methods.

7.3.5.1.1.2. Results

If a participant has a selective deficit in the decimal structure analyzer, his error rate in the length-differing pairs should be higher than the control group's. It should also be higher than the participant's own error rate in the substitution pairs. This pattern was observed for HZ and MA (Table 7.10): they had significantly more errors than the control group in the length-differing pairs (HZ had more errors in all stimulus types, indicating a general difficulty in this task, but the difference was most evident in the length-differing pairs). They also had more errors in length-differing pairs than in substitution pairs (HZ: χ^2 = 13.1, one-tailed p < .001; MA: χ^2 = 9.84, one-tailed p < .001). The control group had similar error rates in the length-differing pairs and the substitution pairs (paired t(19) = 0.62, two-tailed p = .54, Cohen’s d = 0.28).

Table 7.10. Percentage of errors in Experiment 7.8 – same-different decision. If number-length detection in the visual analyzer is impaired, the participant's error rate in length-differing pairs should be higher than their error rate in the other types of pairs, and higher than the control group's error rate in length-differing pairs. Only HZ and MA showed this pattern.

	Length difference	Digit substitution	Identical
HZ (keyboard answer)	70^{+++}	23^{***}	15^{***}
HZ (verbal answer)	18^{+++}	8^{**}	3
MA	20^{+++}	2	5^{+}
ED	5	7^{*}	5^{+}
NL	0	0	1
OZ	3	2	8^{***}
UN	22^{+++}	37^{***}	18^{***}
Control group (SD)	1.9 (2.4)	2.3 (2.0)	2.5 (1.5)

Comparison to control group: $^{+}$ $p < .1$ * $p < .05$ ** $p < .01$ *** $p \leq .001$
$^{+++}$ Errors ≥ 7%, control group ≤ 2%

The other participants – ED, NL, OZ, and UN – did not show this pattern of results. None of them had more errors in length-differing pairs than in substitution pairs, and none had more length errors than the control group (UN did have more errors than the control group, but in all pair types rather than just in the length-differing pairs). These findings indicate that HZ and MA, but not ED, NL, OZ, and UN, had a deficit in encoding number length encoding in the visual analyzer.

7.3.5.1.2. Number matching

This task, described in Section 7.3.3.1.3 (Experiment 7.4), required comparing several numbers to a fixed sample number. The number could be identical with the sample, or differ from it in the number of digits (number length), the order of digits, or the identity of digits. People with a deficit in number length encoding in the visual analyzer are expected to show a higher error rate in length-differing targets than in substitution targets. We also expect their error rate in the length-differing targets to be higher than the control group's. Such a pattern was observed only for HZ (Table 7.6; length-differing targets vs. substitution targets: $\chi^2 = 11.05$, one-tailed $p < .001$). MA and ED showed a partial match to this pattern: they had more errors in length-differing targets than in substitution targets ($\chi^2 > 5.08$, one-tailed $p \leq .02$), but their length error rate did not exceed the 7% criterion that we set as the threshold to count as significantly worse than the control group. NL and OZ did not have many length-related errors ($p \geq .25$ for length vs. substitution, and neither had more length errors than the control group).

Thus, this task indicates that HZ, and perhaps MA and ED too, had impaired number length encoding in the visual analyzer, but NL and OZ did not.

7.3.5.1.3. Interim summary: first-digit shift errors in the visual analyzer

We examined number-length encoding in the visual analyzer using two tasks. In each task, we used two criteria for impaired performance: having more number-length errors than other error types, and having more number-length errors than the control group. This resulted in four statistical tests for the participants' number length encoding (two tasks, two tests per task). HZ had a high rate of number-length errors according to all four tests, MA had high rates of number-length errors according to three tests, and ED showed a high rate of number length errors only in one test. The remaining participants – NL, OZ, and UN did not show high rates of number-length errors in either of the tasks. These results indicate that HZ and MA, but not the other participants, had impaired encoding of number length. Both tasks tapped the visual analyzer, indicating that this was the locus of the number-length encoding deficit.

7.3.5.2. Can decimal shift errors result from impaired triplet parsing in the visual analyzer?

Another possible reason for making first-digit shift errors is a deficit in triplet parsing. For example, if the digits of 54321, which should be grouped as 54,321, were grouped as 543,21, the result would be a first-digit shift – saying "five hundred" instead of "fifty". We reasoned that if this was the source of the participants' decimal shift errors in number reading, the errors should disappear if we provide them with explicit cues about the correct way to parse the number into triplets. As a parsing cue, we used a standard comma separator between the hundreds and thousands digits.

7.3.5.2.1. Experiment 7.9: Reading numbers with a comma separator

The participants read aloud the 120 numbers that were presented in Experiment 7.1, but unlike Experiment 7.1, here they were presented with a comma separator between the thousands and hundreds digits (e.g., 54,321, whereas in Experiment 7.1 it was 54321). We reasoned that the comma separator would provide a bypass strategy for parsing triplets if the participant's visual analyzer had difficulties in doing that. If a participant's decimal shifts in reading aloud originate in a triplet parsing deficit, the comma separator should provide him with a visual cue to improve the parsing, and the participant should therefore make fewer first-digit shift errors here than in Experiment 7.1. The comma separator may also help participants with impaired number-length encoding in the visual analyzer, because this visual cue could help estimating the number length. In contrast, the visual manipulation of adding a comma is not expected to help participants whose first-digit shift errors originate in impaired production processes. We also hypothesized that the comma separator would have no effect on digit-order encoding, and consequently would not decrease the rate of transpositions.

Table 7.11. Error percentages in reading numbers with 4-6 digits with comma separator (e.g., 12,345, Experiment 7.9) and without comma (12345, Experiment 7.1). The visual manipulation of adding the comma, which presumably affects only the visual analyzer, improved the reading of HZ, MA, and ED, but did not help NL, OZ, and UN.

	With comma (Experiment 7.9)				Without comma (Experiment 7.1)			
	1st-digit shift	Order	Thousand[a]	All	1st-digit shift	Order	Thousand[a]	All
HZ	8***	18	0	51	33	21	0	52
MA	4***	4	4	13*	20	4	0	24
ED	0***	2	1	3***	21	3	6	28
NL	9	3	0	16	11	4	1	20
OZ	16	0	3	29	19	1	8	39
UN	18	1	16	39	18	1	8	49

Comparison between conditions: * $p \leq .05$ *** $p \leq .001$

[a] The rate of errors related with the decimal word "thousand" was counted out of the 52 numbers that had a sufficient number of digits (5 or 6).

Table 7.11 compares the participants' reading with a comma separator versus their reading without it, using McNemar test. Only the 90 numbers that can include a comma (4-6 digits) were analyzed. The addition of comma separator clearly reduced the first-digit shift error rate for HZ, MA, and ED, but not for NL, OZ and UN. This indicates that the deficit of HZ, MA, and ED was in the visual analyzer, either in encoding the number length or in parsing the triplets. Because we already concluded above that ED does not have a number-length encoding deficit in the visual analyzer, the present results indicate that she had impaired parsing of triplet in the visual analyzer.

NL, OZ, and UN did not gain from the addition of a comma separator. This finding can be interpreted in two ways: either their deficit was not visual, or they had a double deficit – a visual deficit and another deficit – and the second deficit made them err even when the numbers were presented with a comma separator. We resolve this ambiguity later below (Section 7.3.5.4.3) by considering the results in the present task in conjunction with other tasks.

HZ's performance in this task is interesting also from another respect. The addition of a comma separator significantly decreased her first-digit shift errors but not her digit order errors. The difference between the comma's effects on the two error types was significant (an analysis of the Experiment x Error Type x Success contingency table showed a three-way interaction: $\chi^2(4) = 7.64$, one-tailed $p = .05$). This within-participant dissociation between order errors and first-digit shifts further supports our earlier conclusion that order errors and first-digit shift errors originate in two distinct processes.

7.3.5.3. Can decimal shift errors result from impaired 0 detection in the visual analyzer?

Another possibility is that the decimal shift errors result from impaired 0 detection in the visual analyzer. According to this view, decimal shifts occurred because the participants incorrectly encoded the presence of 0 in the number: ignoring a 0, or encoding an excessive 0, results in encoding a number as having too many or too few digits. In essence, the argument here is very similar to the possibility of a number-length encoding deficit, but it assumes that the change in number length did not result from a number length error per-se, but is the indirect result of incorrect detection of 0's. We hereby examine several specific predictions derived from this view. As we shall see, none of the participants fulfilled these predictions.

First, if a person's decimal shifts result from a visual encoding deficit, that person should show more decimal shift errors in numbers with 0 than in numbers without 0. In the number reading task (Experiment 7.1), no participant showed this pattern (Table 7.12).

Table 7.12. Error percentages of decimal shift errors in Experiment 7.1 (number reading). Contrary to the view that these errors result from erroneous encoding of 0 positions, no participant showed: (a) more decimal shifts in numbers with 0; or (b) a tendency of first-digit shift errors to lengthen numbers without 0 (which could be explained as an addition of 0) rather than to shorten these numbers (which is unexplained as a 0 effect).

	Decimal shifts in numbers…		Numbers without 0: first-digit-shifts resulted in…	
	with 0	without 0	longer number	shorter number
HZ	25	30	11	17
MA	9	21	10	11
ED	14	19	10	10
NL	9	14	5	10
OZ	16	14	6	6
UN	16	21	6	11

The second prediction starts with recognizing that an impaired 0 detector can cause a first-digit shift error only by omitting a 0 or encoding an excessive 0. If the target number does not include 0, it is obviously impossible to omit a 0, so an impaired 0 detector can only create first-digit shift errors that make the number longer. A clear prediction follows: in numbers without 0, such a person would have more first-digit shifts that make the number longer than the target and fewer shifts that make the number shorter than the target. This prediction was no affirmed for any of the participants – in fact, for all participants, the number of “longer” errors was smaller than or equal to the number of “shorter” errors (Table 7.12).

Third, a person whose decimal shifts originate in impaired 0 encoding should perform well in tasks that do not include numbers with 0, e.g. the number comparison tasks that we used – number matching and same-different (Experiments 7.4 and 7.8). Contrary to this prediction, MA had number-length errors in Experiment 7.8, and HZ had such errors both in Experiment 7.4 and in Experiment 7.8. These errors cannot be explained by a 0-encoding deficit – they can be explained only as a number-length encoding deficit.

Finally, explaining decimal shifts as resulting from bad 0 detection does not explain why, for some participants, decimal shifts were nearly eliminated by the addition of a comma separator (Experiment 7.9), because there is no clear reason why the comma separator should facilitate the detection of the presence of 0 in the number or the detection of 0 positions (especially given that the comma did not facilitate the order encoding for other digits).

All these findings indicate that erroneous 0 detection does not account for the decimal shift errors of any of the participants in this study. However, it is still possible that other individuals, who may have a selective deficit in 0 detection in the visual analyzer, would make decimal shift errors as a result. In such cases, we would expect an error pattern different from the ones observed for our participants: (1) there would be more decimal shift errors in numbers with 0 than in numbers without 0; (2) only lengthening errors would occur in numbers without 0; (3) the person would succeed in the number comparison tasks that we used here; and (4) the addition of a comma separator would not reduce the number of decimal shifts. Finally, our findings still allow the possibility of a double deficit – i.e., participants in this study, whose decimal shifts are explained by another impairment, may still have a 0 detection deficit on top of that impairment.

7.3.5.4. *Can decimal shift errors result from impaired generation of number word frames in verbal production?*

The last locus of deficit we considered as a possible origin for first-digit shift errors was the generation of number word frames in verbal production. Generation of incorrect frames could result in decimal shift errors of all kinds, including first-digit shifts. We assessed this generation process with three tasks that allow for first-digit shift errors. Two tasks – number repetition and multiply/divide by 10 – involved verbal production of number words that were not presented visually. We predicted that individuals with impaired generation of number word frames would perform poorly in these tasks, but individuals with a selective deficit in the visual analyzer would perform well. The third task (Experiment 7.11) was number reading, with a manipulation that was designed to improve the reading of participants with impaired generation of number word frames.

7.3.5.4.1. Number repetition

The number repetition task was already described above (Section 7.3.3.2, Experiment 7.5). All participants but UN did not make any first-digit shift errors in this task, nor did they have other decimal shifts (Table 7.7). However, this does not necessarily indicate good processing of number length in the verbal production stage: the number repetition task may be too easy and may allow for alternative strategies – e.g., because the participants hear the number's verbal structure and do not have to produce it on their own. Indeed, in another study we observed a patient with a deficit in verbal production of numbers, who nevertheless managed to repeat numbers correctly (Chapter 9).

UN was the only participant who had many first-digit shift errors in number repetition, suggesting he had a deficit in verbal production. Unlike the reading task, here UN's errors were not restricted to first-digit shifts (13%) – he also had many shifts in the beginning of the second triplet (18%, and only 5% errors in mid-triplet digits). His first-digit shift errors can be interpreted in two ways. One possibility is that he had impaired generation of number word frames. According to this interpretation, unlike the other participants, UN was unable to use bypass strategies such as word-by-word repetition (perhaps because of his severely impaired working memory) or morphological cueing (perhaps due to his morphological deficit). A second interpretation is that UN had a later deficit – in phonological retrieval – which corrupted the lexical class information (ones, tens, teens etc.; Dotan & Friedmann, 2015; patient JG in McCloskey et al., 1986). We revisit these possibilities in the Discussion of this chapter.

7.3.5.4.2. Experiment 7.10: Multiply / divide by 10

In this task, the participant saw simple exercises of multiplication or division by 10, read the exercise aloud, and then said the result. This task taps the production processes for several reasons. First, we presented the numbers with a comma separator, which helps an impaired visual analyzer. Second, because the participants read the exercise aloud, we could rely on correct reading of the exercise as an index to good visual analysis of that exercise. Third, because the produced number was different from the one printed on paper, the information about the number to produce did not arrive directly from the visual analyzer, but from the calculation mechanism (i.e., the task did not involve the standard digit-to-verbal transcoding pathway). In the supplementary online material, we further discuss some methodological aspects of this task.

7.3.5.4.2.1. Method

The participants saw a list of 28 multiplication problems mixed with 28 division problems. The numbers were printed with a comma separator between the hundreds and thousands digits. The first operand had 3-5 digits, with two non-zero digits and then zeros, and the second operand was always 10 (e.g., “6,500 x 10”, “740 ÷ 10”), i.e., the results had 2-6 digits. The participants read aloud each problem and then said the result.

7.3.5.4.2.2. Results

The participants had some errors in reading the exercises, but most of these errors were self-corrected prior to providing the answer. Importantly, except UN, there was not even a single case of an uncorrected reading error followed by an incorrect answer. For UN there were 8 such cases, but in none of them could the erroneous result be explained by the reading error. Thus, the errors reported below originate in a production difficulty and not in a visual analysis difficulty.

Table 7.13. Error percentages in the multiply/divide-by-10 task (Experiment 7.10), which specifically taps the verbal output processes. To examine the processing of number length, we inspected the first-digit shift errors. NL, OZ, and UN had high rates of such errors, whereas the other participants did not.

	First-digit shift	Substitution	Thousand [a]	Transposition	All errors [b]
HZ	4	0	3	2	7 $^{+}$
MA	2	0	5	0	5
ED	5	0	0	0	5
NL	20 ***	0	0	0	20 ***
OZ	9 **	0	0	2	11 **
UN	43 ***	20 $^{+++}$	5	0	57 ***
Controls (SD)	2.0 (2.7)	0	0.1 (0.6)	0	2.9 (3.0)

Comparison to control group: $^{+}$ $p < .1$ ** $p \leq .01$ *** $p \leq .001$ $^{+++}$ Errors ≥ 7%, control group ≤ 2%

[a] The rate of errors related with the decimal word "thousand" was counted out of the 37 numbers that had a sufficient number of digits (5 or 6).

[b] The percentage of items with any error.

NL, OZ, and UN had high rate of first-digit shift errors (Table 7.13), indicating that they had a number-length processing deficit in verbal production. The other participants did not have a high rate of first-digit shift errors, indicating spared verbal production processes.

7.3.5.4.3. Experiments 7.11 and 7.12: Reading numbers as triplets

The last pair of experiment to assess verbal production was a variation of number reading. In a way, they are the verbal correlate of Experiment 7.9, where we used a visual manipulation (comma separator) to help participants with impaired visual analyzer. Here, Experiment 7.11 used a verbal manipulation designed to help participants with impaired generation of number word frame (but not participants with a visual analyzer deficit). Numbers were presented exactly as in the number reading experiments, but the required verbal output was simplified: participants were asked to read each number as two shorter numbers, up to 3 digits long (e.g., the number 54321 was to be read as “fifty four *and then* three hundred and twenty one”). In this reading mode, participants never had to produce a number longer than 3 digits, so they would only need to generate short number word frames (e.g., for a 5-digit number they would generate a 2-digit frame and a 3-digit frame). The visual analysis in this task, however, is as demanding as in Experiment 7.1. If a participant has decimal shift errors in standard reading (Experiment 7.1) but not here, this would indicate that the decimal shift errors originate in a verbal production deficit.

Note that even if a person does have a production deficit that yields first-digit shifts, the verbal manipulation of Experiment 7.11 may fail to eliminate decimal shift errors if this person has an additional impairment that yields such errors – e.g., a visual analyzer deficit in the number length encoder or in parsing the number to triplets. To address such cases, Experiment 7.12 combined the manipulations of Experiments 7.11 and 7.9: the participants saw the numbers with a comma separator (which should help coping with the visual impairment) and read them as pairs of shorter numbers (which should help coping with the verbal impairment).

7.3.5.4.3.1. Method

The participants saw the 120 numbers from Experiment 7.1 and read aloud each number as described above: 3-digit numbers were read like in Experiment 7.1, and each longer number was produced as two shorter numbers, separated by "and then" (the single word /ve-az/ in Hebrew). Participants were instructed to split the numbers in two such that the second number would have 3 digits, and we verified (with examples) that they understood these instructions. They were also given examples for each number length between 3 and 6 digits. In Experiment 7.11 the numbers were presented without a comma separator (like in Experiment 7.1), and in Experiment 7.12 they were presented with a comma separator (like in Experiment 7.9). The results were compared against the participants' reading in Experiment 7.1 using McNemar test. Only the 90 numbers with 4-6 digits were analyzed, because shorter numbers are produced in the same manner in the two experiments.

7.3.5.4.3.2. Results

Reading numbers as triplets clearly helped OZ (Table 7.14): his first-digit shift error rate in Experiment 7.11 was no longer significantly higher than the control group's, and was lower than when reading the same digit strings as whole numbers (in Experiment 7.1). This pattern indicates that OZ's first-digit shift errors originated in a production deficit.

Reading as triplets also helped HZ, but to a lesser extent: she had fewer first-digit shift errors in Experiment 7.11 than in Experiment 7.1, indicating that at least some of her first-digit shift errors originate in impaired production processes. Nevertheless, even when reading as triplets, she still had more first-digit shift errors than the control group, indicating that some of her first-digit shift errors originated in another process – presumably in her visual analyzer deficit. Indeed, in Experiment 7.12, where we used both the visual and verbal easing manipulations, her error rate was even lower (Experiment 7.11 versus 7.12: McNemar $\chi^2 = 9.8$, one-tailed $p = .001$).

Thus, HZ had a double deficit, in the visual analyzer as well in the production stage. Her remaining errors in Experiment 7.12 can be explained by the severity of her deficit – even when she read numbers with only 1-3 digits, she had 6% first-digit shifts, which is similar to her error rate in Experiment 7.12 (χ^2 = 1.43, one-tailed p = .12).

Table 7.14. Error percentages in reading numbers with 4-6 digits – as a whole number ("12 thousand, 345", Experiment 7.1) or when saying each number as two shorter numbers ("12 and then 345", Experiment 7.11). This manipulation, designed to ease on an impaired production process, helped OZ but not ED and NL. HZ had fewer errors in reading as triplets than in whole-number reading, but still more errors than the control group.

	As triplets, no comma (Experiment 7.11)[a]		As triplets, with comma (Experiment 7.12)		Whole number (Experiment 7.1)	
	1st digit shift	All	1st digit shift	All	1st-digit shift	All
HZ	20 +++	41	4	24	33 +++	52
ED	13 +++	21	3	7	21 +++	28
NL	18 +++	19	1	6	11 +++	20
OZ	6	10	7	8	19 +++	39
Controls (SD)	1.5 (1.7)	3.2 (2.5)	–	–	1.4 (1.2)	3.5 (1.6)

Comparison to the control group: +++ Errors ≥ 7%, control group ≤ 2% errors

[a] Comparison of 1st-digit shifts between Experiments 7.1 and 7.11: $p < .02$ for HZ, $p = .004$ for OZ, no significant effect for ED and NL

ED did not gain from reading as triplets – her first-digit shift error rate in the Experiment 7.11 was not significantly lower than in Experiment 7.1, and was higher than the control group's. This indicates that her first-digit shifts originate in another, pre-production process, in perfect agreement with our earlier conclusion that she had a visual analyzer deficit. The present results cannot indicate whether she had a production deficit on top of her visual deficit or not. Based on her good performance in the other production task (Experiment 7.10), we assume that she did not.

NL, ED's sister, also did not gain from reading as triplets (Experiment 7.11). Remember that unlike ED, she also did not gain from the visual manipulation of displaying the number with comma separator (Experiment 7.9). However, when both manipulations were used in conjunction – i.e., when the numbers were presented with a comma separator and she read them as triplets (Experiment 7.12), she had merely one first-digit shift error (significantly fewer than in Experiment 7.1, McNemar χ^2 = 10.29, p = .001) and no other decimal shift. This pattern indicates that NL's first-digit shift errors originated in a double deficit: in the visual analyzer and in verbal production. The comma separator helped the visual deficit and reading as triplets helped the verbal. Neither manipulation on its own was sufficient to improve her performance, because neither addressed the full problem. Only applying both manipulations in conjunction helped her.

7.4. Summary: the participants' impairments, dissociations, and loci of deficit

All participants in this study had difficulties in number reading. We ran a series of number processing tasks to identify the functional locus of deficit underlying the number reading difficulties of each participant. The results of these tasks for each participant, and our conclusions about the functional locus of deficit of each participant, are summarized below and in Table 7.15.

Table 7.15. The locus of deficit for each participant

	Visual analysis					Verbal production	
	Digit identity	Digit order	Number length	0 positions	Triplet structure	Verbal structure	Phonological retrieval
EY	✓	✗	✓	✓	✓	✓	✓
HZ	✓	✗	✗	✗	?	Mild(?)	✓
MA	✓	✓	✗	?	?	✓	✓
ED	✓	✓	✓	?	✗	✓	✓
NL	✓	✓	✓	?	✗	✗	✓
OZ	✓	Mild(?)	✓	✓	✓	✗	✓
UN	✓	✓	✓	✓	✓	✗	✓

EY had many digit order errors when she read numbers aloud. She made digit order errors in number reading and in tasks that involved visual digit input without verbal production (hereby "visual input tasks") – sequence identification (Experiment 7.2) and same-different decision (Experiment 7.3). Conversely, she did not have order errors in a number repetition task (Experiment 7.5), which involved verbal output without visual digit input (hereby "verbal production task"). This pattern indicates that she had impaired digit order encoding in the visual analyzer. Order errors were absent from numbers with 0, i.e., the digit-order encoding deficit did not interfere with her ability to encode the positions of 0. This pattern suggests the existence of an additional process that specifically detects zeros and their positions. EY did not have decimal shift errors, indicating that her processing of the number's decimal and verbal structure was intact. In particular, her digit-order encoding deficit did not interfere with her ability to encode the number length.

HZ had many digit order errors. Similarly to EY, these errors occurred in number reading and in the visual input tasks – sequence identification (Experiment 7.2), same-different decision (Experiment 7.3), and number matching (Experiment 7.4) – but not in the verbal production task (number repetition, Experiment 7.5). This indicates that she too had a deficit in digit order encoding in the visual analyzer. Unlike EY, she had digit order errors even in numbers with 0: namely, whereas EY's dissociation suggests the existence of a dedicated "0 detector", HZ's performance indicates that she had an impairment in this 0 detector.

HZ also had many first-digit shift errors in number reading – decimal shifts of the first (leftmost) digits of the numbers. These decimal shift errors originated in a visual analyzer deficit: they occurred in visual input tasks (same-different decision, Experiment 7.8, and number matching, Experiment 7.4), and the visual manipulation of adding a comma separator (Experiment 7.9) reduced the rate of decimal shifts. The specific deficit that can explain decimal shifts is an impairment in number length encoding or in triplet parsing. The finding of number-length errors in the visual input tasks indicates that HZ had a number length encoding deficit. Our findings are insufficient to tell whether she had impaired triplet parsing too. HZ did not have decimal shifts in the verbal production tasks (number repetition, Experiment 7.5, and multiply/divide by 10, Experiment 7.10), but her decimal shift rate decreased by a verbal manipulation aimed to improve reading in case of impaired production (Experiment 7.11). Thus, it is possible that some of her decimal shift errors originated in a mild production deficit.

MA had many first-digit shift errors in number reading. Her performance indicates that her first-digit shifts originated in a visual analysis deficit: the errors appeared in a visual input task (as number-length errors in same-different decision, Experiment 7.8), and their rate dropped when we introduced the visual manipulation of adding a comma separator (Experiment 7.9). Conversely, she did not make decimal shifts in verbal production tasks (number repetition, Experiment 7.5, and multiply/divide by 10, Experiment 7.10), and she did not gain from the verbal manipulation designed to help in case of impaired production (Experiment 7.11). The type of errors – first-digit shifts in the reading task, and number length errors in the visual input tasks – indicates that the impaired visual analyzer sub-process was the number length encoding. Our findings are insufficient to tell whether MA had impaired triplet parsing too. Importantly, she only had decimal shift errors (similarly perhaps to Noël & Seron, 1993), and did not have many errors of other types, in particular digit order errors. Thus, her digit order encoding was spared – a dissociation pattern opposite to EY's. Together, MA and EY show double dissociation between two visual analyzer sub-processes: digit order encoding and number-length encoding.

ED had many first-digit shift errors in number reading. These errors did not originate in a production deficit: she performed well in the verbal production tasks (number repetition, Experiment 7.5, and multiply/divide by 10, Experiment 7.10), and she did not gain from reading the numbers separated to triplets (Experiment 7.11) – a verbal manipulation designed to ease on impaired production processes. Her first-digit errors also did not originate in impaired encoding of number length by the visual analyzer, because she succeeded in the visual input tasks that tap number length encoding (same-different decision, Experiment 7.8, and number matching, Experiment 7.4). Her first-digit shift error rate dropped when she read numbers with a comma separator (Experiment 7.9) – a visual manipulation designed to ease on impaired parsing of triplets in the visual analyzer. We concluded that her deficit was in a process that parses digit strings into triplets in the visual analyzer.

NL, ED's sister, also had many first-digit shift errors in number reading. At least some of these errors originated in a production deficit: she made first-digit shift errors in a verbal production task (multiply/divide by 10, Experiment 7.10). Her success in the visual input tasks clearly indicates that her visual analyzer was intact in terms of processing the digit identity, digit order, and number length. She did not gain from the visual manipulation of adding a comma separator (Experiment 7.9), which was designed to ease on a visual analyzer deficit in number length encoding or parsing to triplets, nor did she gain from the verbal manipulation of reading the numbers separated to triplets (Experiment 7.11), which was designed to ease on impaired processing of the number's verbal structure in the production stage; but she had no first-digit shifts when both manipulations were used in conjunction (Experiment 7.12). We concluded that she had a double deficit: in parsing the number to triplets in the visual analyzer, and in the number word frame generation in verbal production. Adding a comma separator addressed the former deficit, reading the number as triplets addressed the latter, and only applying both manipulation in conjunction was sufficient to reduce her decimal shift errors.

OZ too had mainly decimal shift errors in number reading. These errors did not originate in impaired number-length encoding in the visual analyzer, because he succeeded in visual input tasks that tap this process without requiring verbal production (same-different decision, Experiment 7.8, and number matching, Experiment 7.4). His errors also did not originate in impaired triplet parsing in the visual analyzer: the rate of decimal shifts did not decrease following the visual manipulation of adding a comma separator (Experiment 7.10). Rather, his decimal shifts originated in impaired production processes: he had many decimal shift errors in a verbal production task (multiply/divide by 10, Experiment 7.10), and the rate of decimal shifts dropped when he read each number as two shorter numbers (Experiment 7.11) – a verbal manipulation designed to ease on number reading in case of impaired production. Within verbal production, OZ's deficit was not in the phonological retrieval process. Impaired phonological retrieval should cause random substitution of words, which should result in decimal shifts in all decimal positions, as well as in producing invalid number names (e.g., 32 → "thirty and twenty"). This was not the case for OZ: his decimal shifts occurred almost exclusively in the first digit/s of a triplet, and he did not produce invalid number names. Thus, his impairment was not in phonological retrieval, but in the generation of number word frames.

OZ had many digit order errors in one of his visual input tasks (sequence identification, Experiment 7.2), but we believe that he did not have a digit order impairment, or at least that it was very mild: first, his digit-order error rate in this task, although higher than the control group's, was lower than the other order-impaired participants (EY and HZ). Second, OZ did not have digit order errors in any other task, neither visual nor verbal: number reading, same-different decision (Experiment 7.3), number matching (Experiment 7.4), number repetition (Experiment 7.5), and multiply/divide by 10 (Experiment 7.10).

UN had many first-digit shift errors in number reading and in a verbal production task (number repetition, Experiment 7.5), but not in a visual input task (sequence identification, Experiment 7.2), indicating that his first-digit shifts originated in a production deficit. Like OZ, he did not have mid-triplet decimal shifts and rarely produced invalid number names, indicating that his deficit was not in phonological retrieval but in the generation of number word frames.

UN also had high rate of digit substitution errors, which appeared in number reading and in verbal production tasks but not in visual input tasks. Thus, his errors originated in impaired production processes. UN's type of errors – substitution of the digit value in production tasks only – resembles patient HY reported by McCloskey et al. (1986). It is possible that UN had, on top of his deficit in number word frame generation, a deficit similar to HY's – in transferring the digit identities to the phonological retrieval stage.

7.5. Discussion of Chapter 7

7.5.1. Processes involved in number reading: conclusions from the participants’ performance patterns

This study investigated the number reading of seven individuals with impaired reading aloud of multi-digit Arabic numbers. A series of experiments showed that different participants had deficits in different processes of number reading. The assessment results, summarized in the previous section, lead to the following conclusions, which any cognitive model of number reading should be able to explain:

(1) The visual analysis of digit strings and the verbal production of number words are handled by separate processes, as concluded by several previous studies (Benson & Denckla, 1969; Cohen & Dehaene, 1995; Cohen et al., 1997; Dehaene & Cohen, 1995; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Friedmann, Dotan, & Rahamim, 2010; Marangolo et al., 2004, 2005; McCloskey et al., 1986; Noël & Seron, 1993). In support of this assertion, we observed that visual analysis and verbal production can be selectively impaired: EY, MA, and ED were impaired only in visual analysis, whereas OZ and UN were impaired only in verbal production (except perhaps for a minor visual analysis deficit for OZ, in digit order encoding).

(2) Within visual analysis, separate sub-processes encode the digit order and the digit identity (Friedmann, Dotan, & Rahamim, 2010). This is shown by the dissociation in EY’s and HZ’s patterns of results – good digit identity encoding and impaired digit order encoding.

(3) Within visual analysis, separate sub-processes encode the digit order and the number length. This conclusion is supported by the double dissociation between EY and MA: EY had impaired digit order encoding and spared number-length encoding, and MA showed exactly the opposite pattern. Both dissociation directions meet the criteria for classical dissociation (Crawford, Garthwaite, & Gray, 2003; the dissociation could not be further assessed using the Crawford et al. formula in this case, because the dissociation was based on several tasks rather than one, and because the control participants were not the same ones in all tasks). Number production was intact for both of them, indicating that both digit order encoding and number-length encoding exist as a part of the visual analysis stage.

This conclusion has direct implication on error analysis in number processing tasks. The dissociation between EY and MA clearly shows that decimal shifts and digit transpositions (digit order errors) should be treated as two different kinds of errors rather than as two exemplars of the same error type (digit in an incorrect decimal position). Our findings further indicate that even this distinction between two error types is not the end of the story, because decimal shifts can originate in several different loci of impairments: number length encoding in the visual analyzer, as is the case for MA; triplet parsing in the visual analyzer, ED; verbal production, OZ; and perhaps also from impaired 0 detection in the visual analyzer (Section 7.3.5.3).

(4) Our findings strongly suggest that within the visual analyzer, the digit order encoder identifies the relative order of digits rather than their absolute positions. This can be deduced from MA's performance: her digit order encoder was intact (Section 7.3.3), but even with the digit order information available, she did not succeed distinguishing between numbers such as 9949 and 99499 (same-different task, Experiment 7.8). If the visual analyzer was encoding absolute positions, MA should have been able to distinguish between 9949-99499 by relying on the spared position encoding of the digit 4 as decades or hundreds (or between 9949-99949, if the position is encoded relative to the left side of the number). Her inability to do so suggests that the visual analyzer does not encode the absolute positions of the digits but rather their relative order. In any case, the digit order is specific to numbers and is not responsible for identifying the position of letters within words, as shown by previous dissociations (Friedmann, Dotan, et al., 2010). This dissociation also implies that the order encoding considers the abstract digits rather than retinotopic locations.

(5) A dedicated visual analyzer sub-process is responsible for parsing triplets. Supporting this conclusion, ED had a selective deficit in parsing of triplets, with spared encoding of digit order and number length. Her deficit was in the visual analyzer, as revealed by her sensitivity to a visual manipulation (adding a comma separator, Experiment 7.9).

(6) The order of digits is encoded by the digit order encoder in the visual analyzer, but the presence of 0 and its positions are encoded by a separate process. This conclusion is supported by EY's performance pattern: she had impaired digit order encoding but this did not affect numbers with 0, suggesting that 0 positions were encoded by another process, which was functioning correctly. This 0-position encoder is a part of the visual analyzer, because we observed the facilitating effect of 0 not only when she read numbers aloud, but also in a task that required only visual analysis, without verbal production (same-different decision, Experiment 7.7).

(7) The reading system handles 0 and 1 in different ways. The positions of 0 are encoded by a dedicated visual analyzer sub-process, as described in the previous paragraph, but the positions of 1 are not. This conclusion is again supported EY's performance: the presence of 0 in a number eliminated her digit order errors, but the presence of 1 did not have this effect: in the visual-only task (same-different) the digit 1 had no effect at all, and in the reading task the facilitating effect of 1 was much smaller than that of 0.

(8) Within verbal production, the number structure is handled by a dedicated process. This conclusion is supported by OZ's and UN's performance pattern: they had many decimal shift errors, indicating a deficit in a process that handles the number structure; and these errors occurred exclusively in verbal production tasks, indicating that this structural process was a verbal production process – presumably, the generation of number word frames. The specific type of errors, first-digit shifts, further indicates that this verbal production process specifically represents the number length and/or the triplet structure.

7.5.2. A revised model of number reading

On the basis of these findings and their implications for the number reading process, we propose the following cognitive model of number reading (Fig. 7.2). This model is a refinement of the model presented in the Introduction. Within visual analysis, the model postulates several processes: digit identity encoding, digit order encoding, and three processes that extract the number's decimal structure – length encoding, 0 identification, and triplet parsing. The output of these processes is sent to the verbal production stage. The decimal structure (length, triplets, 0's) is used to generate a number word frame, and the ordered digits are bound with the constituents of the number word frame to retrieve the phonological forms of each number word. We hereby describe in detail each of the components in the model.

7.5.2.1. Visual analysis

The assumption of separate processes that encode the digit identity and order on the one hand, and the number's decimal structure (length, 0, triplets) on the other hand, is based on the finding of double dissociations between the two kinds of information: EY showed impaired digit order and spared decimal structure; MA, ED, and NL showed spared digit order and a deficit in specific bits of the decimal structure (number length, triplet structure). Conceivably, one could have hypothesized a model where the decimal structure is extracted from the digit order information, but the dissociations we observed clearly refute this possibility. Thus, the decimal structure and the digit order are encoded by separate processes, and this separation is rigid – an intact encoding of digit identity and order cannot overtake an impaired decimal structure analyzer, and vice versa.

The model assumes that the number's decimal structure is not just sent to the verbal production processes, but is also used within the visual analyzer itself, to help the digit identity and order encoders. For example, the positions of 0 may help the digit encoders skip 0's and avoid sending them as digit identities to the production stage.

7.5.2.2. The number word frame

The number word frame (hereby, NWF) represents the number's verbal structure. It specifies the sequence of words in the verbal number, excluding the information about specific digit identities. Concretely, the NWF is a sequence of word specifiers of 3 different types: lexical classes of number words (ones, teens, tens, etc.), decimal words ("thousand", "hundred", etc.), and function words (the word "and"). For example, the NWF of 5,050 is *[_:ones] [thousand] [and] [_:tens]*. The NWF, in conjunction with the 1-9 values of each digit, provide sufficient information for the next processing stage to retrieve the phonological forms of all words in the verbal number.

Which information determines the NWF? In a fully-regular language such as Chinese, the NWF is unambiguously identified by the number's decimal structure – length, 0 positions, and triplet structure. Other languages, however, have various irregularities in the verbal structure of numbers. In Hebrew, English, and French, for example, the presence of 1 in the decade position results in a teen number word (teens do not exist in regular languages, e.g., Chinese 14 → 十四, *Shí sì*, literally "ten four"). In French, the NWF is also affected by the presence of 7 or 9 in the decade position (e.g., 73 → *soixante-treize*, literally "sixty and thirteen"; similarly, 93 → "eighty and thirteen"). Our model therefore assumes that the process creating the NWF receives not only the number's decimal structure (length, 0, triplets), but also the "structure-modifying digits" (1 in Hebrew and English; 1, 7, 9 in French, etc.).

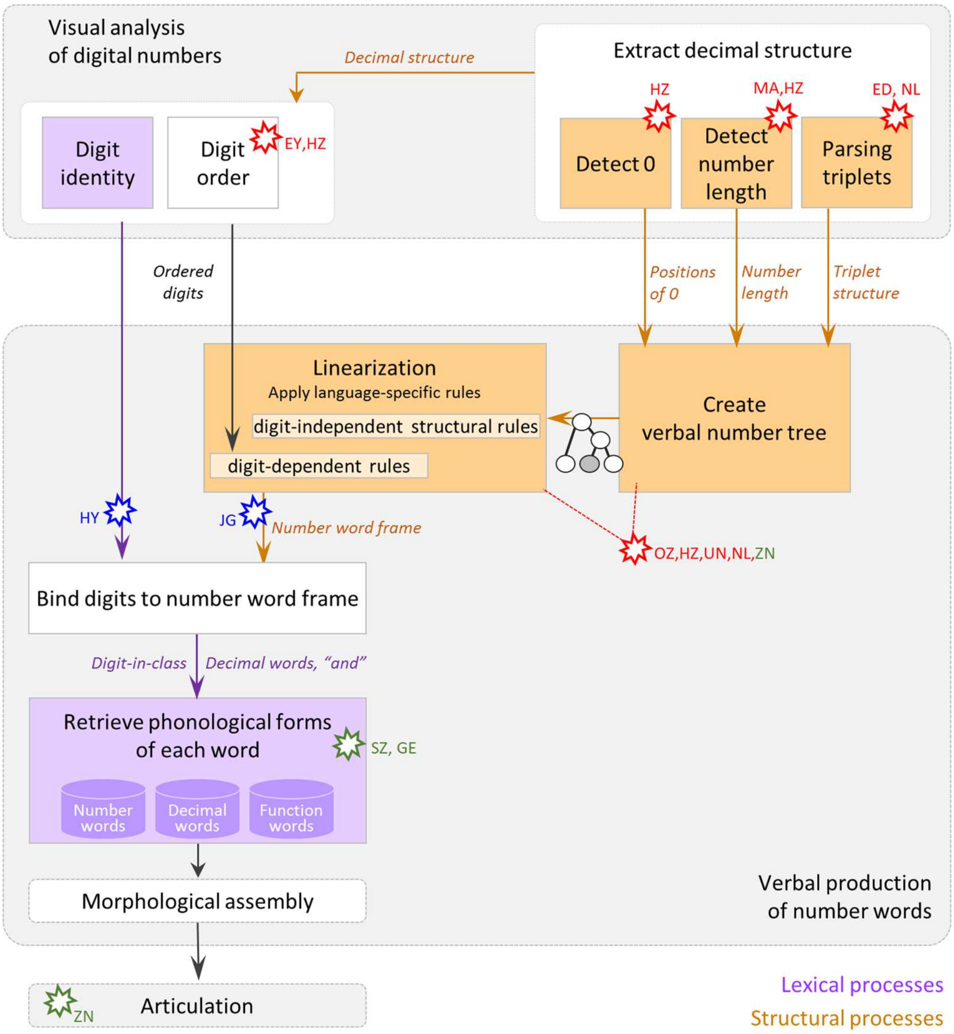

Fig. 7.2. Proposed model for reading multi-digit numbers – an extension of the model shown in Fig. 7.1. The extraction of decimal structure in the visual analyzer involves 3 sub-processes: detecting 0's and their positions, detecting the number length, and parsing the number into triplets. This information is sent not only to verbal production, but also to the encoders of digit identity & order. The number's decimal structure is used to create a verbal, language-independent, representation of the number in the form of a syntactic tree. This representation is then serialized into a linear form – the number word frame – according to language-dependent rules: some rules depend only on the language (e.g., in German and Arabic, the units word precedes the decades word), and some depend also on specific structure-modifying digits in the number (e.g., in English, 1 in the decade position yields an x-teen word). Finally, the number word frame is bound with the digits and used to retrieve the phonological form of each word from dedicated phonological stores. Blast icons indicate the locus of deficit of each participant: red = participants in this study; green = individuals we reported elsewhere (Dotan & Friedmann, 2015; Chapter 9); blue = individuals from other research groups (McCloskey et al., 1986).

We assume that structure-modifying digits are assigned a special status in verbal production but not in the visual analyzer. Two motivations drive this assumption: first, the visual analyzer is presumably language-independent (Bahnmueller, Moeller, Mann, & Nuerk, 2015), and as such should not be aware of the language currently being spoken, but structure-modifying digits are language-dependent. Second, structure-modifying digits are relevant only for digit-to-verbal transcoding, and not for other tasks such as digit-to-quantity conversion, yet the model assumes that the visual analyzer does not depend on the output modality.

Note that both these motivations are not applicable to the number's decimal structure (length, triplet structure, 0 positions). First, the decimal structure affects the NWF in all languages. Second, the decimal structure is relevant even in non-linguistic context such as digit-to-quantity conversion: the digit 0 may have a special status when quantifying single digits (Pinhas & Tzelgov, 2012), which is likely to be a part of quantifying even longer numbers (Meyerhoff et al., 2012; Moeller, Nuerk, et al., 2009; Nuerk & Willmes, 2005; Chapter 4). The number length, another component of the decimal structure, may play a central role in converting numbers to quantity (Chapter 4), and may serve as a cue to approximate a number's magnitude. Thus, the visual analyzer does not become language-dependent by having dedicated processes to extract the decimal structure.

7.5.2.3. Generating the number word frame

Two main findings illuminate on how the NWF is generated. First, a deficit in NWF generation (OZ and UN) resulted in many errors related with the decimal word "thousand" (mostly omissions), whereas such errors were rare in the control group and in the number production of participants with other impairments. Second, the pattern of decimal shift errors depended on their origin: the participants with a visual analyzer deficit (HZ, MA, ED, NL) had mainly first-digit shift errors, whereas the participants whose decimal shifts originated in a production deficit (OZ, and to a lesser extent UN) had decimal shifts also in the beginning of the second triplet (Table 7.5) – e.g., reading 1200 as "one thousand and twenty".

Both findings can be explained if the NWF is generated in a hierarchical manner. For example, the NWF of a 6-digit number may consist of two "sub-NWFs", one per triplet. Certain impairments in verbal production may prevent the person from generating the full NWF of the 6-digit number, but still allow them to generate shorter NWFs. In such cases, the person may resort to processing the long numbers in parts – e.g., one triplet at a time – because this method requires shorter NWFs, which he can still create. This approach could result in omissions of the decimal word "thousand": this word is a part of the 6-digit number NWF, but it is not a part of the NWF of either triplet. Furthermore, if the person's impaired verbal production causes first-digit shifts, and each triplet is being processed as a separate NWFs (and a separate number), then decimal shifts could occur not only in the beginning of the first triplet but also in the beginning of the second triplet, as observed for OZ. Importantly, both findings show that multi-triplet NWFs result in errors specifically around the triplet boundaries, suggesting that the verbal production system was not just splitting long numbers randomly, but specifically into triplets.

We hypothesize that the hierarchical processing in NWF generation is not merely in separation to triplets, but goes deeper and involves a fully hierarchical representation of the number's verbal structure. Specifically, we propose that NWF generation is done in two stages: first, a hierarchical representation of the verbal number is created as a tree-like structure, analogous in a way to the syntactic trees that represent the syntactic structure of sentences. Then, this tree is serialized into a linear form, which is the NWF. This numerical-verbal syntactic tree is hereby explained in detail.

The first stage is creating the tree, which reflects the number's verbal structure: for example, a two-digit number would be represented by three nodes: a decades node, a units node, and a higher-level node that merges them. A three-digit number would be represented by three nodes for hundreds, decades, and units, which are merged by two higher-level nodes. The tree of a 5-digit number such as 17,406 would include one sub-tree for 17, another sub-tree for 406, and a top-level node that merges the two sub-trees (Fig. 7.3, top part).

The tree is a purely structural representation, and its creation requires only the number's decimal structure. The number length determines the height of the tree and the size of the leftmost triplet's sub-tree (for a 5-digit number such as the one in Fig. 7.3, the leftmost triplet yields a 2-digit number sub-tree). The model assumes that the numerical-verbal syntactic tree does not depend on a particular language: it does not reflect language-specific properties such as the order of words, neither does it reflect language-specific irregularities such as teens. There is merely one exception to this language-independency: the tree depends on the structuring of verbal numbers in triplets. In languages where this is not the case, the tree would be different. For example, Japanese verbal numbers are structured in myriads (4-digit chunks). The number 10,000 is a single Japanese word (万, /man/), and a number such as 200,000 is verbalized /ni-jū man/, literally "twenty ten-thousand" (even the digit notation in Japanese conforms to this verbal structure: 20,0000). In Indian, there is a decimal word not only for 10,000 but even for

100,000 (/lakh/). Thus, a particular number would have the same numerical-verbal syntactic tree in all triplet-based languages (English, French, Hebrew, etc.) but a different tree in Japanese and yet another tree in Indian. Note that although the tree is almost independent of a specific language, it is nevertheless a verbal representation – it is used only for production of number words. Thus, the numerical-verbal syntactic tree is not the abstract representation hypothesized by some number processing models (Cipolotti & Butterworth, 1995; McCloskey, 1992).

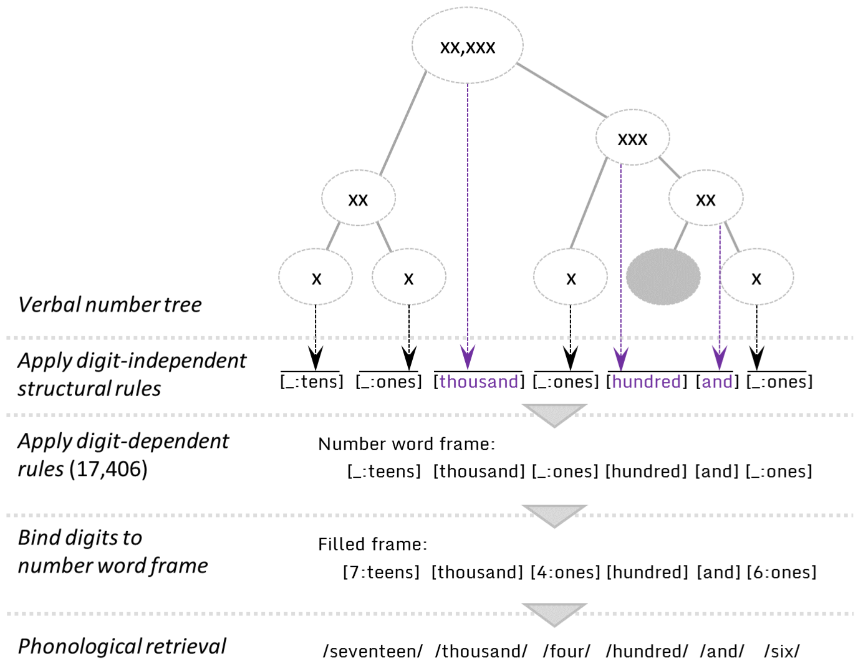

Fig. 7.3. The verbal number representations during production according to our proposed model (Fig. 7.2), demonstrated here for the number 17,406 in English. First, a tree-like representation is created based on the number's decimal structure. This verbal representation depends on the number's decimal structure but not on the specific language. The presence of 0 in the number results in some nodes being disabled (grey color). The tree is converted into a number word frame – a linear representation of the number's verbal structure. The linearization is done by applying language-specific rules. Some of these rules require only the tree representation (e.g., the order of words), and other rules depend also on the structure-modifying digits (e.g., 1 decades yields a teens word). In English, this linearization converts each 1st level (bottom) node into a number word lexical class, each 3rd level node (triplet node) into "hundred and" or "hundred", and each 4th level node into "thousand". In Chinese, a language with purely regular verbal number system, each node yields exactly one word. The linearization results in a number word frame, in which each element is a lexical class of a number word, a decimal word (thousand, hundred, etc.), or "and". The lexical classes are then bound with the number's digits, resulting in information sufficient to retrieve the phonological form of each number word.

The next stage is linearization – applying structural rules that convert the numerical-verbal syntactic tree into a linear representation, the number word frame. These conversion rules depend on the specific language. Some of them are general rules of the language; they require knowing the syntactic tree, but do not rely on specific digits. An example for such rule is the ordering of words: in most languages, the number words are ordered by decimal roles (e.g., for three digit numbers, the hundreds word precedes the decades word, which precedes the units word), but in some languages the unit word precedes the decade word (26 → "six and twenty") – e.g., German (Blanken, Dorn, & Sinn, 1997; Zuber, Pixner, Moeller, & Nuerk, 2009), Arabic, and old English (Berg & Neubauer, 2014). Another example for a language-general rule is proper embedding of the function word "and", which is needed only in some languages. Other structural rules depend not only on the tree but also on specific digits in the number – the structure-modifying digits. One example for such rule is the teens irregularity: a decade+unit sub-tree usually translates into *[_:tens] [_:teens]*, but it translates into *[_:teens]* when the decade digit is 1. Another example is the French rule that converts a decade+unit sub-tree into *[_:tens] [_:teens]* when the decade digit is 7 or 9.

The model assumes that linearization is the only stage affected by language-specific structures. Thus, the visual analyzer, which is language-independent, does not have to consider the language-dependent structure-modifying digits separately from the other digits (Section 7.5.2.2). The model also correctly predicts that structure-modifying digits would not have a special status in nonverbal tasks, where the NWF generation processes are inactive (this is the pattern observed with respect to the effect of 1 on EY's performance, see item 7 in Section 7.5.1).

An alternative model, which assumes that even the numerical-verbal syntactic tree depends on the specific language, seems less likely. Such a model would imply that the tree depends not only on the number's decimal structure (its length, triplet structure, and the positions of 0) but also on structure-modifying digits. This virtually annuls the benefit of having the number's decimal structure as a distinct representation: if the creation of the syntactic tree must anyway await the structure-modifying digits information, it unclear why the visual analyzer dedicates specific processes to identifying the decimal structure but not the structure-modifying digits (see Section 7.5.2.2).

A deficit in the creation of the syntactic tree may impair a person's ability to represent high-level tree nodes, and limit their representational ability to trees up to a certain height (Friedmann

& Grodzinsky, 1997). For example, such deficit may prevent the speaker from representing the top-level node, thereby leaving him able to process only one-triplet numbers. This may have been the case for OZ and UN. A more severe deficit may restrict the person to even shallower trees – e.g., to single-node trees – rendering the person unable to say multi-word numbers. This may have been the case for patient ZN (Chapter 9), who could hardly produce even two-digit numbers.

This model clearly explains why OZ, who had many "thousand"-related errors in number reading, did not have corresponding errors when he read each of these numbers as separate triplets (combining each pair of triplets with "and then" rather than with the decimal word "thousand", Experiment 7.11). The two modes of reading create minimal pairs of numbers with almost identical surface structure (in Hebrew, both "thousand" and "and then" are single words with 2 syllables and 4 phonemes), but with completely different internal representations: when reading the number in a standard manner, OZ would attempt to create a tree structure like the one depicted in Fig. 7.3. Conversely, reading the number as two separate triplets does not require the top node, and can be accomplished by two separate syntactic trees, each of which is shallower (and therefore easier).

7.5.2.4. Binding the number word frame with the digits

Verbal numbers include three types of words: number words ("five", "eleven"), decimal words ("thousand"), and function words ("and"). The NWF identifies unambiguously the decimal words and the function words. Number words, however, are under-specified – the NWF merely specifies their lexical class. To obtain a full specification of the number word, the lexical class must be bound with the value of the corresponding digit. This binding process takes as input the NWF, provided by the tree linearization process, and the sequence of ordered digits, provided by the digit identity and order encoders in the visual analyzer. The bound NWF contains the full information required for retrieving the phonological forms of each word.

The existence of a dedicated digit-NWF binding process within verbal production solves a potential problem of synchronization. Number words are retrieved from the phonological store one at a time, based on two parameters – the identity of a digit (1-9) and a lexical class (ones, tens, teens, etc.). The digits arrive from the digit identity and order encoders, and the lexical class arrives via a different pathway – from the NWF. For successful retrieval, the two pathways must be synchronized. This synchronization challenge can be easily solved if a single process (the binding) activates both the digit and the corresponding lexical class. The dedicated binding process can also explain the modification needed for the French numbers 71-79 and 91-99: saying 75 as *soixante-et-quinze*, “sixty and fifteen”, requires not only creating the irregular NWF *[_:tens] [_:teens]*; it also requires that the binding process would change the digit 7 into 6 (to get the word *soixante*, sixty) and the digit 9 into 8.

The model postulates that the binding process is triggered by the verbal system: the verbal system does not passively receive the ordered digits from the visual analyzer and passes them straight on to phonological retrieval, but rather it actively picks the digits in the appropriate order. This verbal-triggered architecture allows picking the digits in the order imposed by the verbal structure of numbers in the particular language – an order that is presumably unknown to the visual analyzer. An alternative possibility, that the number words are ordered *after* phonological retrieval, is unlikely: the retrieved phonological forms are immediately sent to articulation, without the mediation of a phonological short-term memory store that might have done this reordering (Dotan & Friedmann, 2015; Shalev et al., 2014).

To allow the verbal-triggered architecture, the model postulates the existence of a digit buffer, a short-term storage of digits. The visual analyzer, in particular the digit identity and order encoders, update this buffer, and the binding process picks digits from the buffer. The NWF linearization too picks structure-modifying digits from the same buffer. The existence of such buffer could explain UN’s high rate of digit substitution errors: his low memory capacity (Table 7.3) may have affected this short-term buffer too, resulting in a high rate of digit substitutions. We are inclined to assume that this buffer is visual rather than verbal, for two reasons. First, the buffer is presumably updated by the visual analyzer and not by any verbal process, so the information it contains reflects the ordered digits per-se, and in this sense the information is of visual nature. Second, a visual buffer can explain a peculiar pattern in EY’s performance: the presence of the digit 1 in the number reduced her digit order errors in number reading, but not in a visual-only task. Presumably, the linearization stage explicitly looked up “1 decades” in the digit buffer, which interacted with the visual analyzer and prompted it to improve the position encoding when the number contained 1. When the task was nonverbal, this feedback loop was inactive, so position encoding was not as intense.

7.5.2.5. Phonological retrieval and articulation

The digit-NWF binding process produces an exact specification of the sequence of words in the verbal number. This specification is now used to retrieve the phonological form of each word. Unlike ordinary content words, the phonological forms of the verbal number’s words are not retrieved from the phonological output lexicon (Friedmann et al., 2013) but from a dedicated phonological store (Dotan & Friedmann, 2015). Verbal numbers include three types of words: number words, decimal words (e.g., “thousand”, “hundred”), and the function word “and”. Each of these word types is stored in a separate phonological store (or alternatively, the phonological store is strictly organized in a category-based manner, Dotan & Friedmann, 2015).

The dedicated phonological stores maintain number and function words with their phonemes already assembled, so they can be directly sent to the articulation mechanisms without the need for an additional stage of phoneme assembly (Dotan & Friedmann, 2015). Still, the phonological forms may undergo *morpho-phonological* assembly – e.g., in Hebrew, “and” is a clitic, a bound function word, and should be assembled as the prefix of a number word. After this assembly, the phonological sequences are sent to the articulation mechanisms, which are presumably language-general and not specific to numbers (Shalev et al., 2014).

7.5.2.6. Additional processes

Our model postulates that information is directly sent from visual processes to verbal processes. Our findings can be fully explained without resorting to an additional conversion process that transforms the data from one format to another, however, such an intermediate transformation process is still possible (this would resemble the mechanisms of grapheme-to-phoneme — letter-to-sound — conversion in word reading, Coltheart et al., 2001). Future studies may specifically examine this point.

Visually presented numbers are not used only in the context of reading aloud. Perhaps more often than not, we merely need to comprehend them: either comprehend a concept they represent (e.g., “1984”, “100%”) or the quantity they represent. Dissecting these comprehension processes was not in the scope of the present study. Nevertheless, the processes described by our model – the visual analyzer and the verbal production processes – are presumably used whenever we need to comprehend a digit string or say a verbal number. This assumption is supported by our participants’ pattern of performance, at least with respect to several simple tasks (same-different decision, number matching, sequence identification, saying the result of an arithmetic exercise). It is also in agreement with the model presented in Chapter 6: number-length encoder in the visual analyzer may play an important role in converting digit strings to quantity, in particular in creating the syntactic frame for the number’s quantity.

7.5.3. Relation to other studies and specific types of impairments

The elements of our model can be classified into lexical pathways and processes, which handle single digits or number words (purple color in Fig. 7.2), and syntactic/structural pathways and processes, which handle the number's decimal and verbal structure (orange color in Fig. 7.2). The model therefore complies with the classical lexical-syntactic distinction theorized in several previous transcoding models (Cappelletti et al., 2005; Cipolotti, 1995; Cipolotti et al., 1995; Delazer & Bartha, 2001; Deloche & Seron, 1982; McCloskey et al., 1985; Noël & Seron, 1993, 1995; Sokol & McCloskey, 1988). Nevertheless, our model goes beyond the lexical-syntactic distinction by offering a finer level of granularity: it distinguishes between visual and verbal processes, describes the internal structure and for each of those, separates between specific lexical and syntactic processes, and proposes an accurate account of the information flow.

Our model can account for several cases reported in the literature. SZ and GE, two individuals who we reported previously (Dotan & Friedmann, 2015), made substitutions of number words in number reading and in verbal production tasks. We diagnosed their deficit as localized in the retrieval from the phonological storage of number words.

In another study we reported ZN, an aphasic patient with articulation deficit who was also completely unable to say multi-digit numbers (Chapter 9). Interestingly, ZN's difficulty was observed only when he had to generate a number word frame (e.g., when reading numbers and when saying a calculation result), whereas he performed quite well when the number word frame was explicitly provided to him (e.g., in a number repetition task). We therefore diagnosed his deficit as localized in the NWF generator – similarly to OZ and UN, yet more severe.

McCloskey et al.'s (1986) patient JG, who made only class errors, was apparently impaired in handling the number word lexical class information during NWF-digit binding, or in transferring it from the NWF to the binding stage. Their patient HY was apparently impaired in transferring the digit identities to the binding stage.

Cipolotti (1995) reported patient SF, who made errors in number reading but not in comprehension-only or production-only tasks. His errors were mainly first-digit shifts, but also other decimal shifts and substitutions. Cipolotti concluded that he was impaired in the digit-to-verbal transcoding pathway, with spared visual analyzer and verbal production. Translating this conclusion to our model would point to the *decimal structure analyzer* → *tree generation* pathway as SF's locus of deficit.

The model also makes predictions about specific performance patterns that should be observed given impairments in specific stages of NWF generation. A deficit in creating the numerical-verbal syntactic tree should result in incorrect verbal structure, and in some cases may involve errors in merely a single structural feature. Such single-feature errors may have been the case for OZ and UN: their verbal impairment caused errors in the number length information, but not in other structural aspects of the number (e.g., 0 positions). More specifically, an inability to represent trees with sufficient levels ("pruning the tree", Friedmann & Grodzinsky, 1997) may be the cause for omissions of the "thousand" decimal word. Alternatively, such deficit may result in the creation of undersized trees (e.g., a 3-digit tree for a 5-digit number), yielding too-short NWFs with which only some of the digits would be bound, e.g., 23,456 may be read as 356.

A deficit in the digit-independent linearization rules may yield similar errors, but may also result in language-specific errors such as failing to reorder the decade and units words in German and Arabic.

A deficit in the digit-dependent linearization rules should yield errors in the language-specific irregularities handled by this process – e.g., applying a teen word when the decade digit is 1, and using the appropriate French words for numbers in the range 71-79 and 91-99.

A deficit in the digit-NWF binding stage may result in two kinds of order errors: picking incorrect digits from the digit buffer may result in within-triplet digit order errors. Errors in synchronizing each digit with the lexical class may result in "class order errors" – number words would be produced with an incorrect lexical class, but the erroneous class would be one that exists elsewhere in the number (e.g., 317 may be incorrectly produced as "thirteen hundred and seventeen", but is less likely to be produced as "thirty hundred and seventeen", because the target number has no [Tens] word).

7.5.4. The role of peripheral versus central processes in implementing cognitive operations

Reading multi-digit numbers is a complex process. The conversion of numbers from one representation to another is not merely a simple symbol-to-word conversion: the existence of 0's, 1's, and other structure-modifying digits in the number creates a structural complexity, often referred to as "syntactic". The model we presented here provides a detailed, low-level account of how this syntactic complexity is addressed by the cognitive system. Note that the model puts a lot of weight on the encoding stage. This is not a trivial assumption. Hypothetically, it could have been the case that the visual analyzer extracts only the minimum required information – the identity and position of each digit – and a later, more central mechanism identifies the number's structure in order to convert it from digit format to verbal format. Number processing models along these lines were indeed offered in the past (McCloskey et al., 1986). Based on the performance of several patients with impairments in the visual analysis stage, the present study suggests that this is not the case: the visual analyzer, although it is a visual process, extracts the number's structure and allocates dedicated mechanisms to encode it. Similarly, even peripheral stages in verbal production allocate dedicated mechanisms to handle numbers as high-level representations: the phonological output buffer, the last processing stage before articulation, handles sequence of phonemes for most words, but whole-word representations in the case of number words (Dotan & Friedmann, 2015). The existence of these higher-level representations in peripheral processes is perhaps useful as they may simplify the format conversion and make it more efficient.

High-level representations in peripheral stages exist in other domains too. In word reading, like in number reading, the visual analyzer, together with the graphemic input buffer following it, not only encodes letter identities and positions but also extracts morphological information (Beyersmann et al., 2011; Friedmann & Gvion, 2012; Friedmann, Gvion, & Nisim, 2015; Friedmann, Kerbel, & Shvimer, 2010; Longtin & Meunier, 2005; Rastle & Davis, 2008; Rastle et al., 2004; Reznick & Friedmann, 2009; Sternberg & Friedmann, 2007; Velan & Frost, 2011). Morphological information also has dedicated processing mechanisms in the peripheral-orthographic stages of writing (Badecker, Hillis, & Caramazza, 1990; Badecker, Rapp, & Caramazza, 1996; Yachini & Friedmann, 2008), as well as in the peripheral post-lexical stages of speech (Job & Sartori, 1984; Kohn & Melvold, 2000; Patterson, 1982). These post-lexical stages of speech also handle lexical-syntactic information in the case of function words (Dotan & Friedmann, 2015), and may even perform syntactic sentence-level operations such as verb movement (Chomsky, 1995, 2001; Dotan & Friedmann, 2015; Friedmann et al., 2013; Zwart, 2001). Taken together, this body of research does not suggest a centralized system with sophisticated central processes and simple peripheral processes. Rather, it suggests a distributed system, in which peripheral processes communicate high-level information to one another.

8. Separate mechanisms for number reading and word reading°

Abstract. How specific are the cognitive mechanisms involved in the reading of words and numbers? Several neuropsychological and brain imaging studies show dissociations between the two reading mechanisms. Here, we tackle the question of word-number dissociation in high granularity: we ask which specific cognitive sub-processes serve both word reading and number reading and which are separate. As a framework for this high-granularity comparison, we describe a cognitive model for word reading and another model for number reading. We propose a possible homology between specific sub-processes of the two models, and review specific word-number dissociations in light of this homology. We then report two women with selective deficits in sub-processes that handle the number's structure: parsing the number into triplets (which is a part of encoding the number's decimal structure during visual analysis) and generating the number's verbal structure (number word frame). The deficits were specific to reading numbers: the two women showed good word reading abilities, even in reading morphologically complex words and in morphological tasks that specifically tap the processing of word structure. Together with previous dissociations, this indicates that the word and number reading pathways are largely separate. We propose that differences in the morpho-syntactic structure of words and numbers may underlie this separation.

8.1. Introduction

Reading is a complex cognitive operation. It involves orthographic, phonological, morphological, syntactic, and semantic processes, all of which need to operate in coordination. The degree of specificity of these processes is a major theoretical question. Reading could be implemented by a set of highly specialized mechanisms, dedicated to the processing of words. Alternatively, it could be accomplished by domain-general mechanisms, which serve not only the processing of words but also other functions. Identifying the range of functions supported by the reading mechanisms could improve our understanding of reading, of its developmental and evolutionary origins, and may shed light on the role of domain-specific versus domain-general mechanisms in implementing complex cognitive functions (Dehaene et al., 2003; Hauser, Chomsky, & Fitch, 2002; van de Cavey & Hartsuiker, 2016; Whorf, 1940; Wilson et al., 2015). From a clinical perspective, understanding the relation between word reading and number reading could be crucial for a more accurate assessment and treatment of reading disorders – dyslexia.

° This chapter was submitted to the journal *Cognitive Neuropsychology*. The text here is identical with the submitted manuscript, except reformatting and removing some parts that would, if remained, repeat other sections of this dissertation.

The present study examined the relation between the cognitive mechanisms involved in word reading and number reading. Words and numbers have much in common: both are written as character strings and must comply with certain structural rules, and both types of strings – letters and digits – can be converted to a verbal-phonological format. At the same time, words and numbers are also quite different: the language of numbers is merely a small subset of natural language, and its syntax is simpler. Correspondingly, the string-to-verbal conversion rules of numbers are simpler than those of words: we can formulate a relatively simple algorithm, a set of rules, to convert any digit string to words or vice versa (e.g., Deloche & Seron, 1987), but formulating such rules for translating letter strings to sounds is much more complex. Letter and digit strings are also different in the way they are used: words as well as numbers may refer to semantic concepts ("orange", "Peugeot 205"), but numbers also have meaning as quantities, which can be extracted from any number and do not depend on lexical knowledge of specific digit strings (Nuerk & Willmes, 2005). Indeed, it has been suggested that letter strings and digit strings are processed in different ways and in different brain circuits because the subsequent processing stages that use them are different (Hannagan et al., 2015).

One way to examine the relation between word and number reading is by investigating the reading performance of individuals with reading difficulties. Impairments in processes that serve only words or only numbers should affect only the reading of the relevant stimulus type, but impairment in a shared process would affect both stimulus types. Thus, the extent of the behavioral deficits can inform us about the underlying cognitive processes. In many cases, cognitive impairments affect the reading of words as well as the reading of numbers (Cohen et al., 1994; Denes & Signorini, 2001; Friedmann, Dotan, & Rahamim, 2010; Katz & Sevush, 1989; Shen et al., 2012; Starrfelt, Habekost, & Gerlach, 2010). Indeed, dyslexia and dyscalculia often co-occur in the same people (Wilson et al., 2015). This could suggest that word reading and number reading are implemented, at least in part, by shared cognitive mechanisms (Denes & Signorini, 2001). However, in other cases, dissociations were observed between word reading and number reading, supporting the notion of two distinct reading mechanisms. Disorders of word reading sometimes spare number reading (Anderson, Damasio, & Damasio, 1990; Cohen & Dehaene, 1995; Friedmann & Nachman-Katz, 2004; Friedmann, Dotan, & Rahamim, 2010; Hécaen & Kremin, 1976; Leff et al., 2001; Lühdorf & Paulson, 1977; Nachman-Katz & Friedmann, 2007; Sakurai, Yagishita, Goto, Ohtsu, & Mannen, 2006; Starrfelt, 2007; Temple, 2006), and disorders of word comprehension sometimes spare number comprehension (Cohen

& Dehaene, 2000; Dalmás & Dansilio, 2000; Miozzo & Caramazza, 1998; but some such dissociation were criticized as statistically unconvincing, Starrfelt & Behrmann, 2011). The opposite is also true: disorders of number reading sometimes spare word reading (Basso & Beschin, 2000; Cipolotti, 1995; Cipolotti et al., 1995; Marangolo et al., 2004; Priftis, Albanese, Meneghello, & Pitteri, 2013; Temple, 1989). In the present study, we report two more individuals with difficulties in number reading, whose word reading was spared.

Comparing word reading versus number reading holistically, i.e., treating each kind of reading as a whole (as we did in the previous paragraphs), is merely the first step. Both word reading and number reading are complex cognitive operations, and each involves many sub-processes. It may very well be that some of these sub-processes are shared between word reading and number reading whereas other sub-processes are not. Thus, the next step is to examine the question of shared versus separate mechanisms with respect to specific cognitive sub-processes involved in reading. This is the approach taken in the present study.

In the remaining part of this introduction, we first describe the cognitive mechanisms involved in word reading and number reading, and point to potential parallels between them. Then, we review studies that showed dissociations and associations between specific, analogous processes of number reading and word reading. Finally, we identify the gaps where our knowledge about the word-number relation is incomplete, and explain how the present study fills some of these gaps.

8.1.1. Cognitive mechanisms of reading words and numbers

We begin by describing the processes involved in word reading and number reading. We describe these reading mechanisms in a moderate level of granularity, as we see fit for the goal of comparing the two cases. Both cognitive models, that of word reading and that of number reading, can also be described in further detail. Such descriptions are available elsewhere, both for word reading (Friedmann, Biran, & Dotan, 2013; Friedmann & Coltheart, in press) and for number reading (in Chapter 7).

8.1.1.1. The cognitive architecture of word reading

The dual-route model of word reading (Coltheart et al., 2001; Coslett, 1991; Ellis & Young, 1988; Forster & Chambers, 1973; Frederiksen & Kroll, 1976; Friedmann & Gvion, 2001; Funnell, 1983; Jackson & Coltheart, 2001; Marshall & Newcombe, 1973; Paap & Noel, 1991; Patterson & Morton, 1985) describes several different processes involved in reading single words (Fig. 8.1). Reading begins by visual analysis of the sequence of letters: encoding their identities, their relative positions within a word, and their association to words (Coltheart, 1981; Ellis, 1993; Ellis, Flude, & Young, 1987; Ellis & Young, 1996; Friedmann, Biran, & Gvion, 2012; Friedmann & Gvion, 2001; Friedmann & Haddad-Hanna, 2012, 2014; Humphreys et al., 1990; Humphreys & Mayall, 2001; Kezilas et al., 2014; Lambon-Ralph & Ellis, 1997; Marshall & Newcombe, 1973; Peressotti & Grainger, 1995; Saffran & Coslett, 1996). The orthographic-visual analyzer also performs an initial morphological decomposition of the word (Beyersmann et al., 2011; Friedmann et al., 2015; Friedmann, Kerbel, et al., 2010; Longtin & Meunier, 2005; McCormick, Rastle, & Davis, 2008, 2009; Rastle et al., 2004; Reznick & Friedmann, 2015; Taft & Forster, 1975). The reading process then continues in two parallel pathways: in the lexical pathway, the word is first found in a lexicon that contains the orthographic form of all familiar words (Coltheart & Funnell, 1987; Friedmann & Lukov, 2008). The lexical entry is used to retrieve the word's phonological components (phonemes, metric structure) from a phonological output lexicon. These phonological components are merged in the phonological output buffer, and the word is finally articulated (Butterworth, 1992; Dell, 1988; Friedmann et al., 2013; Laganaro & Zimmermann, 2010; Levelt, 1992; Levelt, Roelofs, & Meyer, 1999; Nickels, 1997). The second pathway of word reading, the sub-lexical pathway, does not rely on lexicons: the sequence of letters is translated into a phonological sequence by the grapheme-to-phoneme converter, which relies on language-general conversion rules (Coltheart, 1978; Friedmann & Lukov, 2008; Schmalz, Marinus, Coltheart, & Castles, 2015). The phonological sequence is then merged in the phonological output buffer and sent to articulation. Thus, the last stages in the word production pathway (phonological output buffer, articulation) serve both the lexical and the sub-lexical routes.

A partially separate processing pathway handles comprehension. After identifying a word in the orthographic input lexicon, we use this information to access the semantic system and get the word's meaning. The semantic system, in turn, can directly access the phonological output lexicon and the rest of the production pathway, even if the word was not presented visually – this is what happens when we just talk (Friedmann et al., 2013). When both the lexical and sub-lexical pathways are blocked, reading proceeds via the semantic pathway, a situation known as deep dyslexia (Coltheart, Patterson, & Marshall, 1987; Stuart & Howard, 1995). Because the semantic pathway conveys the meaning of the word rather than a specific entry in the lexicon, deep dyslexia is characterized by semantic errors (e.g., fruit → apple) that presumably result from insufficiently accurate representations of the word meanings.

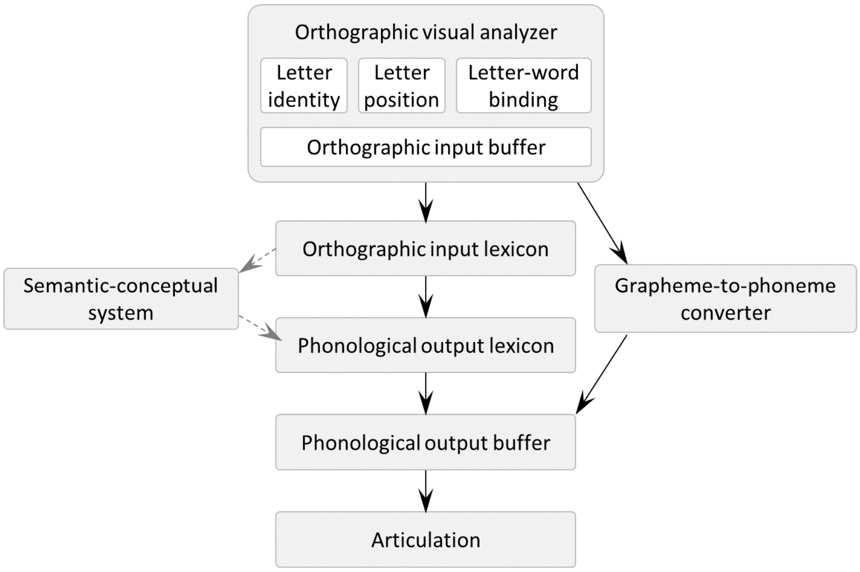

Fig. 8.1. A cognitive model of word reading (Friedmann & Coltheart, in press). The orthographic-visual analyzer extracts the letter identity and order from the letter string, and binds letters to words. In the lexical pathway (middle column), the letters are used to identify the word in the orthographic input lexicon. The phonological output lexicon, which contains a corresponding lexical entry, provides the phonological components of the word. These components are merged in the phonological output buffer and then articulated. In the sub-lexical pathway (right), the lexicons are bypassed by directly converting graphemes (letters or letter groups) to phonemes. The semantic pathway (left) allows comprehension of the word without producing it, as well as production of words that were not visually presented.

8.1.1.2. The cognitive architecture of number reading

When reading numbers, we see a sequence of digits and translate it into a sequence of number words. The visual analysis of digits and the verbal production of number words are implemented by separate cognitive mechanisms, as indicated by several neuropsychological case studies (Benson & Denckla, 1969; Cohen & Dehaene, 1995; Cohen et al., 1997; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Friedmann, Dotan, & Rahamim, 2010; Marangolo et al., 2004, 2005; McCloskey et al., 1986; Noël & Seron, 1993; Chapter 9) and brain imaging studies (Dehaene & Cohen, 1995; Dehaene et al., 2003; Shum et al., 2013).

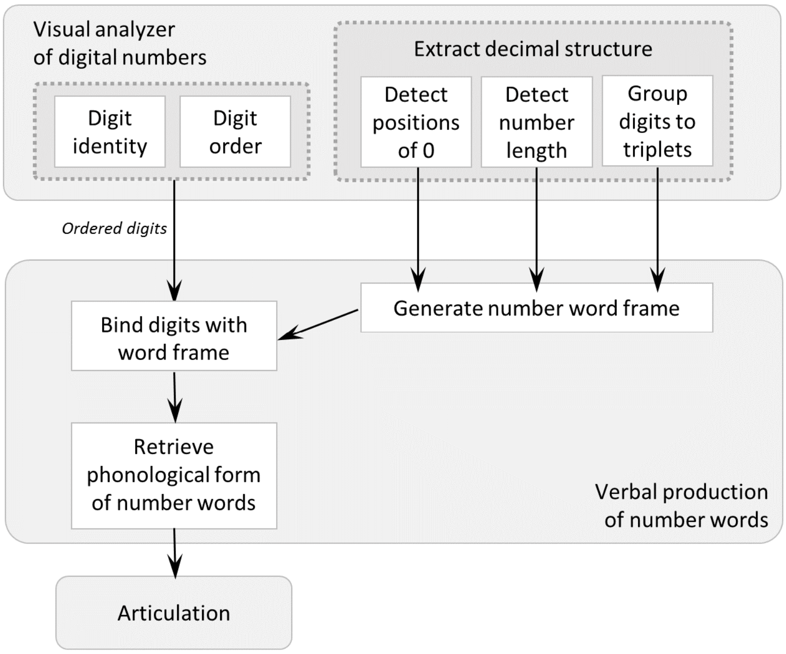

Fig. 8.2. A cognitive model for number reading. The digit string is parsed by the numeric-visual analyzer, which identifies the digit identity, the digit order, and several aspects of the number's decimal structure: its length, the positions of 0, and its triplet structure. The decimal structural information is used to obtain a number word frame – an almost-full specification of the sequence of number words to produce, that lacks only the specific digit values (e.g., the word frame for 5012 is [_:ones] [thousand] [and] [_:teens]). The word frame is bound with the corresponding digit identities ("5" and "2" in this example), resulting in a full specification of the words to produce. The phonological form of each word is then retrieved and sent to articulation.

A more detailed model of number reading was proposed in Chapter 7. Here, we re-describe the main components in this model (Fig. 8.2). The model postulates that within visual analysis of digits, the identities and order of digits are encoded by two separate processes. This separation is supported by studies that found selective impairments in each of the two processes (Cohen & Dehaene, 1991; Friedmann, Dotan, & Rahamim, 2010). Another set of numeric-visual analysis sub-processes extracts the number's decimal structure – how many digits it has, the positions of 0, and how digits are grouped to triplets. The decimal structure is used by the verbal production processes to generate the *number word frame* – a representation of the number's verbal structure. The number word frame is a sequence of word specifiers, each of which can be the lexical class of a number word (e.g., ones, tens, teens), a decimal word ("thousand", "hundred") or a function word ("and"). Thus, the word frame specifies the verbal number fully except the specific digit values. For example, the word frame for 5012 is *[_:ones] [thousand] [and] [_:teens]*. To obtain the words, the word frame is first bound with the specific digit identities, provided by the digit identity encoder and digit order encoder sub-processes of the numeric-visual analyzer. This yields a complete specification of the words to produce: *[5:ones] [thousand] [and] [2:teens]*. This full specification is used to retrieve the words from dedicated phonological stores (Dotan & Friedmann, 2015; McCloskey et al., 1986).

The model describes how visually-presented digit strings are read aloud. Another question is how we comprehend numbers. This question has at least two answers, because numbers have at least two different kinds of categorically-different meanings. One meaning refers to a familiar digit string as a lexical entry, and may resemble the meaning of words – e.g., “Peugeot 306”, George Orwell’s “1984”, and the use of “100%” to express absolute certainty. Another kind of meaning is the quantity represented by the number. This quantity, represented in the Approximate Number System (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene et al., 2003; Feigenson, Dehaene, & Spelke, 2004; Mou & VanMarle, 2014; Nieder, 2013; Piazza, 2010), can be extracted from any digit string by a set of dedicated cognitive processes (Nuerk & Willmes, 2005; and see Section A of this dissertation). The digits-to-quantity pathway presumably involves the numeric-visual analyzer presented above, but not the verbal production mechanisms of numbers (Chapter 9).

8.1.2. Possible parallels between word reading and number reading mechanisms

Perhaps the clearest parallel between words and numbers is that both involve visual / orthographic input processes (orthographic or numeric visual analyzer, orthographic input lexicon) and verbal / phonological production processes (phonological retrieval, phonological output buffer, generation of number word frames).

Parallels can be found also for higher-granularity processes. For words and numbers alike, visual analysis of the character string involves specific processes to encode the identity of characters and their relative order. Another parallel concerns the distinction between lexical and structural processes. Models of symbolic number processing typically categorize processes as “lexical”, handling the identities of single elements (digits and number words), or as “syntactic”, handling the relations between these elements, i.e., the number’s decimal or verbal structure[7]. This distinction, single-element processing versus structure processing, may apply to word reading too: some processes handle single letters or phonemes, whereas other processes handle

[7] Note that the term “lexical” is used with different meanings in the literatures of word and number processing: for numbers, “lexical” refers to processing the identity of a single digit or number word; for words, “lexical” denotes familiar words, for which we store some information in a lexicon.

the word's morphological structure. The so-called syntactic processes in number reading may therefore parallel morphological processes in word reading. In support of this putative parallel between "number syntax" and morphology, the phonological output buffer appears to process number words and morphemes in similar manners, storing them as phonological building blocks, larger than a single phoneme and already assembled and ready for articulation (Dotan & Friedmann, 2015). Similarly, morphological/structural information is extracted by the orthographic-visual analyzer of words (Friedmann et al., 2015; Reznick & Friedmann, 2009, 2015) as well as by the numeric-visual analyzer of numbers (Chapter 7).

In spite of these similarities between word and number reading, the parallel is not perfect. Two issues stand out as major differences between word and number reading. First, a letter string is converted to one word, whereas a digit string is converted to multiple words. Second, any digit string yields a valid number (except leading 0's), but letter string are subject to lexical and morphological restrictions. Most digit strings are not lexically familiar and are not processed as whole lexical units (Chapter 4), but most words are lexically familiar and enter orthographic and phonological lexicons. In this respect, number reading may parallel the sub-lexical route of word reading (Denes & Signorini, 2001). On top of these two differences, even when word and number reading involve potentially parallel processes, these processes seem to be different when examined in detail. For example, both word reading and number reading involve orthographic/numeric visual analyzers that extract structural/morphological information about the letter string or digit string, but the nature of this information is different in the two cases: morphemes for words, decimal structure for numbers.

8.1.3. Dissociations and associations between specific processes of word and number reading

We now turn to review studies that compared word reading and number reading (see a review in Starrfelt & Behrmann, 2011). Per our approach in the present study, we restrict this review to studies that compared specific sub-processes of reading.

8.1.3.1. Visual analysis of letters and digits

Current research clearly shows a stage of orthographic-visual analysis for words, and numeric-visual analysis for numbers. Is there one mechanism responsible for these two functions, or are there two separate visual analyzers?

In support of the separate-mechanisms possibility, several dissociations were reported between the orthographic-visual analyzer and the numeric-visual analyzer. Both word reading and number reading involve a character-position encoding process as part of the visual analysis. However, Friedmann, Dotan, et al. (2010) reported 10 individuals who had letter position encoding impairment but whose digit position encoding was normal. This suggests that the position encoders of letters and digits are separate. Letter identity dyslexia (sometimes referred to as visual dyslexia), a selective deficit in letter identification, can sometimes affect digit identification too (and even other symbol types, Sinn & Blanken, 1999), but importantly, there are also reports of individuals with this dyslexia whose digit identification was intact (Crutch & Warrington, 2007). Neglect dyslexia, another disorder that affects the visual processing of the character string, can impair number processing while sparing words (Priftis et al., 2013) or the other way around (Friedmann & Nachman-Katz, 2004). At the neural level, the word-number separation in visual processing is supported by studies that showed different brain activity patterns following exposure to letters and digits (Abboud et al., 2015; Baker et al., 2007; Grotheer, Herrmann, & Kovacs, 2016; Hannagan et al., 2015; Park, Hebrank, Polk, & Park, 2012; Shum et al., 2013). All these evidence lead to the conclusion that there are separate visual analysis processes for words and numbers.

Dissociations between words and numbers were not yet reported with respect to the structural components of the orthographic/numeric visual analyzers – decimal structure analysis of digit strings and morphological analysis of letter strings; and with respect to the binding of letters to words (it is even unknown whether a corresponding digit-to-number binding process exists for numbers). In the present study, we will report one such dissociation – we show a selective deficit in one sub-process of the decimal structure analysis, without a corresponding deficit in word reading (in fact, without any deficit in word reading).

Several visual processes take place before the orthographic or numeric visual analyzer can identify the abstract identity of the letters or digits (e.g., processing the visual image, encoding visual features, etc.). For these processes too, no dissociation was reported. McCloskey and Schubert (2014) suggested that these mechanisms are shared for words and numbers.

8.1.3.2. Verbal production

Verbal production of words and numbers is also at least partially separate. Temple (1989) and Marangolo et al. (2004) reported individuals who had impaired number production alongside spared word production, and Bencini et al. (2011) reported the opposite pattern. Going to higher granularity and even more specific sub-processes, there appear to be dissociations between the phonological retrieval mechanisms of words and numbers: the phonological form of words is retrieved from the phonological output lexicon, whereas the phonological form of numbers is retrieved from a dedicated and separate phonological store (Dotan & Friedmann, 2015). Indeed, certain types of aphasia result in different types of errors in words and numbers (Cohen et al., 1997; Delazer & Bartha, 2001; Dotan & Friedmann, 2015; Girelli & Delazer, 1999; Marangolo et al., 2004, 2005). However, the essence of this separation does not seem to be about words versus numbers, because some other word categories – in particular, function words and morphological affixes – are also retrieved from dedicated phonological stores, similarly to number words (Dotan & Friedmann, 2015; Marangolo et al., 2005). It seems that the phonological retrieval of some word categories, including number words, is handled by a set of dedicated processes, whereas the phonological output lexicon handles the remaining words, which are the majority of words.

An interesting dissociation of syntactic/structural processes in speech production was reported by Marangolo et al. (2004): their patient F.A. had syntactic errors in numbers, in particular errors in the number's decimal structure (e.g., 5,300 $\rightarrow$ 500,300), but his word production was spared. A possible explanation of this dissociation is a selective deficit in the generation of number word frame, and that this process is involved in number production but not in word production. This would imply separate verbal production mechanisms for words and numbers, at least with respect to structural processing. However, Marangolo et al.'s dissociation may have other explanation too, e.g., the syntactic errors in number reading could result from impaired phonological retrieval.

Articulation mechanisms, which handle oral production of the already-retrieved phonological forms, may serve words and numbers alike. In support of this view, word-number dissociations were observed for pre-articulation deficits, but an articulation disorder (apraxia) seems to have similar effects on number words and non-number words (Shalev, Ophir, Gvion, Gil, & Friedmann, 2014; Chapter 9).

At the neural level, verbal numbers are associated with activity in the left angular gyrus, a region also activated in several language tasks (see a review in Dehaene et al., 2003). We are not aware of any brain imaging study that directly compared verbal production of numbers with verbal production of non-number words.

8.1.3.3. Structural processes

Another interesting word-number dissociation was reported by Cipolotti (1995). This dissociation specifically concerned the structural mechanisms of number processing: Cipolotti's patient SF made errors in number reading but his word reading was spared, i.e., he had a deficit in a process that specifically serves number reading. His number reading errors were primarily syntactic, indicating that the impaired, number-specific process was a structural process – one that handles the number's decimal or verbal structure. This dissociation is interesting because it is the only clear evidence for a selective deficit in a structural component of number reading with spared word reading. Such dissociation pattern is important because this is the kind of dissociation required to show that the structural processes involved in number reading are dedicated to numbers and do not serve words. The present study shows another such structural dissociation: we report two women with deficits in specific structural processes of number reading, whose word reading is spared.

8.2. Case descriptions

ED and NL are two sisters with developmental difficulties as detailed below. Both are right handed and have corrected-to-normal vision. At the time of examination, NL was a B.A. student and ED worked in an administrative job and had an undergraduate degree. Their performance in number reading was reported in detail in Chapter 7.

Table 8.1. Background information and performance in general tasks.

	ED	NL
Age	31	24
Education years	15	14
Memory spans		
Digit (free recall)	5^+	5^+
Digit (matching)	7	7
Word (free recall)	4½	5
Word (matching)	7	7
Dictations (% errors)		
Writing 50 words	2%	2%
Writing 50 numbers	2%	20%

Comparison to control group: $^+$ $p < .1$

Table 8.1 shows their background information and their normal-level performance in phonological short-term memory tasks (FriGvi battery, Friedmann & Gvion, 2002). They also performed well in writing words to dictation (TILTAN battery, Friedmann et al., 2007). In writing numbers (digit strings) to dictation (MAYIM battery, Dotan & Friedmann, 2014), NL had many errors (20%), all of which were syntactic (number length, the position of the digit 0). ED did not have many errors (2%), but she often hesitated when writing the digit 0, suggesting a difficulty similar to NL's.

8.3. Procedure

The participants were tested in a series of 1- to 2-hour sessions in a quiet room in our lab. In the reading tasks, stimuli were presented on paper with no time limit. An error followed by a self-correction was classified as an error in our coding of responses.

Control participants with outlier error rates were excluded. The threshold for outlier was defined per task as exceeding the 75th percentile of error rates by more than 150% the inter-quartile distance. Individual participants were compared to control groups using Crawford and Garthwaite's (2002) one-tailed t-test. In cases of a control group ceiling effect (mean error rate ≤ 2%), the low variance does not allow for a reliable statistical comparison so we arbitrarily decided that 7% errors or more would be considered as impaired performance.

8.4. Experimental investigation

ED's and NL's number reading was described in detail in Chapter 7. The detailed assessment showed that both of them had impaired number reading. In particular, both were impaired in the numeric-visual analyzer, in the triplet parsing function of the structural analysis. NL was also impaired in the structural-verbal production stage, in the generation of number word frames.

Do these components, which were found impaired in our participants' number reading, serve both number reading and word reading? If so, they should affect word reading as well. If, however, our participants show a deficit only in number reading, this would indicate separate components for number and word reading.

To answer this question, we examined ED's and NL's word reading using several tasks. First, because both ED and NL were impaired in the numeric-visual analyzer, we examined their visual analysis processes of word reading. Their impairment in the visual analysis of numbers was in the structural analysis component, so we made sure to use tasks that tap the structural functions of the orthographic-visual analyzer, in particular the analysis of the morphological structure of words.

Second, because NL was impaired also in verbal production of numbers, we also examined the participants' verbal production of words. Here too NL's deficit in numbers was in structural processes (number word frame generation), so we used word production tasks that tap structural processes, in particular the oral production of morphologically complex words.

8.4.1. Orthographic-visual analyzer

To examine ED's and NL's orthographic-visual analyzer, we used two kinds of tasks: one type of task required reading aloud of words. These tasks rely on the orthographic-visual analyzer as well as on verbal production. Another set type of task involved presentation of written words but did not require verbal production.

8.4.1.1. Method

ED and NL read aloud a total of 928 words and 40 nonwords, administered as several tests. First, we used oral reading screening tasks from the TILTAN battery for the identification of subtypes of dyslexia (Friedmann & Gvion, 2003): oral reading of single words, word pairs, and nonwords. These tasks include words of various types, which can reveal different types of dyslexia (and, specifically for our purpose in the current study, can assess the performance of the various components of the word reading process): irregular words and potentiophones[8] for the identification of surface dyslexia (and assessment of the performance of the lexical route); nonwords for the identification of phonological and deep dyslexia (and for the assessment of the performance of the sub-lexical route); morphologically complex words for identifying deep dyslexia, orthographic input buffer or phonological output buffer deficits and for the assessment of morphological decomposition and composition (Cohen et al., 1994; Dotan & Friedmann, 2015; Job & Sartori, 1984; Reznick & Friedmann, 2009, 2015; Stuart & Howard, 1995; Temple, 2003); words (and nonwords) that can be read as other words by neglecting one side of the word, for the identification of neglect dyslexia (Friedmann & Nachman-Katz, 2004; Haywood & Coltheart, 2001; Patterson & Wilson, 1990; Reznick & Friedmann, 2015); words with many orthographic neighbors for visual dyslexia (Cuetos & Ellis, 1999; Friedmann et al., 2012; Lambon-Ralph & Ellis, 1997; Marshall & Newcombe, 1973); word pairs in which between-word migration creates other existing words for attentional dyslexia (Friedmann, Kerbel, et al., 2010; Hall, Humphreys, & Cooper, 2001; Humphreys & Mayall, 2001; Mayall & Humphreys, 2002; Saffran & Coslett, 1996; Shallice & Warrington, 1977b); words that allow for vowel

[8] *Potentiophones* are pairs of words contain homophonic letters (and are usually underspecified for vowels). If read solely via the grapheme-to-phoneme conversion route, one word can be read aloud instead of the other. For example, the English word *now*, when read aloud via the sublexical route, may erroneously be pronounced like the words *no* and *know* (Friedmann & Lukov, 2008).

errors, for vowel letter dyslexia (Khentov-Krauss & Friedmann, 2011); and migratable words and nonwords, for the identification of letter position dyslexia[9] (Friedmann, Dotan, & Rahamim, 2010; Friedmann & Gvion, 2001; Friedmann & Haddad-Hanna, 2012; Friedmann & Rahamim, 2007; Peressotti & Grainger, 1995).

ED and NL read two additional lists of words. One list included migratable words, to tap letter position encoding. Another list was a test design to assess the effect of morphologically structure of the target word on letter transpositions (Friedmann et al., 2015). This list included 500 words, most of which were morphologically complex.

On top of the reading aloud tasks, ED and NL also performed two lexical decision tasks, which require orthographic-visual analysis but do not require verbal production. One task focused on letter position encoding – it included 30 words, 15 migratable nonwords, and 15 non-migratable nonwords. Another task focused on morphological encoding – it included 45 words and 60 nonwords, all morphologically complex. In both tasks, words were printed on paper and the participants were asked to circle the existing words.

8.4.1.2. Results

In all the word reading tasks, both ED and NL performed very well, and their error rates were within the norm for their age (Table 8.2). This forms a clear dissociation between their visual analysis of digit strings, which was impaired, and their visual analysis of letter strings, which was completely normal. This dissociation was demonstrated both by tasks that specifically tapped the visual analysis stage and by tasks that required oral reading. NL's reading patterns further show another dissociation, between her impaired verbal production of numbers and her intact verbal production of words; this dissociation is discussed in the next section.

The dissociation can clearly be tracked back to structural processes. ED's and NL's numeric-visual analyzer impairment was in structural processing. Their good performance in the words tasks demonstrated that their orthographic-visual analyzer was able to process correctly the morphological structure of words. As a Semitic language, Hebrew has a rich morphology – all verbs and most nouns and adjectives in Hebrew are constructed from a root and a morphological template and/or inflection. Hebrew also has deep orthography, including many degrees of freedom that are derived from fact that vowels are only partially represented in the orthographic forms, stress is not represented at all, and 13 letters can be converted to more than one phoneme

[9] In migratable words/nonwords, interior letters can be transposed in a way that results in another existing word. Such words are prone for errors in case of letter position dyslexia.

(Friedmann & Lukov, 2008). Hebrew's deep morphology and deep orthography make it virtually impossible to read morphologically complex words correctly without processing their morphological structure. Still, ED and NL were able to do this easily: each of them read aloud 712 morphologically complex words, and they made no more errors on these words than did the control participants.

Table 8.2. Error percentages in tasks that tap orthographic-visual analysis and verbal production of words. Both participants had low error rates in all tasks.

	Task	No. of items[a]	ED	NL	Control group Errors (SD)	n	Age (SD)
Reading	Read words	136 (76)	1	2	1.7 (1.5)	372	28;7 (7;0)
	Read nonwords	40	0	8	4.1 (4.2)	372	28;7 (7;0)
	Read word pairs	30x2 (52)	0	3	2.5 (2.4)	372	28;7 (7;0)
	Read migratable words	232 (182)	2	1	2.4 (1.8)	192	18+
	Read morphologically complex words	500 (402)	2	2	1.8 (0.4)	10	30;5 (13)
Visual	Lexical decision						
	Migratable	60 (11)	0	3	0.3 (0.6)	19	18+
	Morphologically complex	105 (45)	0	0	1.0 (0.9)	24	28;8 (4;2)
Verbal	Picture naming	100 (27)	2	3	2.3 (1.7)	87	20-40
	Verb elicitation	24 (24)	0	0	0.2 (0.6)	18	38;11 (14;5)
	Nonword repetition	48	2	2	4.4 (3.5)	20	31;2 (8;9)

[a] The parentheses indicate the number of morphologically complex words in the task

8.4.2. Verbal production of words

8.4.2.1. Method

Because NL had a deficit in the verbal production mechanism of numbers, we wanted also to examine directly her verbal production of words. Her good oral reading was already a strong indication that her verbal production was intact. To examine it further we used three tasks that involve verbal production without reading. In a picture naming task (SHEMESH test, Biran & Friedmann, 2004), the participants were presented with 100 object pictures and asked to retrieve their names. In a nonword repetition task (from the BLIP battery, Friedmann, 2003), they repeated 53 nonwords with 1-4 syllables. Finally, in the inflected verb elicitation task, they had to orally complete a missing verb in a sentence by inflecting the verb to agree with the sentence on tense, gender, and plural (BAFLA, Friedmann, 1998). This third task was used to test directly their production of morphologically complex words.

8.4.2.2. Results

Again, ED and NL performed well in the word production tasks: their error rates were within the norm for their age (Table 8.2). These results show another dissociation between numbers and words: NL had a deficit in number production, in a structural process (the generation of number word frames), but she showed good production of words, even in the task that taps structural (morphological) processing. This provides a specific dissociation between the structural processing of numbers and words.

8.5. Discussion of Chapter 8

The comparison of number reading with word reading showed clear dissociations. First, both ED and NL were impaired in the visual analysis of digit strings, but their orthographic-visual analysis of letter strings was perfectly intact. Second, NL was impaired in verbal production of numbers, but had good verbal production of words (and nonwords). ED and NL therefore join the small group of reported cases with number-specific reading deficits, which do not affect word reading (Basso & Beschin, 2000; Cipolotti, 1995; Cipolotti et al., 1995; Marangolo et al., 2004; Priftis et al., 2013; Temple, 1989).

Both number impairments specifically affected structural processing. In the visual analysis of digit strings, ED's and NL's deficit was in a sub-process that encodes the number's decimal structure (in particular, triplet parsing). In verbal production, NL's deficit was in the sub-process that generates number word frames – the number's verbal structure. The finding that these deficits dissociated from word reading shows that these two structural processes are specialized for numbers and do not handle words. The parallel structural process for word reading would be morphological decomposition and composition for morphologically complex words, and these processes were intact for both participants. Importantly, both specific dissociations were not yet reported in previous studies, and only a single case of word-number dissociation in specific structural processes was previously reported (Cipolotti, 1995). This suggests that the structural processing of numbers is done by dedicated processes, which do not serve word reading, and in particular do not process the morphological structure of words.

Our findings, in conjunction with previous reports on word-number dissociations, indicate that most of the processes involved in number reading, from visual analysis to phonological retrieval, are dedicated to numbers and are not a part of the word-reading pathway. In the visual analysis stage, position encoding is separate for letters and digits (Friedmann, Dotan, & Rahamim, 2010), and so are digit identity encoding (Abboud et al., 2015; Baker et al., 2007; Grotheer et al., 2016; Hannagan et al., 2015; Park et al., 2012; Shum et al., 2013) and triplet parsing (ED, NL). In verbal production, the generation of number word frames is a number-specific process (NL), and phonological retrieval is done in different ways for number words and other words (Cohen et al., 1997; Dotan & Friedmann, 2015; Marangolo et al., 2004, 2005). The only number-processing components for which dissociations with word processing have *not* yet been studied are the numeric-visual analyzer sub-processes that encode the 0 positions and the number length. Some possible analogous processes in the visual analysis of words may be the encoding of word length (analogous to the number length) and the detection of vowel letters (analogous to the detection of 0's in a number, see Khentov-Krauss & Friedmann, 2011 for vowel-specific processing in reading). If future dissociations are found between 0 detection and number length on the one hand and word reading on the other, this would indicate completely separate processing pathways for word reading and number reading.

From a clinical perspective, our findings indicate that word reading disorders (dyslexia) should be treated as a separate clinical situation from number reading disorders (numbers-dyslexia, which may be termed “diglexia”): a person may have dyslexia for words, for numbers, or for both. The two situations should be assessed separately of each other – we cannot automatically conclude from the ability to read numbers to the ability to read words and vice versa. Similarly, different treatment methods may apply to reading of words and numbers, and we cannot make the assumption that treating one would generalize to the other.

Given that word reading and number reading are done in separate cognitive pathways, it could be informative to examine the similarities and differences between these two pathways. One important difference between words and numbers is the existence of lexicons. Word reading heavily relies on lexical knowledge, stored in orthographic and phonological lexicons. Conversely, there are no lexicons in the number reading model, except the knowledge of single digits and the phonological storage of single number words. This view is supported by studies showing that in digit-to-quantity conversion of numbers as short as two digits, the number is still processed as separate digits rather than being recognized as one lexical unit (Nuerk & Willmes, 2005; Chapter 4). Still, an extreme assumption, according to which number reading involves no lexical knowledge whatsoever, is apparently incorrect: It seems that at least some multidigit numbers that have a particular meaning (year of birth, car model, etc.) may be stored and identified as a whole. Cohen et al. (1994) reported a brain-injured patient with severe difficulties in number reading, who could still read familiar numbers – presumably via a semantic pathway (visual analysis → semantic system → verbal production). This finding still does not show the existence of phonological/orthographic lexicons for multi-digit numbers, such as the lexicons we have for words, yet it does demonstrate a different aspect of lexical processing of numbers. Future research would be required to determine the exact role of lexical knowledge in number processing.

Another potential difference between word reading and number reading concerns the way letters or digits are associated with the multi-character string to which they belong. When reading words, the visual orthographic-analyzer includes a dedicated process that binds each letter to the appropriate word (Coltheart, 1981; Ellis, 1993; Ellis & Young, 1996). Malfunctions of this process yield letter migrations between words, e.g., reading "**r**ear **d**ock" as "**d**ear **r**ock", a situation known as attentional dyslexia (Friedmann, Kerbel, & Shvimer, 2010; Humphreys & Mayall, 2001; Saffran & Coslett, 1996). An open question is whether an analogous digit-to-number binding process exists in number reading. On one hand, the high degree of homology between the orthographic-visual analyzer and the numeric-visual analyzer (both have identity, position, and structure encoders) raises the possibility that the letter-to-word binding process would also have a homologous counterpart in number reading. On the other hand, it is completely unclear whether number reading really requires a binding process of this kind, because reading words involves rapid scanning of a long sequence of words, whereas rapid scanning of a sequence of numbers is not a common activity. It is also an open question whether, if such digit-to-number binding function does exist, it is one and the same with the letter-to-word binding function that applies in word reading. To date, no study examined the notion of digit-to-number binding, but preliminary data from our lab indicate that digit-to-number binding can be intact even for individuals with attentional dyslexia, whose letter-to-word binding is impaired (Lukov & Friedmann, unpublished data).

An important point that stands out from the comparison between word reading and number reading mechanisms is the role of structural processing. For words as well as for numbers, the structure of the character string (morphological or decimal) is extracted during an early stage of visual analysis (Beyersmann et al., 2011; Longtin & Meunier, 2005; McCormick et al., 2008, 2009; Rastle et al., 2004; Reznick & Friedmann, 2015; Taft & Forster, 1975; Chapter 7). This structural information may be used to help the remaining visual analysis processes, and could also be communicated immediately to the production processes. Similarly, the structure of the verbal stimulus (morphological for words, decimal for multi-digit verbal numbers) is encoded in the verbal production processes as late as phonological retrieval. This role of structural processing suggests that reading is not a simple mechanism that merely scans a series of characters in visual input and processes a series of phonemes in verbal output. Rather, reading has dedicated processes to represent visual and verbal structures, and these processes are tailored to the type of stimulus being processed – words or numbers.

Why does our cognitive system allocate two separate processing pathways to words and numbers, two cultural inventions that are very recent in evolutionary terms? Amedi, Dehaene, and collaborators (Abboud et al., 2015; Hannagan et al., 2015) considered this question with respect to the mechanisms of visual analysis of words and numbers which, as they showed, are implemented in different brain areas – the so-called visual word form area (VWFA) and the visual number form area (VNFA). They pointed out that the reason for this neural separation is unlikely to be the visual properties of letters versus digits, because letters and digits are visually relatively similar (and in their experiment, the stimuli were identical). They proposed that the reason for neural separation between VWFA and VNFA is the connectivity patterns of these brain areas with the rest of the brain, in particular with the regions that make use of the parsed visual information. The VWFA has better connectivity with language areas, which require the parsed letters information, whereas the VNFA has better connectivity with quantity representation areas (IPS), which require the parsed digits information. The architecture we proposed here, where reading is dominated by structural processing, offers a complementary explanation for the separation of words and numbers. Although the visual properties of letters and digits are quite similar to each other, the structural properties of letter strings and digit strings are very different from each other: the decimal structure of digits is completely different from the morphological structure of words. Consequently, a dedicated visual analysis process that extracts the morphological structure of words could be very different from a dedicated visual analysis process that extracts the decimal structure of numbers. These differences may motivate the allocation of different cognitive and neural circuits to the visual analysis of words and numbers. When a processing stage is structure-insensitive, it may be shared for words and numbers, as seems to be the case for the processes that precede the numeric/orthographic visual analyzers (McCloskey & Schubert, 2014) and for post-phonological-retrieval processes (Shalev et al., 2014). Note that the structural differences merely provide motivation for separating words from numbers – they do not necessarily favor the allocation of words and number processing to specific brain regions. The allocation of a specific cognitive function to a specific brain area may be driven by other factors, such as neural connectivity patterns.

One thing is clear: the specialization of different cognitive processes to words and numbers is quite rigid. The growing number of word-number dissociations demonstrates that at least in some cases, well-functioning processing of words cannot overtake an impaired processing of numbers, and vice versa, even when the impairment is developmental and presumably existed from birth. This rigidity of word-number separation accords with the rigidity we observe within each of these domains: an intact process is sometimes unable to compensate for an impaired process, even when two processes encode information that appears to be redundant (Chapter 7). Future studies may elaborate further on the cognitive and neural factors that drive the development of this neural and cognitive specialization.

9. Breaking down number syntax: Spared comprehension of multi-digit numbers in a patient with impaired digit-to-word conversion°

Abstract. Can the meaning of two-digit Arabic numbers be accessed independently of their verbal-phonological representations? To answer this question we explored the number processing of ZN, an aphasic patient with a syntactic deficit in digit-to-verbal transcoding, who could hardly read aloud two-digit numbers, but could read them as single digits ("four, two"). Neuropsychological examination showed that ZN's deficit was neither in digit input nor in phonological output processes, as he could copy and repeat two-digit numbers. His deficit thus lied in a central process that converts digits to abstract number words and sends this information to phonological retrieval processes. Crucially, in spite of this deficit in number transcoding, ZN's two-digit comprehension was spared in several ways: (1) he could calculate two-digit additions; (2) he showed good performance in a two-digit comparison task, and a continuous distance effect; and (3) his performance in a task of mapping numbers to positions on an unmarked number line showed a logarithmic (nonlinear) factor, indicating that he represented two-digit Arabic numbers as holistic two-digit quantities. Thus, at least these aspects of number comprehension can be performed without converting the two-digit number from digits to verbal representation.

9.1. Introduction

Benjamin Lee Whorf suggested that language lies at the core of human thought and shapes our concepts (Whorf, 1940). In the domain of arithmetic, this view has been explicitly refuted by showing that a broad array of numerical abilities are spontaneously present even without verbal representation of numbers. This has been demonstrated in animals, preverbal infants, and adults from language communities with a reduced lexicon of number words (Brannon & Terrace, 2000; Dehaene et al., 2008; Dehaene, Molko, Cohen, & Wilson, 2004; Feigenson et al., 2004; Hauser, Carey, & Hauser, 2000; Nieder & Dehaene, 2009; Viswanathan & Nieder, 2013). Yet a narrower hypothesis may still be tenable, according to which some higher mathematical abilities are tightly coupled with language: a specifically human recursive computation mechanism may underlie syntactic processes, not only in language, but in other cognitive processes, including the way we represent multi-digit numbers and mathematical expressions (Hauser et al., 2002; Houdé & Tzourio-Mazoyer, 2003). Even this view of "global

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syntax”, however, is challenged by certain findings: brain areas and functional processes that characterize language syntax are dissociable from those that support many combinatorial mathematical processes, including the processing of algebraic operations (Monti, Parsons, & Osherson, 2012) and mathematical expressions (Maruyama, Pallier, Jobert, Sigman, & Dehaene, 2012), multi-digit number naming (Brysbaert, Fias, & Noël, 1998), transcoding, and multi-digit calculation (Varley, Klessinger, Romanowski, & Siegal, 2005).

The present chapter aims to further probe this issue by analyzing dissociations between different syntactic processes within the domain of number cognition in an aphasic patient with impaired conversion of Arabic numbers to words. Specifically, we asked whether the meaning of two-digit Arabic numbers can be accessed independently of their verbal representations when the syntactic mechanisms converting numbers to words are impaired. Our goal is to examine in detail the locus and nature of the patient's impairment in transcoding, and to evaluate his number meaning abilities and syntactic processes of number comprehension using various tasks.

Numbers have three distinct representations: they can be coded in digits as Arabic numerals (68), as number words (sixty-eight), or as quantities, the dominant "meaning" of the number. These different cognitive representations are dissociable (Gordon, 2004; Lemer et al., 2003), can be selectively impaired (Cohen & Dehaene, 2000), and are implemented in different brain areas (Dehaene et al., 2003). However, these representations are tightly related: symbolic representations of numbers (words, digits) are associated with the corresponding quantities, which can be represented spatially along a left-to-right mental number line (Dehaene et al., 1993; Loetscher et al., 2010; Moyer & Landauer, 1967; Ruiz Fernández et al., 2011; Shaki et al., 2009).

Multi-digit Arabic numerals enter into several types of internal conversion processes. Converting multi-digit Arabic numbers to number words is a syntactic process that requires encoding the relative positions of the digits according to the base-10 system, converting each digit to a word according to its position, and combining the words, sometimes with the addition of coordination markers (“and”). This syntactic sub-process can be selectively impaired (Cipolotti, 1995; Noël & Seron, 1993). But multi-digit Arabic numbers can also be quickly converted into the corresponding quantity (Dehaene et al., 1990; Reynvoet & Brysbaert, 1999; and Section A of this dissertation). Computing the quantity associated with an Arabic multi-digit number requires encoding the relative positions of the digits and combining their quantities according to the base-10 principles. Thus, syntactic operations are required when converting multi-digit Arabic numerals either to verbal number words or to quantities. Is a single syntactic process involved in both conversion processes? The present study addresses one aspect of this question: we asked whether the conversion of a two-digit Arabic number into a quantity can be spared when the syntactic operation involved in forming a verbal representation of the number is impaired.

The triple-code model of number processing (Dehaene, 1992; Dehaene & Cohen, 1995) predicts that there is a direct conversion route from Arabic inputs to the quantity representation, independent of the Arabic-to-verbal route. However, the verbal representation of numbers is also thought to play a crucial role even in tasks that do not necessarily involve overt comprehension and production of verbal numbers, e.g., memorization of arithmetic facts (Cohen & Dehaene, 2000; Dehaene, 1992; Dehaene & Cohen, 1997). The relation between verbal representations and quantity is sometimes surprisingly complex, to the extent that quantity encoding may be affected by the language in which a verbal number is presented, even in the same person: Dehaene et al. (2008) investigated individuals from an Amazonian culture with little or no formal mathematical education, and found that their quantity processing showed a more linear pattern when numbers were presented in a Western second language (Portuguese) than in their native tongue, Mundurucu, where numerals yielded a more logarithmic pattern.

To separate these two possibilities, and probe whether Arabic-to-quantity conversion makes use of a syntactic process that is also needed for Arabic-to-verbal conversion, we examined the various number abilities of ZN, an aphasic patient who has a selective deficit in verbal number production. This deficit prevents him from converting multi-digit numbers into verbal-phonological forms, and renders him almost completely unable to say them aloud. We tested whether, in spite of this deficit, he can encode the holistic quantity of two-digit Arabic numbers.

Another question addressed in this study is whether multi-digit addition depends upon verbal-phonological forms of number words. Rote knowledge of arithmetic facts relies on the verbal representation of numbers (Cohen & Dehaene, 2000; Dehaene, 1992; Dehaene & Cohen, 1997; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). Multi-digit addition too can be affected by verbal factors such as the grammatical structure of number words (Colomé, Laka, & Sebastián-Gallés, 2010), though not always (Brysbaert et al., 1998). Phonological representations may be involved in multi-digit addition, but are probably not necessary for it (Klessinger, Szczerbinski, & Varley, 2012). The present case study of ZN extends Klessinger et al.'s conclusions by examining a dissociation between multi-digit addition and the verbal representation of numbers: we tested whether ZN could solve addition problems that involve two-digit numbers in spite of his severe impairment in the conversion of Arabic numbers to verbal representation.

9.2. Case Description

9.2.1. Overview

ZN, a 73 years old man, used to work as an engineer, a job that involved a lot of number processing. When he was 72 he had a subacute infarct in the left corona radiata, following which he lost much of his speech, and was diagnosed with aphasia, severe apraxia of speech, impaired comprehension, dyslexia, dysgraphia, and agrammatism. He started language rehabilitation that focused on articulation, lexical retrieval, and grammatical processing. By the time we met him, eleven months after his stroke, he still had severe difficulties in comprehension and production of speech. Until that time he was neither diagnosed nor treated for number processing.

ZN is right-handed and wears reading glasses. His mother tongue is Hebrew, and all tests were conducted in this language. He is an engineer with B.Sc. degree. We tested him in a series of 45-minute sessions that took place in a quiet room in his home. The sequence of sessions lasted several months, but crucial tasks (hereby described) of number reading and comprehension were administered in intertwined sessions within a short period of less than 3 months.

9.2.2. Language Assessment

ZN’s lexical retrieval was impaired. When asked to name 100 objects in a picture naming task (SHEMESH, Biran & Friedmann, 2004), he made phonological errors and neologisms in 45/100 items. These could be explained by his apraxia of speech, but he also failed to make any verbal response to 40 items, and made 13 semantic errors – a finding that indicates that on top of his apraxia, he also had a deficit in an earlier stage of lexical retrieval (Friedmann et al., 2013).

His working memory was assessed in a digit span task (Friedmann & Gvion, 2002; Gvion & Friedmann, 2012) in which he answered by pointing to the digits 0-9 on paper. His digit span was 3, significantly lower than an age-matched control group (z = -3.06, p = .001; control data from Gvion and Friedmann, 2012).

ZN had severe morpho-syntactic difficulties. In a picture-to-sentence matching task (BAFLA, Friedmann, 1998), his performance was at chance level not only in object and subject relative clauses (18/40 errors), but even in simple subject-verb-object sentences (14/30 errors). In a sentence completion task, which required him to inflect verbs for tense and agreement (BAFLA, Friedmann, 1998), he had inflection errors in 10/24 items and failed to respond to 5 items. The morphological difficulty was present not only in verbs but also in nouns: in a picture-naming task that required ZN to inflect morphologically complex nouns, he made morphological errors (substitution or omission of the morphological affix) in 7/20 items and failed to respond to 2 items.

His reading aloud was impaired too. In reading 49 words from the TILTAN dyslexia screening test (TILTAN, Friedmann & Gvion, 2003), he made 37 errors and failed to respond to 6 additional words. His errors were phonological paraphasias (phonemic and formal, in 20 items), neologisms (10 items), morphological errors (8 items), and sublexical reading (surface-dyslexia-like errors, 4 items). His word writing was severely impaired too (8/11 errors).

9.3. Assessment of symbolic number processing

9.3.1. Input and output of Arabic numbers

ZN's ability to read and write numbers as digits was assessed using two tasks: number dictation and delayed copying of numbers.

Number dictation. The experimenter said aloud the 40 numbers between 1 and 40 in random order, one at a time, and ZN wrote them as Arabic numerals. He performed this task without any error (40/40 correct).

Delayed copying. ZN saw twenty 2-digit numbers on the computer screen, one at a time. Each number was presented for one second, and after it disappeared, ZN was requested to write it down on paper. To eliminate possible verbal rehearsal, ZN was required to say aloud the first two Hebrew alphabet letters (*alef*, *bet*) after the target number disappeared from the screen and before he wrote it down. In this task too, his performance was flawless (20/20 correct).

These two tasks demonstrate ZN's ability to write two-digit numbers in Arabic notation, which stands in contrast to his complete inability to write words (Fisher's $p < .001$ in number dictation vs. word writing). The delayed copying task further shows that his Arabic number input is intact: he correctly encoded the identity and the relative positions of the digits in the two-digit number, and could memorize the number until writing it down. The dictation task shows his preserved ability of converting number words to digits.

9.3.2. Production of verbal number words

Next, we assessed ZN's ability to produce number words orally. In the following tasks we classified responses as correct whenever the target number was produced, even if it was not articulated completely accurately (e.g., 35 → "thirby five"). This is because our focus in this section was not on examining ZN's articulation, which is known to be impaired by his apraxia of speech, but on examining earlier processing stages involved in number processing. Thus, phonological errors were not included in the overall error rates[10].

Reading aloud 2-digit numbers. ZN saw a list of 33 two-digit numbers and was asked to read them aloud. The numbers were administered as three different lists (of eight, five, and 20 numbers) in separate sessions. Five of the numbers were teens and the rest were larger than 20. None of the numbers included the digit zero. The first two lists were printed on paper in Arial 22 font. The third list (of 20 numbers) was read from the computer screen, and the numbers were presented for one second.

In marked contrast to his good number writing, ZN's reading aloud was very poor and he produced correctly only 7 of the 33 numbers. The most striking error pattern was that he read most of the numbers as two separate digits (e.g., *15 → one five*, or *47 → four seven*). This occurred for 23 of the 33 items (70%). He was able to produce the decade name for only 9 of the 28 numbers larger than 21, and could not produce the teen form for any of the five teen numbers that were presented. On top of that, he had other errors too: he omitted a digit in two numbers, made lexical within-class errors in 14 numbers (e.g., *63 → seventy three*), and class errors in 2 numbers (e.g., *14 → forty*; in Hebrew, some class errors are not phonologically similar to the target, and this was the case with these two errors).

Reading 2-digit numbers as single digits. ZN was presented with a list of the 40 numbers from 1 to 40, appearing in random order, and was asked to read them aloud as single digits (e.g., 54 → five, four) – i.e., a total of 71 digit names. The numbers were printed on paper in Arial 22 font. ZN performed well in this task: he made only one lexical error, and another error that could be interpreted either as lexical or as phonological, with a total of 69/71 correct digit names.

Thus, although ZN was able to identify and name the digits in two-digit numbers, he was unable to say aloud two-digit numbers when he was asked to produce the number name with a

[10] In reading aloud 2-digit numbers (the task hereby described), ZN made 15 phonological errors of the 25 numbers that were encoded for phonological errors (we had technical problems with the audio recording of the remaining 8 items, so we do not know how many phonological errors they included). In reading 2-digit numbers as single digits he made phonological errors in 20/71 digits. In number repetition he made phonological errors in 30/40 items.

valid decade+unit syntactic structure (5% vs. 79% errors, $\chi^2 = 41.6$, $p < .001$). This pattern persists even when comparing only the 15 two-digit numbers that appeared in both tasks (one error in the digit reading task, but 13 errors when reading the numbers with a valid syntactic structure, Fisher's $p < .001$). Namely, when he read a number as a two-digit number, he failed, but when he read the exact same number digit by digit, without the need to process its syntax, he succeeded. In the next section, we explore the origin of this multi-digit number production deficit.

9.3.3. The origin of ZN's difficulty in verbal number production

The results show that ZN has a deficit in transcoding Arabic numbers to number words that selectively affects his ability to verbally produce two-digit (and multi-digit) numbers. This deficit is not in the input stages, as demonstrated by ZN's good performance in the delayed copying task, by his ability to read two-digit numbers when required to say only the digit names, and, as we will show below (Section 9.4.1), by his spared ability to perform two-digit additions. One crucial result in this matter is his good production of single digits (e.g., "four, three" for 43). Is it the case that ZN's deficit is in the production stage, and he cannot retrieve teen and decade number words? If this were the case, we would expect him to fail also in other tasks that require multi-digit number production, such as number repetition. We therefore tested his multi-digit number repetition.

Number repetition. The experimenter read aloud the numbers between 1 and 40 in random order and ZN was asked to repeat the number. ZN's ability to repeat numbers correctly (or only with phonological errors) was good, and clearly superior to his performance in the number reading task: he correctly repeated 37 of the 40 numbers, and made one lexical error, one error that may be lexical or syntactic, and one lexical or phonological error. Importantly, in the repetition task ZN never tried to produce the digit names instead of the number name, like he so often did in the number reading task.

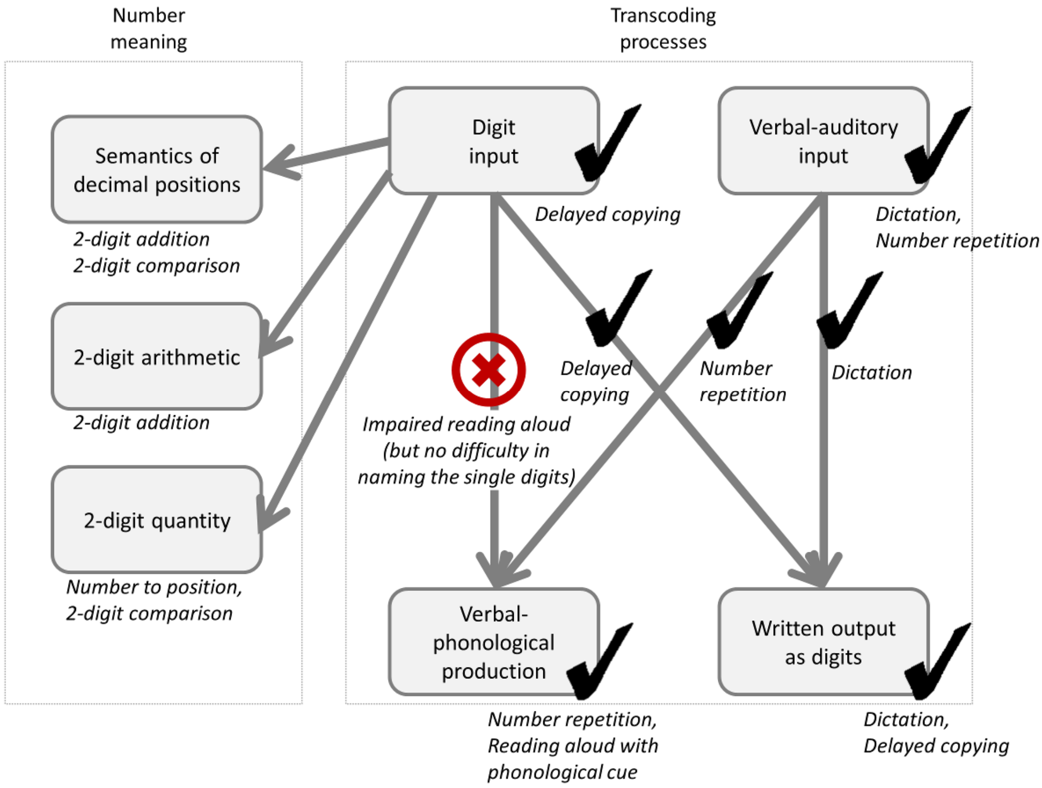

Fig. 9.1. An overview of the numerical processes that were tested in ZN. The italic text denotes the tasks that indicate which processes are spared and which are impaired.

Any task that requires number production involves retrieval of the phonological forms of the number words from a phonological storage in the phonological processing stages (Dotan & Friedmann, 2015; McCloskey et al., 1986; Oppenheim, Dell, & Schwartz, 2010). ZN's good repetition shows that he was able to retrieve the phonological forms of the number words and to articulate them (even if with phonological errors). Thus, his phonological retrieval and articulation stages are not the source of his difficulty to produce two-digit number words. The deficit that caused this difficulty must be in a pre-phonological processing stage, between the input of the written numbers and their production. This conclusion, and the evidence supporting it, is visually summarized in Fig. 9.1.

Number-like nonword repetition. Finally, we ruled out a possibility that ZN's good performance in the number repetition task was due to phoneme-by-phoneme repetition, and therefore may not indicate that he was able to retrieve the phonological representations of numbers. To assess this possibility, we compared his performance in number repetition with a nonword repetition task, which is undoubtedly performed phoneme-by-phoneme, via the sublexical repetition route. For each number word, a corresponding nonword was created with matched length, syllable structure, stress position, morphological structure, CV structure, and consonant clusters. For example, for the number 34, /shloshim ve-arba/, the matched nonword was /frifol ve-umdi/. The order of the stimuli was also the same in both tasks. This created a list of 40 number-like nonwords.

The comparison between the repetition of numbers and number-like nonwords revealed very different patterns. ZN made significantly more consonant substitutions in nonword repetition than in number repetition (13% vs. 4% out of 180 consonants, $\chi^2 = 9.3$, one-tailed $p = .001$). This indicates that the two tasks were performed via different processing pathways: the number repetition task involved a lexical repetition route, which induced a smaller amount of phonological errors than the sublexical repetition route. The findings therefore refute the possibility that ZN repeated numbers sub-lexically, and support the conclusion that he can retrieve number words, including teens and decades, when the access to the phonological number words does not involve reading of multi-digit Arabic number, and hence, does not require digit-to-verbal transcoding.

Another finding that refutes a strictly sublexical number repetition hypothesis concerns the way ZN combined the decade and unit names. In Hebrew, the decade and unit number words are combined by the function word “and” (i.e., we say “thirty and two” rather than “thirty two”). The Hebrew word “and” is generally pronounced as /ve/, but in some phonological contexts it is considered normatively "more correct" or higher register to pronounce it as /u/. In the repetition task, the experimenter used the /u/ pronunciation in three occasions, and in all cases ZN repeated the number using the /ve/ pronunciation, thereby showing that he did not process the function word merely as a sequence of phonemes, but treated it as a lexical/syntactic element. Together with the different patterns of phonological errors in number and nonword repetition, these results indicate that ZN did not use a phoneme-by-phoneme strategy for repeating numbers. Last, several studies of number production mechanisms render the sublexical repetition hypothesis unlikely (Bencini et al., 2011; Cohen et al., 1997; Dotan & Friedmann, 2015): these studies investigated aphasic patients with phonological deficits and showed that even in a very late stage of speech production, the phonological output buffer, whole number words are processed as atomic phonological units rather than as separate phonemes.

The above experiments show that ZN has a deficit in one of the modules along the Arabic-to-verbal transcoding route, which selectively affects his ability to say two-digit (or multi-digit) numbers, while sparing single-digit numbers. The deficit is in a stage later than the Arabic number input modules and earlier than the phonological production stages, i.e., the deficit is in the process of converting the digits to number words (this pathway is marked in Fig. 9.1 with ⊗). Furthermore, the deficit is syntactic as it affects only two-digit numbers (and longer numbers too, although they were not systematically explored in this study) whereas single digits are spared.

At this point we know that ZN's deficit is in a syntactic module in the process that converts multi-digit Arabic numerals into number words. We can think of this process as involving two stages – a "core" stage that converts numbers from a sequence of digits to a set of abstract identities of number words (Cohen & Dehaene, 1991; Dotan & Friedmann, 2015; McCloskey et al., 1986, 1990; Sokol & Mccloskey, 1988), and a subsequent stage that transfers this information to the morpho-phonological production modules. Because his number repetition indicates that the source of his number deficit is not the phonological output itself, we can conclude that the latest possible locus of deficit is in the module that sends the output of the conversion process, namely, the abstract identities of number words, to the phonological modules of lexical retrieval. In terms of the number reading model described in Chapter 7, ZN's deficit may be in any of the verbal-structural processes that result in the number word frame (Fig. 7.2).

Although ZN's deficit is in number syntax, our findings rule out the extreme possibility that he has lost *all* syntactic abilities: in both number reading tasks (as numbers and as single digits) he never said the unit digit before the decade digit. Thus, the relative order of the two digits, which is one kind of syntactic information, was spared. ZN's good dictation of two-digit numbers showed that his verbal-to-digit syntactic processing was also spared. As the next section will show, these were not the only kind of spared syntactic abilities.

9.4. Assessment of number comprehension

We have now reached the main question of this study: we saw that ZN cannot convert two-digit Arabic numbers to the phonological form of number words. Can he still understand two-digit numbers?

We were interested in two general questions about his comprehension of numbers – can he understand two-digit numbers, and what is the nature of the processes he uses to understand numbers. More specifically, the first question we will ask is whether he can apply the process that converts the relative positions of the digits into their abstract decimal roles as decades and units. This question was assessed using a two-digit addition task and a number comparison task (see the left column in Fig. 9.1).

The second question relates to the way he processes the quantity corresponding with two-digit numbers. Two-digit numbers can be encoded as decomposed decade and unit quantities (Meyerhoff et al., 2012; Moeller, Klein, Nuerk, & Willmes, 2013; Moeller, Nuerk, et al., 2009; Nuerk & Willmes, 2005) but they can also be encoded by combining the tens and units values into an appropriate overall holistic quantity (Brysbaert, 1995; Dehaene et al., 1990; Reynvoet & Brysbaert, 1999). Does ZN encode a two-digit number as a holistic quantity? Such a holistic encoding would unequivocally show that he not only managed to categorize the two digits into their two decimal roles and understand the quantity represented by each digit, but he was also able to reach a combined quantity by evaluating the two digits in correct proportions. This second question was assessed using the number comparison task and an Arabic number-to-position mapping task.

9.4.1. Two-digit addition

The two-digit addition task examined ZN's ability to assign the two digits into their decimal roles as decades and units, and apply the procedures required to add them. He was presented with written exercises in which he added single-digit numbers to other single-digit numbers or to two-digit numbers. The exercises were always presented using Arabic numbers and ZN answered in writing. He was shown three single-digit additions (X+Y) with a single-digit result, four round-number additions (X0+Y or X00+Y), and eight two-digit additions (XY+Z), five of which required carry procedure. His performance was flawless.

This performance with written answers is markedly different from his performance when he was required to give oral responses to two-digit additions. He could not answer orally any of the 3 two-digit addition exercises presented to him (XY+Z), and responded by saying only the (correct) unit digit of the result. He performed flawlessly, however, with single digit additions (X+Y; 3/3 correct), and had only one error in 4 round-number additions.

Thus, in spite of his deficit in verbal production of multi-digit numbers, ZN can still perform two-digit addition, even when a carry procedure is required, as long as he is not required to produce the result orally.

9.4.2. Two-digit comparison

A common task to examine holistic processing of numbers is two-digit comparison. In the task that we used, ZN was asked to decide in each trial whether the two-digit number presented on screen is smaller or larger than 55 (“the standard”). Number comparison tasks involve comparison of magnitudes, and reaction times decrease when the target-standard distance increases (Dehaene et al., 1990; Hinrichs & Novick, 1982; Moyer & Landauer, 1967; Nuerk & Willmes, 2005). In the present task, if the participant uses two-digit holistic quantity, we should observe a continuous distance effect (Dehaene et al., 1990).

9.4.2.1. Method

A two-digit number was presented on screen in each trial. ZN was asked to compare each number as quickly as possible to a fixed reference number of 55; he pressed the “Z” key with his left hand to respond “smaller than 55”, or the “.” key with his right hand to respond “larger than 55”. Each of the numbers from 31 to 79, except 55, was shown 4 times. The 192 trials were presented in random order. The experiment was implemented using PsychToolbox with Matlab R2012a on a Macbook Pro laptop with a 13” monitor. The numbers were presented in the center of the screen in black font on gray background. The digits were 2 cm high.

9.4.2.2. Results

ZN had 2.6% errors in this task (5 errors), and these trials were excluded from the analyses. His RT was 877 ± 273 ms, with no significant difference between hands (right: 909 ± 346 ms; left: 844 ± 161 ms, $t(185) = 1.66$, two-tailed $p = .10$)[11].

To analyze the effect of target-standard distance, the trials were grouped into 3 groups by their distance from 55 (distances of 1-8, 9-16, or 17-23). The RT was submitted to ANOVA with the distance group and response type (smaller/larger than 55) as factors. Both factors had a significant main effect (distance group: $F(2,181) = 4.2$, two-tailed $p = .02$; response type: $F(1,181) = 2.76$, one-tailed $p = .05$) and there was no interaction ($F(2,181) = 1.11$, $p = .33$). The mean RTs of the three distance groups were 954, 865, and 814 ms respectively, and the linear contrast was significant ($F(1,181) = 8.18$, $p = .005$), which confirms the predicted distance effect.

[11] The variance in right-hand responses was larger than in the left hand ($F(90,95) = 4.64$, $p < .0001$), perhaps as a result of the left-hemisphere brain damage.

The RTs were then submitted to a regression analysis. 14 outlier trials were removed (a trial was defined as an outlier with respect to the four trials of the same target number, if removing this trial decreased the standard deviation to 33% or less). The predictors were log(absolute distance between target and standard), the response type (1 or -1), and the product of these two (to assess interaction). Only log(distance) had a significant effect (p = .001), confirming again the distance effect. The two other predictors were not significant (p > .23).

To study the contribution of the unit digit to the distance effect, we used two regression analyses introduced by Dehaene et al. (1990). In the first analysis, the predictors were *LogDiz*, which represents the decade distance, and *Dunit*, which assesses the unit digit contribution[12]. Both predictors were significant (p < .04), which shows that both digits affected the comparison. The second analysis was run only on trials outside the standard's decade. The predictors were log(absolute distance between target and standard), response type, their product, and *Dunit*. Only log(distance) had significant contribution (p < .02, and $p \geq$ .13 for the other predictors), showing that the holistic distance is a sufficient predictor of the distance effect, and that the decomposed unit makes no additional observable contribution to the RT.

9.4.2.3. Discussion of the two-digit comparison task

ZN's high accuracy on this task clearly indicates that he was able to understand two-digit numbers and assign the digits to their appropriate decimal roles as decades and units. Furthermore, his performance is in accord with the assumption that he used holistic encoding of the two-digit quantities, as we observed a target-standard distance effect that extended beyond the standard's decade, with no additional contribution of the unit digit.

9.4.3. Number-to-position

The number-to-position task, discussed at length in the first section of this dissertation, is another common paradigm to investigate quantity representation. ZN performed our iPad-based version of this task, so we could tap his two-digit quantity representation in detail.

9.4.3.1. Method

The number-to-position task was administered as described in Section 2.2. Each target number between 0 and 40 was presented four times, all in random order.

[12] Let D_s, U_s, D_t, and U_t be respectively the decades and units digits of the standard and the target ($D_s = U_s = 5$). *LogDiz* is the logarithm of $1 + |D_t - D_s|$. *Dunit* equals zero for targets within the standard's decade (i.e., when $D_t = D_s$). Outside the standard's decade, Dunit equals U_t - 4.5 for targets smaller than the standard and 4.5 - U_t for targets larger than the standard.

ZN's results were compared with a control group of 15 right-handed individuals matched for age (70;7 ± 4;2, from 66;6 to 79;7), language (native Hebrew speakers), education (BA or MA degree), and occupation (they all worked, like ZN, in number-oriented jobs: 9 engineers, 3 math teachers in high or junior high schools, 2 economists, and one accountant).

9.4.3.2. Results

ZN's movement time was 1320 ± 220 ms from target onset to reaching the number line, which is very similar to the control participants (1290 ± 210 ms, Crawford & Garthwaite's (2002) and Crawford & Howell's (1998) t(14) = .14, one-tailed p = .45). Fig. 9.2a shows ZN's finger trajectories (for each target number, a median trajectory was calculated by re-sampling the raw trajectories into equally time points and finding the median coordinate per time point).

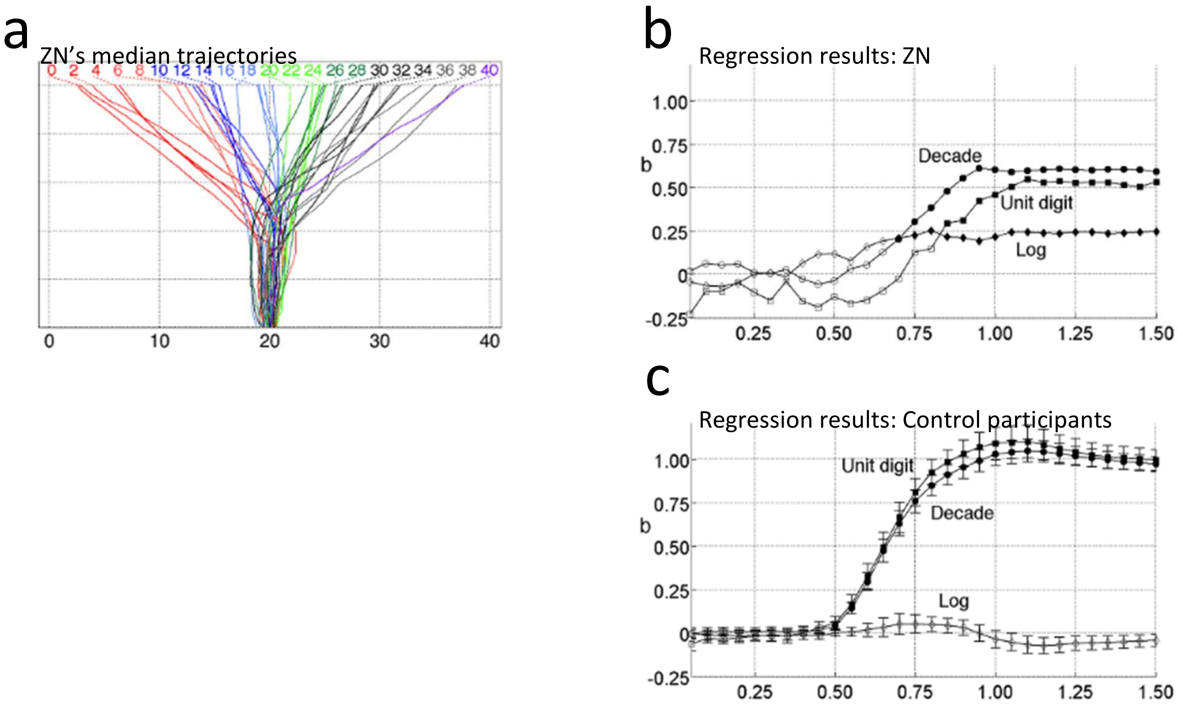

Fig 9.2. Number-to-position task results. (a) ZN's median trajectories per target. (b-c) The b values of the regressions that capture the results of the number-to-position task. Each group of 3 vertically-aligned points represents a single regression of a specific post-stimulus-onset time (t = 0 is the stimulus onset). Significant b values ($p \leq .05$) are represented by black markers.

The quantity representation in this task can be investigated by finding which factors govern the finger's horizontal movement. This was done using a regression analysis. The dependent variable was the trajectory endpoints (the positions marked by ZN on the number line), and there were three predictors. The first two predictors account for the linear quantity representation: the target number's decade (0, 10, 20, 30, or 40) and the unit digit. The two digits were entered as separate predictors to account for the possibility that the contributions of the decomposed decade and unit quantities deviate from a 1:10 ratio. The third predictor was log(target+1), linearly rescaled to the 0-40 range. This predictor taps holistic-logarithmic representation of quantity. The regression showed significant contribution of the all three predictors ($p < .001$)[13].

A similar regression analysis was performed to assess how ZN's quantity representation evolves throughout a trial (from stimulus onset until he touches the number line). One regression was run per post-stimulus-onset time point, in 50 ms intervals. The same three predictors were used (decade, unit, log) and the dependent variable was the *implied endpoint* – the position on the number line that the finger would hit if it keeps moving in its current direction θ_t. This θ_t was defined as the direction vector between the finger x,y coordinates at times t - 50 ms and t. The implied endpoint was also cropped to the range [-2, 42] and was undefined when the finger moved sideways ($|\theta| > 80°$). The regression was also run for each of the control participants, and the significance of each b value in the control group (per predictor and time point) was assessed by comparing the group's b values with 0 using t-test. One-tailed p values were used for average(b) > 0, and two-tailed p values for average(b) < 0.

This sequence of regressions (Fig. 9.2b) showed that ZN had significant contributions of the decade predictor from 700 ms post-stimulus-onset and in all subsequent time points, of the units from 850 ms, and of the log from 650 ms. The control group (Fig. 9.2c) showed an earlier effect of the decades (from 500 ms) and units (from 550 ms) digits and no significant group-level log effect. We now turn to a detailed analysis and comparison of these effects.

9.4.3.2.1. ZN encodes holistic two-digit quantities

The existence of a significant logarithmic factor clearly shows that ZN represented a holistic quantity that integrated the decade and unit values of the target number. This is because the logarithmic function cannot be represented as a linear combination of the decade and unit quantities, so a logarithmic factor in the regression necessarily reflects a log or log-like function of the whole quantity. Importantly, there was a time window of 100 ms, starting in 650 ms, in which the log predictor was significant but the linear predictors (decade and unit) were not yet

[13] In our experiments with healthy participants, the regression analyses discovered a fourth significant predictor that reflects a spatial aiming strategy in the late trajectory parts ("the spatial reference points" effect, Section 2.3.2.6). However, in ZN's data this predictor had no significant effect on the trajectory endpoints (b < .001, p > .93), nor was it significant in the trajectory analysis described in the next paragraph ($p > .12$ in all time points). Thus, we did not use this predictor here.

significant. This indicates that the holistic-logarithmic quantity representation preceded the linear representation.

We compared ZN's performance pattern with the group of healthy control participants using the three-predictor regression model described above. ZN's b value of the log predictor was higher than the control group in 250 ms and in all subsequent time points, and this difference was marginally significant in 1250 ms and in the subsequent time points (Crawford & Garthwaite's (2002) $t \geq 1.83$, two-tailed $p \leq .1$). A per-participant analysis showed that 4 control participants had a significant log effect in 2 or more time points. ZN's log effect remained quite stable throughout the trajectory and was observable even in the endpoints. This pattern is different from the control participants, for whom the non-significant logarithmic trend was clearly transient (see Fig. 9.2c)[14].

We examined and excluded an alternative explanation according to which ZN employed a decomposed quantity representation of each of the digits using a logarithmic quantity scale. According to such an alternative explanation, the log factor in Fig. 9.2b is an artifact of the correlation between the log(target+1) predictor, which we used in the regression, and the factors that allegedly governed ZN's hand movement: some linear combination of log(decade) and log(unit-digit). To rule out this possibility, another regression analysis was run: the dependent variable was still the implied endpoint, but the logarithms of the decade and unit were added as two new predictors on top of the decade digit, the unit digit, and log(target+1). One such regression was run per post-stimulus-onset time point, in 50 ms intervals. The results showed significant contributions of log(target+1) in 800 ms and in all time points from 900 ms ($p < .05$). Importantly, there was no time point in which any of the single-digit logarithms made a significant contribution. In fact, their regression *b* values were negative in 900 ms and in all subsequent time points.

Another alternative explanation that we ruled out was the possibility that the log effect results from faster encoding of smaller quantities. Faster processing of small-target trials would make their initial finger trajectories farther apart from each other than the trajectories of larger target numbers. To neutralize this differential quantity encoding speed, we aligned trajectories

[14] ZN's log effect was also compared with the 21 younger participants reported in Chapter 2. This comparison too showed that ZN's performance pattern was no less logarithmic than the control group's – in fact, his b[log] was *larger* than the b[log] of this control group in 600 ms and in all subsequent time points, and this difference was significant from 850 ms and onwards (Crawford & Howell's (1998) $t \geq 2.28$, two-tailed $p \leq .04$; and from 1000 ms, $t \geq 4.05$, $p < .001$). The log effect of the younger control participants was also transient, like the older control group (and unlike ZN).

by the time point of the first significant horizontal finger movement (calculated as described in Section 3.2.2.4.1) and re-ran the decade-unit-log regression on the aligned trajectories. Even in these regressions, which neutralize possible differences in velocity onset per trial (and per target), ZN still showed a logarithmic effect (b[log] > .12, one-tailed $p < .05$, in all time points from 700 ms post-velocity-onset, and $p < .07$ from 500 ms), thereby refuting the differential velocity onset as an alternative explanation.

The results show unequivocally that ZN used holistic encoding of two-digit quantities, and that this holistic encoding was not impaired in comparison to the control group.

9.4.3.2.2. Decomposed linear quantity encoding

Fig. 9.2b shows that ZN's effect of the unit digit seems slightly delayed with respect to the decade digit. This difference was statistically assessed by modifying the predictors in the above per time point regression analysis into log(target+1), the target number $N_{0\text{-}40}$, and the unit digit U. In this new set of regressions, the predictor U captures situations in which the relative contributions of the decade and unit digits deviate from a strict 1:10 ratio. Such deviation was indeed found: the unit digit predictor's b value (b[U]) was smaller than zero in all time points, and this difference was significant in a certain time window (two-tailed $p < .05$ in 800 and 900 ms; and $p < .1$ from 650 ms to 950 ms except in 750 ms). These results suggest that ZN was processing the decade and unit digits in decomposed and possibly serial manner.

ZN's delayed processing of the unit digit was not statistically different from the control group: comparing his b[U] in the log+target+unit regression with the control participants showed no significant difference in any time point (even when assuming that his b[U] should be smaller than the controls' and consequently using one-tail *p* values, only a marginally significant difference was found in only 3 time points – 800, 900, and 950 ms – Crawford & Garthwaite's (2002) $t \leq -1.49$, $p < .1$). A per-participant analysis of the control group showed that three participants also showed a significant b[U] < 0 in two or more time points. Thus, even if ZN's processing of the decade and unit digits appeared slightly more sequential than the control group's, this difference was very small.

9.4.3.2.3. Accuracy

ZN's endpoint error – the absolute difference between the judged endpoint and the correct target position – was 3.13 ± 2.43 (using the 0-40 scale). This is less accurate than the control participants, whose mean endpoint error was 1.94 ± .53 (Crawford & Garthwaite's (2002) t(14) = 2.17, p = .02). ZN's endpoint errors were not correlated with the target number (r = -.05, p = .53), nor was there another, non-linear dependency between the target number and the endpoint error (one-way ANOVA, F(40,123) = 1.33, p = .12).

9.4.3.3. Discussion of the number-to-position task

ZN's performance in this task showed that he encoded two-digit quantities holistically. There was no evidence to suggest that the holistic encoding was impaired with respect to healthy participants – in fact, the holistic-logarithmic trend in ZN's result was even slightly higher than in the control group. Given ZN's severe syntactic deficit in converting two-digit numbers from digit to verbal representation, we can reach the most important conclusion in this study: constructing the holistic quantity was performed successfully, independently of the impairment in digit-to-verbal conversion. Furthermore, an analysis of the linear factors in this task suggests that ZN's ability to process the decade and unit digits in parallel was comparable with that of the control participants, or only slightly worse.

9.5. Discussion of chapter 9

This study presented the case of ZN, an aphasic patient who has a selective syntactic deficit in converting two-digit numbers from digit representation to verbal-phonological representation. ZN can read aloud single digits but he has great difficulty in reading aloud two-digit Arabic numbers using a valid decade+unit syntactic structure. A detailed neuropsychological examination showed that ZN's deficit is neither in the Arabic input nor in the phonological output modules, because he could copy multi-digit numbers, write them to dictation, and repeat them. His syntactic deficit therefore lies in the central process that converts the digits into a structured sequence of abstract identities of number words (the *number word frame*, Cohen & Dehaene, 1991; Chapter 7), or in a subsequent stage that uses these abstract identities to access the phonological production modules. This deficit is not a global deficit in processing number syntax: ZN has intact syntactic processing in the opposite pathway – verbal to digit representation – as demonstrated by his good performance in number dictation. This dissociation between digit-to-verbal and verbal-to-digit syntax is in line with previous studies (Cipolotti, 1995).

In spite of his deficit in digit to verbal number conversion, ZN showed spared number comprehension and spared number syntax abilities in several ways. First, he is able to add two-digit numbers with single-digit numbers, even when the addition exercise requires carry operation, as long as verbal output is not required. This shows that he understands the base-10 system, can assign the digits to their decimal roles as decades and units, and can carry out the addition procedure. This finding extends previous studies showing that multi-digit addition does not depend on phonological (Klessinger et al., 2012; Varley et al., 2005) and orthographic (Varley et al., 2005) representations of verbal numbers: whereas those studies showed that addition does not depend on phonological encoding of the numbers, we showed that addition does not depend even on an earlier stage – a syntactic module involved in digit-to-verbal transcoding. In this sense our conclusions resemble Brysbaert et al.'s (1998), who showed that addition is unaffected by the syntactic structure of verbal numbers in a certain language. However, whereas Brysbaert et al.'s conclusion rested on a null effect of language in nonverbal calculation, we managed to show a strict dissociation between spared addition and impaired syntactic processing.

Crucially, ZN's spared comprehension and syntax was also shown by his ability to encode two-digit numbers as holistic quantities. This was demonstrated by the finding of a continuous two-digit distance effect in the two-digit number comparison task, and by the finding of a logarithmic factor in the two-digit number-to-position mapping task. His good performance in these tasks also demonstrated his ability to assign digits to decimal roles. Fig. 9.3 illustrates these conclusions.

These findings lead to interesting conclusions regarding the specificity of the modules that process number syntax. Multi-digit Arabic numbers require syntactic processing when converted to verbal number words, when converted to quantities, and when manipulated in addition exercises. Our results unequivocally show that certain syntactic functions – assigning digits to their decimal roles and converting two-digit Arabic numbers to holistic quantities – are dissociable from at least some of the syntactic processes involved in digit-to-verbal transcoding, because these syntactic functions can be successfully performed even when one of the syntactic digit-to-verbal transcoding processes is impaired.

These results are in line with several previous studies that dissociated between Arabic number comprehension and Arabic-to-verbal transcoding. Several previous patients showed impairments of digit-to-word conversion with spared number comprehension (Cohen & Dehaene, 1995, 2000; Cohen et al., 1994). Other studies specifically reported patients with a *syntactic* deficit in digit-to-word conversion (Cipolotti, 1995, patient SF; Cipolotti & Butterworth, 1995, patient SAM) who could perform certain number comprehension tasks –

number comparison (both patients), multi-digit comparison (SF), and two-digit addition (SAM). The syntactic deficit of SAM and SF still allowed them to assign digits to decimal roles. The present findings replicate and extend these results, particularly using the number-to-line task to demonstrate a fine-grained preservation of the encoding of two-digit numbers as holistic quantities in patient ZN.

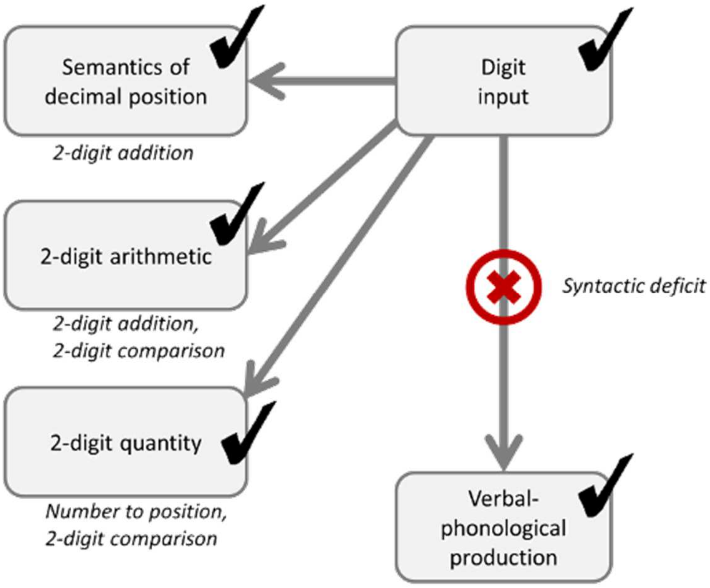

Fig. 9.3. In spite of ZN's syntactic deficit in converting numbers in Arabic notation to a verbal representation, he can assign the digits to their decimal roles, encode two-digit holistic quantities, and perform two-digit additions.

Taken together, such neuropsychological cases indicate that the syntactic processes involved in converting digits to words and digits to quantities are at least partially separate, and that several aspects of two-digit number comprehension can be achieved without transcoding the number to its verbal representation. This conclusion fits with several other findings that dissociated language syntax from several aspects of syntax-dependent mathematical processing (Brysbaert et al., 1998; Maruyama et al., 2012; Monti et al., 2012; Varley et al., 2005). Taken together, this body of evidence weakens the hypothesis that a single global mechanism underlies all kinds of syntactic processes (Hauser et al., 2002; Houdé & Tzourio-Mazoyer, 2003) and promotes a view of several, distributed syntactic processes.

Section C

From digits to arithmetic

10. Reducing interference improves the memorization of calculation facts°

Abstract. Hypersensitivity to interference (HSTI) is a situation where a person is extremely sensitive to verbal interference when trying to memorize verbal information. Individuals with HSTI have difficulty in memorizing verbal items that are similar to each other. This may result in induced dyscalculia: HSTI was shown to be correlated with a difficulty in learning the multiplication table, presumably because the multiplication table, which is typically memorized verbally, has much similarity between the items ("three times four", "three times five", etc.). Here, we show causal evidence that HSTI disrupts the memorization of multiplication facts. We examined DL, a woman with HSTI who had a severe difficulty in memorizing multiplication facts. To examine whether her multiplication difficulty results from interference, we tested whether she could learn multiplication facts when interference was reduced. In a series of merely 12 short sessions over a period of 4 weeks, DL rehearsed 16 multiplication facts – four facts per week. When the 4 facts in a given week were similar to each other, DL's learning was poor. Conversely, when the 4 facts in a given week were dissimilar from each other, DL learned them quickly and easily. The effect of similarity was observed at the end of the 4-week training period, and persisted after two months during which DL received no additional training. These results provide the first causal evidence that hypersensitivity to interference impairs the learning of arithmetic facts. From a clinical perspective, the success of our training method may call for a change in the way multiplication facts are taught in elementary school.

10.1. Introduction

Why is it so hard for some people to learn the multiplication table – single-digit multiplications up to 10*10? In his book *The Number Sense*, Stanislas Dehaene aimed to clarify the difficulty of learning multiplication and addition facts by inviting the reader to the following mental exercise:

> *Arithmetic facts are not arbitrary or independent of each other. On the contrary, they are closely intertwined and teeming with false regularities, misleading rhymes, and confusing puns. What would happen if you had to memorize an address book that looked like this: […]*
>
> *Charlie David works on Albert Bruno Avenue*
> *Charlie George works on Bruno Albert Avenue*
> *George Ernie works on Charlie Ernie Avenue*

° This chapter has supplementary material in Appendix C.

> *Learning these twisted lists would certainly be a nightmare. Yet they are nothing but multiplication table in disguise. They were composed by replacing each of the digits 1, 2, 3, 4, 5, 7 … by a surname – Albert, Bruno, Charlie, David, Ernie, George […]. No wonder we have trouble remembering them: the most amazing thing may well be that we* do *eventually manage to memorize most of them!* (Dehaene, 1997, p. 127).

Dehaene is suggesting that memorizing the multiplication table is as difficult as memorizing a list of arbitrary, highly similar verbal items. The analogy he draws between multiplication facts and verbal facts (names and addresses) is not coincidental: according to his triple-code model of number processing (Dehaene, 1992; Dehaene & Cohen, 1995), multiplication facts are stored using verbal representation. This view was supported by many behavioral and brain imaging studies (for a review, see Dehaene et al., 2003): neuropsychological studies showed that multiplication deficits were associated with verbal impairments (as opposed to subtraction deficits, which were associated with impaired quantity processing; Cohen & Dehaene, 2000; Cohen, Dehaene, Chochon, Lehéricy, & Naccache, 2000; Dagenbach & McCloskey, 1992; Dehaene & Cohen, 1997; Delazer & Benke, 1997; Lampl, Eshel, Gilad, & Sarova-Pinhas, 1994; Lochy, Domahs, Bartha, & Delazer, 2004; Pesenti, Seron, & van der Linden, 1994; van Harskamp & Cipolotti, 2001). Furthermore, several brain imaging studies showed dissociations between multiplication and subtraction (Chochon, Cohen, van de Moortele, & Dehaene, 1999; Cohen et al., 2000; Lee, 2000), and multiplication activated brain areas that are also activated by tasks of language, verbal short-term memory, and phonological processing (Dehaene et al., 1999; Simon, Mangin, Cohen, Le Bihan, & Dehaene, 2002).

Several studies showed that similar verbal items may interfere with one another in memory tasks (Hall, 1971; Nelson, Brooks, & Borden, 1974; Oberauer & Kliegl, 2006; Oberauer & Lange, 2008). Multiplication facts, which are stored verbally, may therefore be subject to this verbal interference, because they are highly similar to each other. Note that low capacity of verbal memory is not sufficient by itself to explain all multiplication difficulties, because there is double dissociation between low memory capacity and impaired knowledge of arithmetic facts (Butterworth, Cipolotti, & Warrington, 1996; Kaufmann, 2002). Interference may be the specific factor that could explain impaired knowledge of arithmetic facts even when memory capacity is unimpaired. In particular, interference might be the reason for operand errors – responding to a multiplication exercise with the result of another multiplication exercise, e.g., 4x5=24 (Ashcraft, 1992; Campbell, 1987; Campbell & Graham, 1985; De Visscher & Noël, 2014b; Kaufmann, 2002; Kaufmann, Lochy, Drexler, & Semenza, 2004; Lemaire & Siegler, 1995; Sokol, McCloskey, Cohen, & Aliminosa, 1991; Stazyk, Ashcraft, & Hamann, 1982; Thibodeau, Lefevre, & Bisanz, 1996).

All of us may sometimes suffer memorization difficulties arising from verbal interference, but for some the difficulty is more severe than for others. De Visscher and Noël (2013, 2014a, 2014b) suggested that some people have hyper-sensitivity to interference – an extreme sensitivity to interference from similar verbal items – and that such individuals may show worse-than-normal knowledge of multiplication facts. In support of their hypothesis, they reported DB, a woman with very poor memory of the multiplication table, and showed that she also had hypersensitivity to interference: she performed poorly in tasks that were sensitive to interference, even when they involved only non-number words. In contrast, she performed normally in tasks that assessed several other potential sources of difficulty in calculation, including verbal working memory capacity. De Visscher and Noël suggested that DB's difficulty in memorizing the multiplication table was a reflection of a more general verbal difficulty – her hyper-sensitivity to interference.

De Visscher and Noël's series of studies is very convincing, yet they only showed correlational relation between hypersensitivity to interference and difficulty in arithmetic facts. In the present study, we wished to strengthen their point by providing casual evidence to the claim that hypersensitivity to interference disrupts the memorization of multiplication facts. To this end, we examined DL – a woman that, similarly DB – had poor memory of multiplication facts and hypersensitivity to interference. To show that hypersensitivity to interference not only correlates with DL's difficulty in multiplication facts but is also the *reason* for this difficulty, we designed an experiment to demonstrate that once interference was taken out of the game, DL would be able to memorize multiplication facts.

The specific idea was as follows. First, we reasoned that even if hypersensitivity to interference impairs DL's ability to memorize *similar* verbal items, she would still be able to memorize *dissimilar* verbal items. This assumption is well supported by the studies of De Visscher and Noël (2013, 2014a, 2014b). We capitalized on the fact that although the multiplication table as a whole has much similarity between the facts, some facts are dissimilar from each other (e.g., 9*9=63 and 7*4=28). Thus, DL may still be able to learn a subset of the multiplication table that consists of dissimilar facts. Second, we assumed that the interfering effect of fact A on a similar fact B depends on A and B being presented within reasonable time from each other. If A and B are learned with sufficient temporal delay between them, they would not interfere with each other. Thus, even if we teach DL the full multiplication table, she might still be able to learn the subset of dissimilar facts if this subset is taught in a time period during which no other multiplication facts are presented.

These two foundations led to the following simple training method. We identified the multiplication facts that DL did not know, and grouped them into small sets of facts. Crucially, different sets of facts had different levels of between-item similarity within the set, i.e., different levels of induced interference. Each set was taught for one week. Importantly, in the week when DL was learning a certain set of facts, she refrained from rehearsing facts from any of the other sets, in order to avoid interference from out-of-set facts. We predicted that DL would have difficulty in learning the multiplication facts in high-similarity sets, but would succeed learning the low-similarity sets.

Our goal in this study was not only theoretical but also clinical. A success of our experiment would suggest a simple scheme to teach the multiplication table to individuals with hypersensitivity to interference. For our training scheme to be valid clinically, we should show not only that item similarity has the predicted effect on learning, but also that this effect persists over time. To this end, DL's knowledge of the multiplication facts was tested not only at the end of the training period, but also after a period of two months during which she received no additional training.

10.2. Case description

DL was a 40-year-old woman who arrived in our lab to assess of her difficulties in math. Initial examination indicated that her main difficulty was a severely impaired knowledge of the multiplication table. Put in her own words, she was "clueless in multiplication". She reported that her difficulties began in elementary school, and persisted in spite of hard work and private tutoring in math during several years. When she finished school after 11 years, her math grades were very low.

Language abilities. DL was assessed in a series of reading tasks (TILTAN, Friedmann & Gvion, 2003). She was flawless in reading of single words and nonwords, and had a single error in reading 30 word pairs (372 adult control participants had an average of 1.52 errors). This shows she had good reading and good lexical retrieval. Her lexical retrieval was further assessed in a picture naming task , where she had only 3 errors in 100 pictures – same as the average of a control group of 102 adults aged 20-50 (SHEMESH, Biran & Friedmann, 2004).

10.3. Assessment of DL's difficulties

10.3.1. Knowledge of multiplication facts

In a screening test, DL was presented with 12 multiplication facts – four rule-based facts (N*0 and N*1) and eight other facts (both operands ≥ 2). She had 2/4 errors in the rule-based facts, 2/8 errors in the non-rule facts, and our subjective impression was that the task was extremely difficult for her, even on trials in which she gave the correct answer. Notably, she erred even in the rule-based facts. At this time, we taught her the rules *N x 0 = 0* and *N x 1 = N*, and in all subsequent testing she never erred again in these rule-based multiplication facts.

We then tested her knowledge of all 55 multiplication facts (the larger operand always appeared first) – 19 rule-based facts and 36 non-rule facts[15]. She was flawless in the rule-based facts, but she had 14/36 errors (39%) in the non-rule facts: 5 operator errors (adding instead of multiplying), 2 within-table errors (saying the result of another multiplication fact), 1 out-of-table error (saying a number that is not the product of any two digits), and 7 “don’t know” responses. This was significantly worse than the performance of 10 age-matched control participants (mean age = 39;4, SD = 3;0), who had 0-4 errors each (mean = 1.5 errors, SD = 1.58; comparing DL with the worst-performing control participant, $\chi^2 = 7.4$, one-tailed $p = .003$). Thus, DL clearly had impaired knowledge of the multiplication table. We next examined potential origins for this impairment.

10.3.2. Other aspects of calculation

DL was presented with 15 single-digit addition exercises and was asked to say the result verbally. She performed flawlessly, although she hesitated in some exercises. She was also flawless in 8 subtraction exercises in which the first operand was 0-20 and the second operand and the result were 0-10.

We then presented DL with 9 two-digit calculation exercises (three additions, three subtractions, three multiplications). She easily applied correct calculation procedures and solved all the exercises (except one case, in which she used the correct procedure but was not sure about the result of a single-digit subtraction fact).

[15] This test is in fact the first pretest session defined in Section 10.4.1.1.

Thus, DL did not have a general difficulty in all aspects of calculation – her difficulty was restricted to arithmetic facts, and especially multiplication facts.

10.3.3. Verbal memory

As reviewed in the Introduction, multiplication facts are stored verbally, so we examined whether DL's difficulty in multiplication facts results from a verbal memory deficit: low capacity of verbal-phonological short-term memory, or a difficulty in storing verbal information in long-term memory or retrieving it.

Table 10.1. DL's performance in tasks that assess possible origins of her difficulty in multiplication facts. She showed good performance tasks sensitive to phonological working memory, and in symbolic number processing.

Task	No. of items	DL	Control participants Mean (SD)	DL vs. controls [a]
Short-term memory span				
Digit span		7	7.05 (1.28) n=29	t(28) = .04, *p* = .49
Word span		6	5.57 (0.75) n=35	DL was better
Nonword span		3	3.46 (0.54) n=37	t(36) = .84, *p* = .20
Word matching span		5	6.33 (0.98) n=12	t(11) = 1.3, *p* = .11
Digit matching span		7	7 (0) n=10	DL was better
Nonword production (% errors)				
Nonword reading	30	0	4.1 (4.19) n=372	DL was better
Nonword repetition	48	2.1	4.6 (3.5) n=20	DL was better
Symbolic number processing (% errors)				
Number reading aloud	120	3.3	2.8 (1.3) n=21	t(20) = .38, *p* = .36
Number repetition	120	9	4.6 (3.6) n=20	t(19) = 1.25, *p* = .11
Number dictation	68	2.9	2.5 (2.3) n=20	t(19) = 0.21, *p* = .42

[a] One-tailed *p* values are reported.

Low capacity of verbal-phonological short-term memory may stem from a limited phonological input buffer or from a limited phonological output buffer. During comprehension, the phonological input buffer is responsible for maintaining auditory verbal information in memory until it is processed. During speech production, the phonological output buffer is responsible for merging phonological elements into phonological sequences (words and sentences), and for maintaining these sequences until they are produced (Butterworth, 1989, 1992, Dell, 1986, 1988; Franklin, Buerk, & Howard, 2002; Friedmann et al., 2013; Friedmann

& Gvion, 2002; Garrett, 1976, 1992; Gvion & Friedmann, 2012; Kempen & Huijbers, 1983; Levelt, 1989, 1992; Monsell, 1987; Nickels, 1997; Nickels, Howard, & Best, 1997; Patterson & Shewell, 1987; Shallice, Rumiati, & Zadini, 2000; Shallice & Warrington, 1977a).

We used three kinds of verbal short-term memory tasks. To examine the phonological input buffer, we used a word-sequence and a digit-sequence matching span tasks: DL was presented with pairs of sequences of words or digits, in increasing length, and judged whether the two sequences in each pair were identical (e.g., *house-train-star*; *house-train-star*) or differed in the order of items (e.g., *house-train-star*; *house-star-train*). This task requires memorizing the auditory input but does not stress the phonological output mechanisms, so the task specifically taps the phonological input processes, in particular the phonological input buffer. To examine the phonological output buffer, DL was asked to read aloud 40 nonwords (TILTAN, Friedmann & Gvion, 2003) and to repeat 48 nonwords (BLIP, Friedmann, 2003), some of which were long, and phonologically or morphologically complex. She was also tested in serial recall (span) tasks, in which she repeated sequences of digits, words, or nonwords in increasing lengths (FriGvi, Friedmann & Gvion, 2002; aged-matched control data was taken from there). The span tasks require memorizing the auditory input as well as verbal production, so they tap both the input buffer and the output buffer, as well as additional mechanisms (Martin & Lesch, 1996). As shown in Table 10.1, DL's performance in all these tasks was good, i.e., her verbal short-term memory was intact.

DL also performed two memorization tasks that examined her verbal long-term memory. The first task required memorizing a list of arbitrary words that were presented repeatedly. The second required memorizing a short story.

Memorizing a list of words (Vakil, Blachstein, & Sheinman, 1998). This task, which includes 10 sub-tasks, examined DL's ability to memorize words in a context-free manner. The task started as a free recall task: the experimenter read aloud a list of 15 nouns, and DL recalled as many words as she could, in any order. The same list was repeated 5 times, with a recall attempt after each time. DL's performance was within or above norm in all 5 sub-tasks (*z* scores: .57, 1.03, .25, 1.44, -.05). A new list of 15 words was then read aloud to DL, and she recalled this list and then the first list again. These two sub-tasks are aimed to examine verbal-semantic interference between the two word lists, and DL performed well in both (*z* scores: -.68, .56). She then recalled list #1 again after a 20-minute retention interval during which other, non-verbal tasks were administered. Again she performed well (z score = 1.42), thereby showing that she managed to store the words in long-term memory and retrieve them. She was then asked to recognize words of list #1 from a longer list of 50 nouns (the original list and 35 distracters – semantic, phonological, and words from list #2). She was flawless. Finally, she was shown the 15 words of list #1 in random order and was asked to sort them. Again, she performed well (z = .93). Overall, DL's good performance in all sub-tasks demonstrates her good verbal short-term and long-term memory.

Memorizing a short story (Cohen, 1997). The experimenter read aloud two short stories (about 100 words each), one after another. DL repeated each story immediately after its presentation, and again after 30 minutes (during which other tasks were administered). Her performance was on the 34^{th}-40^{th} percentile both in immediate and delayed recall. Again, this result indicates well-functioning short-term and long-term verbal memory.

Overall, DL performed well in all the verbal memory tasks. This shows that her short-term and long-term memory were functioning well. Thus, her difficulty in memorizing multiplication facts does not originate in a general memory deficit. This pattern of results resembles previously-reported dissociations between good memory functions and impaired knowledge of arithmetic facts (Butterworth et al., 1996; Kaufmann, 2002).

10.3.4. Symbolic number processing

To examine the possibility that DL's difficulty was related to a general deficit in symbolic number processing, we assessed her ability to process Arabic numbers and number words and to convert multi-digit numbers from one format to another. In a **number reading task**, she read aloud from paper a list of 120 multi-digit numbers (30, 38, 47, and 5 items with 3, 4, 5, or 6 digits, respectively). The digit 0 appeared in 57 items, the digit 1 appeared in 41 items, and 38 items included neither 0 nor 1. Her performance was compared to 21 control participants (mean age = 25;5, SD = 2;6). In a **number repetition** task, she repeated the same 120 numbers. Her performance was compared to 20 control participants (mean age = 26;1, SD = 4;8). Finally, a **number dictation** task required writing on paper 58 numbers, 3-5 digit long, that were read by the experimenter. The digit 0 appeared in 27 of the numbers. DL’s performance was compared to 20 control participants (mean age = 34;7, SD = 9;5). In all tasks, we excluded control participants with outlier error rates (higher than the 75^{th} percentile by more than 150% the inter-quartile range).

DL’s performance in all the above tasks was good (Table 10.1). Thus, her difficulty in memorizing the multiplication facts did not originate in impaired processing of symbolic numbers.

10.3.5. Sensitivity to interference

We next examined whether DL had hypersensitivity to interference, which may have disrupted the acquisition or retrieval of multiplication facts. The task that examined sensitivity to interference was a "first name – surname – country" memorization task (adapted to Hebrew from De Visscher & Noël, 2013). The task resembles the mental exercise described by Dehaene (1997), cited in the beginning of this chapter, in the sense that it required memorizing a list of verbal, non-numeric facts. To specifically tap sensitivity to interference, DL was required to memorize two lists of items: one list with high between-item similarity, and another list with low between-item similarity. If she has hypersensitivity to interference, she should succeed memorizing the low-similarity list but not the high-similarity list. This was exactly the pattern of results exhibited by the person described in De Visscher and Noël (2013), and as we shall now see – also by DL.

10.3.5.1. Method

DL was asked to memorize a list of 12 fictitious person names (first name + surname) and a country in Africa or Asia where each of them allegedly lived. Unknown to DL, the 12 names were two mixed lists with 6 names in each – a low-similarity list, in which each first name and surname appeared only once; and a high-similarity list, in which there were only 3 first names and 3 surnames, each repeating twice to create the names of 6 different people. The list was provided for memorization in five successive and identical learning stages, at the end of which DL's knowledge was tested. Each learning stage was administered as follows: the experimenter said aloud each list item (name-surname-country) and DL repeated it. Then, the experimenter presented – in random order – each name+surname, and DL said where that person lived. If she made an error, the experimenter corrected her. The 5 learning stages were followed by a final test stage: DL was presented with 24 name-surname-country combinations, and judged whether each combination was correct or not. In this final test, each of the 12 name-surname combinations appeared twice: once with the correct country, and once with the country of one of the 5 other persons in his similarity-level group. Both during learning and during testing, items were presented in semi-random order such that no two subsequent items had the same first name, surname, or country.

DL's performance in this task was compared with 24 age-matched control participants (mean age = 40;3, SD = 5;2, range = 31;6 to 48;5) with no reported cognitive deficits and with normal memory spans (mean digit span = 7.17, SD = 1.19; Friedmann & Gvion, 2002). One additional control participant was excluded due to outlier performance (chance level) in the final test.

10.3.5.2. Results

DL showed a dramatic effect of similarity in this task. In the final test (Table 10.2), she performed almost at ceiling on the low-similarity list, having only a single error (which is not significantly higher than zero errors, Fisher's $p = .50$). This performance was even slightly better than the control group. Conversely, her performance was poor on high-similarity items – significantly worse than the control group, not significantly different from chance level ($\chi^2 = .17$, one-tailed $p = .34$), and significantly worse than her own performance in the low-similarity items ($\chi^2 = 3.56$, one-tailed $p < .03$). Crucially, increasing the similarity level (low-similarity list versus high-similarity list) disrupted DL’s performance significantly more than it disrupted the control group’s performance (dissociation analysis of Crawford, Garthwaite, & Porter, 2010: t(23) = 2.15, one-tailed $p = .02$). The results clearly show that DL was sensitive to the item similarity level significantly more than the control group. Namely, she had hypersensitivity to verbal interference.

Table 10.2. The number of errors (out of 12) in the verbal memorization task (name-surname-country). DL performed poorly in the high-similarity items, which are especially sensitive to interference, but she performed well in the low-similarity items.

Similarity between items	DL	Control participants Mean (SD)	DL vs. controls [a]
Low	1	1.68 (1.44)	DL was better
High	5	2.12 (1.26)	t(23) = 2.24, $p = .02$

[a] One-tailed *p* values are reported.

We note that the control group too was affected by the similarity level of items, even if this effect was smaller than DL’s: their performance in low-similarity items was marginally better than in high-similarity items (paired t(23) = 1.64, one-tailed $p = .06$). Furthermore, during intermediate learning stages, they performed better in low-similarity items than in high-similarity items (e.g., in the last learning stage they recalled 4.21 out of 6 low-similarity items, SD = 1.72, but only 2.79 out of 6 high-similarity items, SD = 1.28; paired t(23) = 4.30, one-tailed p = .0001). These results agree with previous findings of similarity-induced interference in normal population (Corman & Wickens, 1968; Hall, 1971; Mark-Zigdon & Katzoff, 2015; Oberauer, Lewandowsky, Farrell, Jarrold, & Greaves, 2012; Oppenheim et al., 2010; Posner & Konick, 1966; Runquist, 1970, 1971).

10.3.6. Summary of the assessment results

The experiments above showed that DL’s difficulty in solving multiplication facts was not caused by a general memory or language impairment, nor did it originate in deficits in symbolic number processing. Her multiplication difficulties are best explained as hypersensitivity to verbal interference: she demonstrated this hypersensitivity also in a memorization task that did not involve numbers.

Given this conclusion, we now turn to the main question of this study – to examine whether the training method we devised would indeed help DL overcoming her hypersensitivity to interference, and enable her to learn the multiplication table.

10.4. Multiplication facts training

10.4.1. Method

The training program was structured as a pre-training test, a training period, a test at the end of the training period, and a follow-up test after two months. The training was done on the 16 multiplication facts with the lowest pre-training scores. These facts were grouped into four sets with four facts in each. Each set of facts was trained during one week (and only during this week). After this 4-week training period, a post-training test evaluated DL's knowledge of all multiplication facts. Another test was run after 2 months, during which DL received no training. Throughout this 3-month period, DL was asked not to rehearse multiplication on her spare time, and she reported to have followed this instruction. All training and test sessions were performed over the telephone, while DL was in a quiet room in her home. Training and testing were done orally – the repetition of facts, the experimenter's questions, and DL's answers.

Crucially, the four sets of trained multiplication facts differed from each other with respect to the degree of within-set interference: there was one low-interference set, one high-interference set, and two medium-interference sets. DL was aware that the aim of our intervention was to teach her multiplication facts, but she was unaware of the experiment design details, in particular of the manipulation of interference levels. We predicted that the effectivity of training would be influenced by the within-set interference level, namely, that DL would show worse memorization of higher-interference sets. As we shall see below, this prediction was confirmed, which means that at the end of the study DL knew some multiplication facts but not others. Thus, after completing the study (including the follow-up test), we taught DL the remaining facts properly, in low-interference condition, so that by the time she left our lab she knew the multiplication table fully.

10.4.1.1. Testing before and after the training

DL's knowledge of the multiplication facts was tested in 3 time points (hereby, "testing times"): before training, immediately after training, and two months after the training ended. Each testing time included 3 separate testing sessions, administered in 3 separate days of a single week. In each of these testing sessions, DL was asked to solve the 55 multiplication facts (larger operand first). Reaction times were defined as the delay between the experimenter finishing to ask the question and DL beginning to say the result.

Three kinds of responses were classified as errors: (1) incorrect responses, including situations where DL made several response attempts, at least one of which was incorrect. (2) DL did not know the answer. (3) Extremely slow responses – i.e., outlier reaction times. Such slow responses may suggest that DL was employing a calculation strategy rather than retrieving the multiplication fact from memory. Outliers were defined as reaction times that exceeded the 75th percentile by more than 150% the inter-quartile range. Outlier calculation was done within each set of 36*3=108 non-rule facts of a single testing time, excluding items that were classified as errors by one of the two other criteria.

Per testing time, each exercise was given a score between 0 and 3 based on the 3 testing sessions. The 16 facts with lowest pre-training scores were selected for training (10, 3, and 3 facts with score = 0, 1, and 2, respectively.

10.4.1.2. Grouping the trained facts into sets

The 16 trained facts were grouped into four sets (four facts per set) that differed from each other in the level of within-set similarity. We calculated a similarity index per set, using De Visscher and Noël's (2014b) method: first, the similarity between each two multiplication facts was defined as the number of identical digit pairs that appear anywhere in the two facts. For example, the facts 8*7=56 and 8*3=24 have no common digit pair (only the digit 8 appears in both) so their similarity index is 0. The facts 3*4=12 and 3*7=21 have three common digit pairs (1-2, 2-3, and 1-3) so their similarity index is 3. The similarity index of each 4-facts set was calculated as the sum of the similarity indices of each of the 6 pairs of multiplication facts in the set (see Appendix A for comparison of this similarity index with other possible indices).

Of the four sets of facts, one set had high similarity index (7*4=28, 7*6=42, 8*4=32, 9*4=36, similarity = 18). The three other sets had lower similarity indices – very low similarity in one set (4*4=16, 8*3=24, 8*7=56, 5*3=15, similarity = 0) and moderate similarity in two sets (8*8=64, 9*7=63, 6*2=12, 8*6=48, similarity = 6 and 9*6=54, 6*5=30, 8*5=40, 7*5=35, similarity = 8).

10.4.1.3. The training program

Each set of four facts was trained over 3 sessions, in 3 separate days of a single week. A fourth session, administered after the weekend, was dedicated to testing DL's knowledge of all facts that she learned since the beginning of the 4-week training period. This design implies that on one hand, earlier sets received a bit more training in retrieval; on the other hand, on the time of the final test the memory of later-learned sets might have been fresher. The high-similarity set was scheduled for the second week of training, packed between lower-similarity sets.

Training sessions: Each training session lasted about 5 minutes. The session started with a pretest, in which DL was asked to solve each of the 4 trained exercises. After the pretest, the experimenter corrected DL's errors. Next, three memorization-and-recall phases were done. In each phase, the experimenter said each multiplication fact (exercise and result, e.g., "four times five, twenty") and DL repeated it immediately. The four facts were presented in a fixed order. After this memorization, DL recalled the four facts in free recall. The experimenter immediately corrected her when she gave an incorrect answer or when she did not know the answer, and reminded her of exercises that she forgot to mention. At the end of the session, a post-test was administered in the same way as the pretest.

Testing sessions: The first session in each week (except the first week) was dedicated to testing DL's knowledge of all the facts she learned since the beginning of the training program. Namely, 4 facts were tested in the beginning of the 2nd week, and all 16 facts were tested in the beginning of the 5th week. No teaching was done during these test sessions. Rather, each fact was presented 3 times in pseudo-random order: the same fact never appeared twice in a row, and the question a*b was never followed by the questions a*(b±1) or (a±1)*b. No feedback was provided for specific exercises, but DL was told her total number of errors at the end of the testing session.

10.4.2. Results

10.4.2.1. Effectiveness of training: performance over all facts

Table 10.3 shows DL's average performance in each testing time, separating between rule-based facts (N x 0 and N x 1) and non-rule facts. Outlier reaction times of the non-rule facts were defined as explained in Section 10.4.1.1: slower than 2485 ms in the pre-training test, 3165 ms in the post-training test, and 4040 ms in the follow-up test. To avoid over-representation of some facts over others (due to excluding errors and RT outliers of some facts more than of others), in each testing time we calculated the average RT per fact, and the RT analyses were based on these averages.

DL solved the rule-based exercises flawlessly in all three testing times. This was better than her performance in the non-rule facts ($\chi^2 > 5.41$, one-tailed $p \leq .01$). In the pre-training test, the rule-based facts were also solved more quickly than the non-rule facts ($t(113) = 4.14$, one-tailed $p < .0001$).

Table 10.3. DL's overall performance before and after training. The error rate in the trained facts significantly dropped following the training, and this improvement persisted two months later, in the follow-up test.

		Before training	After training	Follow-up
Rule-based facts				
	% Errors	2	11	4
	RT (ms)	753 (312)	1114 (512) ***	665 (263)
Non-rule facts				
All	% Errors	45	26***	19***
	RT (ms)	1033 (306)	1097 (413)	1616 (1044)***
All - matched[a]	RT (ms)	1047 (303)	1029 (338)	1285 (538)
Trained	% Errors	81	42***	35***
	RT (ms)	1114 (432)	1276 (463)	2194 (1087)*
Untrained	% Errors	17	13	7*
	RT (ms)	1009 (268)	971 (330)	1241 (529)

Comparison with pre-training test (unpaired t-test, one-tailed p for errors, two-tailed p for reaction times): * $p < .05$ *** $p < .002$

[a] Facts that were answered correctly at least once in each of the testing times. The per-fact mean RTs were compared with paired t-test.

The error rates in the non-rule facts significantly dropped from the pre-training test to the post-training test, demonstrating that the training was effective. As predicted, this overall improvement was driven by a significant improvement in the trained facts, with no significant improvement in the untrained facts[16]. Reaction times increased in the follow-up testing, but this was mostly an artifact: before training, many facts were excluded from the RT statistics because DL did not answer, or gave an incorrect answer. After training, more facts were answered correctly, but sometimes slowly. When analyzing only the facts that were answered correctly at least once in all three testing times, the RT increase in the follow-up test was not significant for the trained facts (paired $t(4) = .82$, two-tailed $p = .46$) and marginally significant for the trained+untrained facts (paired $t(24) = 19.97$, two-tailed $p = .06$).

10.4.2.2. Effect of within-set interference on the post-training knowledge

Table 10.4 presents DL's detailed performance per exercise in each of the 3 testing times. Our main prediction was that the training would be more effective for sets with lower within-set similarity. To test this prediction, we examined the point biserial correlation between the within-set similarity level (1, 2, or 3) and the success/failure in each answer attempt in the post-test. To eliminate a possible effect of prior knowledge, only the 10 exercises with pre-training score = 0 were analyzed (but including all items yielded essentially the same results). The correlation between similarity level and success was significant ($r_{pb} = .30$, one-tailed $p = .05$), confirming the predicted effect of similarity. This effect persisted in the follow-up test after two months ($r_{pb} = .35$, one-tailed $p = .03$).

A possible concern is that the effect of interference is actually a problem size effect in disguise. Knowledge of multiplication facts is typically better when the operands are smaller (Zbrodoff & Logan, 2005; Zimmerman et al., 2016). The correlation between the average operand size and the within-set similarity level, although mild and non-significant ($r = .10$, $p = 0.60$), may perhaps explain the above findings. However, contrary to this explanation, the post-training performance did not correlate with the average operand size (point biserial correlation = -.02), indicating that the results were not an artifact of a problem size effect. As for the follow-up results, they did correlate with problem size (point biserial correlation: $r = -.34$, one-tailed $p = .001$). Thus, to assess a possible problem size artifact, the follow-up test scores of the 10 exercises with pre-training score = 0 (one score per exercise) were submitted to logistic regression with two predictors: the average operand size and the within-set similarity

[16] Conceivably, the reduced error rate in trained facts could be explained as regression to the mean, because the pretest data presented here was also used to select the trained facts. However, we doubt that regression to the mean could convincingly account for a difference of 39% in error rates. Furthermore, the regression to the mean account predicts a corresponding *increase* in error rates in the untrained facts, but no such increase was observed.

level (1, 2, or 3). Both predictors had significant contribution (b[similarity] = -1.24, p = .03; b[operand size] = -.34, p = .05), refuting the problem-size artifact interpretation, and confirming that lower within-set similarity resulted in better performance.

Table 10.4. DL's performance score per trained fact (scale: 0-3) and per testing time (scale: 0-12). After training, she performed better in the low-similarity set than in the high-similarity set. This difference persisted when she was tested 2 months later.

	Within-set similarity		Score			Answers during training (each cell is an answer attempt)													
			Before training	After training	Follow-up	Day 1				Day 2					Day 3				
Week 1	Low	4 * 4 = 16	0	3	3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	(S = 0)	8 * 3 = 24	0	2	2	x	x	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		8 * 7 = 56	0	1	0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		5 * 3 = 15	2	3	3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		Total	2	9	8														
Week 2	High	7 * 4 = 28	0	2	1	✓	✓	✓	✓	x	✓	x	x	✓	✓	x	✓	✓	✓
	(S = 18)	7 * 6 = 42	0	1	0	x	x	✓	x	✓	✓	✓	x	✓	✓	✓	✓	✓	✓
		8 * 4 = 32	0	0	0	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	x	✓	x	x
		9 * 4 = 36	0	1	1	x	x	x	x	✓	x	x	✓	x	✓	✓	✓	✓	✓
		Total	0	4	2														
Week 3	Medium	8 * 8 = 64	0	3	2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	(S = 6)	9 * 7 = 63	0	2	2	x	✓	x	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		6 * 2 = 12	1	1	3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		8 * 6 = 48	2	3	3	✓	✓	✓	✓	✓	✓	x	✓	✓	✓	✓	✓	✓	✓
		Total	3	9	10														
Week 4	Medium	9 * 6 = 54	0	2	2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	(S = 8)	6 * 5 = 30	1	0	3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		8 * 5 = 40	1	3	3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		7 * 5 = 35	2	1	3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
		Total	4	6	11														

Red/Green = below/above 50% performance.

A second analysis was restricted to the low-similarity and high-similarity sets (and excluded the medium-similarity sets). Per testing time and per set, we counted the number of correct answers of the 12 answer attempts. The post-training accuracy in the high-similarity set (25%) was lower than in the low-similarity set (67%, χ^2 = 4.2, one-tailed p = .02), even when excluding the single exercise with high pre-training score (χ^2 = 2.29, p < .07). Again, this pattern persisted after two months: the follow-up test accuracy in the high-similarity set (33%) was lower than in the low-similarity set (83%, χ^2 = 6.17, one-tailed p = .006), even when excluding the single exercise with high pre-training score (χ^2 = 3.5, one-tailed p = .03).

10.4.2.3. DL's progress during the training sessions

The effect of within-set similarity was observed not only in the post-training test results but also in DL's performance during training. Table 10.4 shows each of DL's answer attempts during training. For low- and medium-similarity sets, she reached a ceiling level by the end of the first or second training day, whereas the high-similarity set continued posing difficulty even by the end of the third training day. To quantify this difference, we defined the "last error day" per fact – the last training day in which DL made at least one error in that fact (0 if no errors). The 4 facts with earliest last-error-days were exactly the 4 facts in the low-similarity set (an event with a random probability 1 to $\binom{8}{4}$ – i.e., $p < .02$), confirming that DL learned the facts in the low-similarity set more quickly than in the high-similarity set.

The effect of similarity did not go unnoticed by DL herself: during the 2nd week of training, when she learned the high-similarity set, she commented more than once that "it is hard for me to learn these exercises because of all these 4's that repeat over and over again" – an accurate description of her sensitivity to interference.

10.5. Discussion of Chapter 10

10.5.1. Hyper-sensitivity to interference as a source for difficulty in memorizing calculation facts

We reported the case of DL, a 40-year-old woman with severe difficulties in memorizing the multiplication table. A series of tasks showed several spared memory functions: DL's short-term memory spans were in the normal range, she performed well in nonword reading and repetition, and she showed good ability to remember an arbitrary list of words and the details of a story. These results indicate good short-term and long-term memory abilities, i.e., DL's difficulties in memorizing multiplication facts did not originate in impaired verbal memory. In contrast, DL performed poorly in a task that taps hypersensitivity to interference: when asked to memorize lists of verbal non-numeric items, she performed poorly only in the list where items were similar to each other. Thus, her difficulty in multiplication is best explained as resulting from hypersensitivity to interference.

To confirm this conclusion, as well as to help DL overcome her difficulty, we devised a training method that controlled the degree of interference. The method was clearly successful: DL managed to memorize multiplication facts as long as in a given week, she only had to learn facts that were relatively dissimilar from each other. In this condition, her learning was virtually immediate: in the set with lowest similarity, she reached perfect performance after merely two (!) exposures to each fact, and in the sets with medium similarity, she reached ceiling performance in the second day of training, i.e., after spending a total of less than 2 minutes per trained fact. This good memorization of multiplication facts was exhibited during the training sessions, when tested at the end of the training period, and even after a retention period of two months, during which DL received no additional training. Conversely, she had much difficulty in the set with high similarity between facts: she made many errors during training, her post-training score was hardly any better than the pre-training score, and this small improvement virtually disappeared two months later.

These results extend the findings of De Visscher and Noël (2013) in two ways. First, the hypersensitivity to interference of the woman they reported, DB, was manifested mostly in slow retrieval of multiplication facts, whereas DL showed not only slow RTs but actually erred in almost half of the multiplication facts. Second, whereas De Visscher and Noël's evidence for interference as the source of multiplication difficulty was correlational, here we showed evidence for a causal relation: manipulating the amount of similarity-induced interference affected the memorization of multiplication facts.

10.5.2. The cognitive mechanisms underlying sensitivity to interference

Our findings clearly show that DL's difficulty in multiplication was the result of hypersensitivity to interference. Still, to understand the effects of interference fully, we would have to identify the exact mechanism that was sensitive to interference. We consider here two aspects of this question.

10.5.2.1. Which memory process is sensitive to interference?

High levels of interference – induced in this study by high similarity between multiplication facts – may take an effect in different processing stages (Bartko, Cowell, Winters, Bussey, & Saksida, 2010; Farrell, 2006; Fernandes & Moscovitch, 2000; Kaufmann et al., 2004; Lochy, Domahs, & Delazer, 2004; van Dyke & McElree, 2006; Wixted, 2004). Interference may disrupt either the encoding and storage of data in memory (Farrell & Lewandowsky, 2002;

Lewandowsky & Farrell, 2008) or the retrieval stage (Burgess & Hitch, 1999; Henson, 1996, 1998). Clearly, the manipulation that we used (changing the within-set similarity) specifically targeted the storage stage, whereas the retrieval stage was done identically for all sets of facts. One interpretation of these findings is therefore that interference disrupts the storage/encoding processes rather than on the retrieval processes (Parkin, Ward, Bindschaedler, Squires, & Powell, 1999), in line with the view of De Visscher and Noël (2013, 2014a, 2014b). However, the findings can also be explained under the assumption that interference affects retrieval processes. For example, high interference may cause over-activation of incorrect facts during retrieval, yet a storage-time manipulation, which perhaps improves the association between a pair of operands and their product, could help an impaired retrieval mechanism. More research would be required to tease apart between storage and retrieval as the mechanisms sensitive to interference. Note, however, that from a clinical/intervention point of view, the picture is clear: a storage-time intervention can help overcoming similarity-induced interference.

10.5.2.2. Which kind of information is sensitive to interference?

The processes impacted by interference can be characterized not only as storage versus retrieval processes, but also by the kind of information they represent. The process sensitive to interference could be phonological (Baddeley, 1966, 1968; Farrell, 2006; Nelson et al., 1974; Runquist, 1970), semantic (Baddeley, 1966; Oppenheim et al., 2010), a number-specific process, or another processes.

In line with the possibility of phonological sensitivity-to-interference, the speed and accuracy of addition fact retrieval was shown to be affected by phonological similarity (Noël, Désert, Aubrun, & Seron, 2001). Further support to the phonological view comes from studies of non-number words, which show that word memorization is affected by their phonological similarity to each other (Nelson et al., 1974; Pajak, Creel, & Levy, 2016; Runquist, 1970). However, interpreting these findings as an explanation to difficulties in memorizing multiplication facts should be done with caution, because at least some phonological mechanisms treat words and numbers differently (Bencini et al., 2011; L. Cohen et al., 1997; Dotan & Friedmann, 2015). Furthermore, the representation of multiplication facts in memory is apparently not purely phonological (Whalen, McCloskey, Lindemann, & Bouton, 2002).

An interesting comparison is between sensitivity to interference of numbers in multiplication facts, as investigated in the present study, and another type of interference phenomenon, observed in sentence processing. In comprehension and production of sentences, some syntactic structures are harder than others. One such example are object relative sentences (*this is the dog that the cat bites*), which are harder to produce and understand than sentences with subject relative (e.g., *this is the dog that bites the cat*). Object relatives are difficult for individuals with acquired and developmental syntactic deficits as well as for children whose syntactic abilities were not yet fully developed. In these cases, upon hearing the sentence *this is the cat that the dog bites*, they may fail to understand whether the cat is biting the dog or vice versa. Why are object relatives more difficult than subject relatives? The reason for the difference lies in the different syntactic structures of the two sentences. In both sentences, the key is to understand that "the dog bites the cat", but this exact phrase appears in neither sentence: in each of the two sentences, one constituent (either the dog or the cat) is missing from the embedded clause because it already appears in the main clause. This phenomenon is known as syntactic movement – a constituent “moved” from the embedded clause to a new location in the main clause. In both (1) and (2) the phrase "the dog" has moved from its original position within the embedded clause (marked with an underline) to an earlier position, in the main clause. The crucial difference between the two sentences is that whereas "the dog" moves in both, it does not cross another noun phrase in its movement in (1), but it does cross the noun phrase "the cat" in sentence (2):

(1) This is the dog that ____ bites the cat

(2) This is the dog that the cat bites ____

Friedmann, Belletti, and Rizzi (2009) (see also Belletti, Friedmann, Brunato, & Rizzi, 2012) suggested that the moved constituent is harder to relate to its original position in (2) than in (1) because in (2), the original position and the new position are separated by an *intervener:* an interfering element (the dog). They further suggested that sentences like (2) are more difficult only when the moved element and the interfering element are syntactically similar to each other. In (2), both elements are noun phrases. In support of this notion, Friedmann et al. showed that children's comprehension was not impaired when the moved and interfering constituents belonged to different syntactic categories. For example, in (3) one constituent is a lexically-restricted noun phrase and the other (*who*) is not, and such sentences were relatively easier. In (4), both constituents were nouns, and indeed such sentences were harder:

Sentence	Moved constituent	Interfering constituent
(3) Who did the cat bite _____?	who	the cat (noun)
(4) Which dog did the cat bite ____?	the dog (lex. rest. noun)	the cat (lex. rest. noun)

Friedmann et al.'s (2009) experiment was about syntactic processing, and DL’s case was about hypersensitivity to interference in calculation. Yet the two cases bear some resemblance: in both situations, a person needs to process the relation between two target items in the presence of an interfering item. Memorizing multiplication facts occurs in the presence of other interfering facts, and representing syntactic movement – the local syntactic relation between a moved constituent and its original position – occurs in the presence of an interfering constituent. In both situations, the key to succeeding is the existence of sufficient dissimilarity between the target items and the interfering item. This analogy suggests that sensitivity to interference is not a property of systems that process single items, but a property of structural (syntactic) systems that process the relations between items.

10.5.3. Clinical implications

The clinical goal of this study was to examine whether a person can learn the multiplication table even when they have hypersensitivity to interference. This was clearly the case – when we maintained a low level of interference, DL easily learned the multiplication facts. This is not trivial: conceivably, one could hypothesize that learning a sequence of facts like 7*4, 8*4, 9*4 would actually be easier – for example, it may allow more transparently to use an addition-based strategy as scaffold for multiplication. The fact that such a set was actually harder to memorize, in spite of the opportunity to use scaffold strategies, emphasizes even further the importance of the within-set similarity as a factor that determines the difficulty of memorization, at least for individuals with hypersensitivity to interference.

Our findings directly imply on preferred practices for teaching the multiplication table. At least for individuals with hypersensitivity to interference, it seems better to teach simultaneously dissimilar rather than similar facts. This is almost the opposite of how multiplication is typically taught at school: very often, children learn the multiplication table in an ordered manner – first the products of 2, then of 3, etc. Although this ordered teaching method might have its advantages, it implies that the children learn similar facts simultaneously, which increases the degree of interference and may therefore create difficulty.

Our findings are in excellent agreement with another study that examined how manipulating the interference level affected the memorization of multiplication facts (Mark-Zigdon & Katzoff, 2015). Mark-Zigdon and Katzoff taught a group of typically-developing 3rd grade children a set of 10 new multiplication facts. They showed that the children's memorization of these facts was disrupted if interference was induced by teaching a new set of multiplication facts immediately after the first set. Thus, like us, Mark-Zigdon and Katzoff showed that high-interference conditions disrupted memorization of multiplication facts. The difference between the two studies is that each of them highlights a slightly different aspect of interference: our study highlights the importance of low interference within a set of learned facts; Mark-Zigdon and Katzoff’s study highlights the importance of avoiding interference from out-of-set facts. Together, the two studies support what we described in the introduction as the two foundations of an interference-reducing training method: grouping dissimilar facts when teaching, and temporally separating one set of facts from another.

Our training method was effective for DL, an adult woman with hypersensitivity to interference, but its clinical implication may be most relevant for children who learn the multiplication table at school, many of whom may have normal sensitivity to interference. Will the same method be effective for all children, including children without hypersensitivity to interference? The findings of Mark-Zigdon and Katzoff (2015), who did not select participants based on sensitivity to interference, suggest that the answer to this question is affirmative. Future research may further examine the effect of interference-based manipulations on individuals without hypersensitivity to interference.

General Discussion

This dissertation investigated the cognitive mechanisms of multi-digit number processing. It focused on the syntactic processes that handle the number structure – encode the relations among the digits, or integrate several digits into a single cognitive structure. Each of the three sections of this dissertation examined a different cognitive process: the conversion of a digit string to quantity – namely, number comprehension; the conversion of a digit string to spoken number words – namely, oral reading of numbers; and the association of pairs of digits with memorized multiplication results. The main conclusions regarding these three processes are hereby described.

Converting a multi-digit string to quantity

The first section of this dissertation examined how two-digit and multi-digit numbers are converted to quantity. We used the number-to-position paradigm: participants saw a number and marked the corresponding position on a number line. Our touchscreen-based version of this paradigm continuously measures the finger position and direction, and this way provides high-temporal-granularity information about the intended responses in intermediate stages of a trial. The two main findings in the series of tasks ran with this paradigm were these: first, the participants' mapping was linear in the trajectory endpoints but had an additional logarithmic effect in intermediate trajectory parts. This log effect could be reduced to an effect of differential first-deviation-time of the trajectories of small versus large numbers. Second, when educated unimpaired adults performed the task with two-digit numbers, the effects of the decade and unit digits on finger movement built up in parallel. A lag induced in the appearance of the decade digit delayed its effect by the same amount, but a lag in the unit digit delayed the unit effect by 35 ms less than the lag duration, indicating the existence of an idle time window in the units processing pathway.

To account for these and the other findings, we proposed a detailed model of the multidigit-to-quantity transcoding process. The model describes several processing stages: the visual identification of the digit symbols and of the number length; the creation of a quantity syntactic frame – a sequence of decimal placeholders; the quantification of each digit according to its decimal role; and merging the per-digit quantities into a single multidigit quantity. Moreover, on top of the conversion to quantity, the number-to-position task involves two additional stages:

decision on a target location; and manual movement – dragging the finger towards that target location.

Looking deeper into the dynamics of the processing stage that decides on target location, we showed that this decision process is in good agreement with the Bayesian notion of a prior that gradually turns into a posterior. Within a trial, three stages could be observed in the finger direction: first, pointing according to a default behavior or the task instructions; second, by a Bayesian prior, determined by the perceived distribution of target numbers in the experiment; and finally, by the trial-specific target number (Bayesian posterior). The transition to the third (target-based) stage is faster for large small numbers than for larger numbers, because the quantity representation allows faster accumulation of evidence for smaller numbers than for large numbers.

Some of the processing stages in this model specifically concern the number structure or the relations between digits. Such is the case for the identification of a number length and for the creation of a syntactic frame. Other processing stages, such as digit quantification, are apparently executed per digit, in accord with studies that showed decomposed processing of decades and units (Meyerhoff et al., 2012; Moeller, Fischer, et al., 2009; Nuerk & Willmes, 2005). However, even these per-digit processes were remarkably synchronized: when the decade and unit digits were presented simultaneously, their effects on finger movement were nearly simultaneous and in 1:10 ratio, even in three-digit numbers. This suggests some kind of dependency –synchronization or coordination – between the quantification processes of the different digits.

This model integrates several existing concepts with new concepts. The creation of a “quantity syntactic frame” is a new notion, introduced in this research for the first time. Other sub-processes were discussed in previous studies: visual identification of the digits (Cohen & Dehaene, 1991; Friedmann, Dotan, & Rahamim, 2010; Starrfelt et al., 2010), visual identification of the number length (Cohen & Dehaene, 1991), per-digit quantification (Meyerhoff et al., 2012; Moeller, Fischer, et al., 2009; Nuerk & Willmes, 2005), multidigit quantity (Brysbaert, 1995; Dehaene et al., 1990), and the decision on a target location, including the Bayesian modeling of this decision process (Cicchini et al., 2014). The present research provided further evidence for these processes, and integrated them into a single model.

The paradigm we developed – number-to-position mapping with trajectory tracking – may be useful to examine several questions in number processing. The analysis methods that we created may be also for other, non-numeric trajectory tracking experiments. Indeed, this paradigm, and our tools, is already being used by several researchers – to examine number processing in very young children (Feldman & Berger, unpublished data), calculation (Pinheiro-Chagas et al., 2017), non-linguistic syntactic processing (Al-Roumi, Dotan, & Dehaene, 2017), and decision making (Dotan et al., 2017). Furthermore, the paradigm may prove as a useful clinical diagnostic tool: its high temporal sensitivity can capture anomalies even on single subject analysis (e.g., ZN's unexpected serial decade-unit pattern, Fig. 9.2b).

Converting a multi-digit string to number words

The second section of this dissertation examined in detail the mechanisms involved in multidigit-to-verbal conversion, namely, number reading. Chapter 7 reported a detailed neuropsychological examination of seven individuals with different deficits in number reading. Based on their impairment patterns and the dissociations they showed, we proposed a detailed model of number reading. This model distinguishes between the visual analysis of the digit string and the verbal production of the corresponding number words. Visual analysis involves several sub-processes: encoding the digit identities, encoding their order, and additional sub-processes that encode several aspects of the number's decimal structure: its length, its triplet structure, and the positions of 0. In verbal production, the model stipulates that a series of processes creates the number's verbal structure. This verbal structure starts with a tree-like representation, created based on the number's decimal structure. This syntactic tree is a verbal representation, yet it does not depend on a particular language. The tree is then converted into a linear representation of the number's verbal structure by applying language-specific rules, some of which depend on the number's decimal structure (e.g., "the digit 0 does not translate to any number word") and some on additional digits (e.g., "1 in the decades position yields an x-teen word"). The result of applying these rules is the number word frame, a list of number word specifiers (e.g., the frame for 750 is *[_:ones] [hundred] [and] [_:tens]*). This frame is bound with the ordered digits, provided by the visual analyzer's identity and order encoders, and the bound frame (*[7:ones] [hundred] [and] [5:tens]*) is used to retrieve the phonological form of each word from a dedicated phonological store.

Similarly to the case of multidigit-to-quantity conversion, here too there are several sub-processes that specifically handle the multi-digit number's structure: the decimal structure encoder in the visual analyzer, and the verbal processes that generate the number word frame. Indeed, an impairment in any of these sub-processes results in errors that were traditionally classified as "syntactic", e.g., decimal shift errors such as 750→7,500 (Cipolotti et al., 1994; Deloche et al., 1992; Deloche & Willmes, 2000; Lochy, Domahs, Bartha, et al., 2004; Noël & Seron, 1993).

The investigation of number reading, on top of its theoretical contribution to the understanding of the processes involved in number reading, also made an important clinical contribution. Our ability to diagnose specific deficits in number processing depends on good understanding of the processes involved in number reading, as well as on the availability of sensitive assessment tasks and accurate methods for fine-grained analysis of error types (e.g., distinguishing between decimal shifts of a leftmost digits and shifts of other digits). An important result of this study was that we developed a set of cognitive tests to diagnose specific types of acquired and developmental number reading disorders. This battery of tests (Dotan & Friedmann, 2014; Appendix E) is already being used in the field, and is being taught in Tel Aviv University's program for assessment of learning disabilities.

Having identified several specific sub-processes of number reading, we examined the degree of specialization of these processes: we showed that number reading is dissociable from word reading (Chapter 8) and from number comprehension (Chapter 9).

To compare word reading with number reading, Chapter 8 described in detail the processes involved in reading words and numbers, and proposed a possible homology between specific sub-processes of word reading and number reading. We reviewed existing literature in light of the detailed reading models and the proposed homology between them, and identified several word-number dissociations in specific sub-processes: in the visual analysis stage, position encoding is separate for letters and digits (Friedmann, Dotan, & Rahamim, 2010), and so is digit identity encoding (Abboud et al., 2015; Baker et al., 2007; Grotheer et al., 2016; Hannagan et al., 2015; Park et al., 2012; Shum et al., 2013). In verbal production, phonological retrieval is done in different ways for number words and other words (Cohen et al., 1997; Dotan & Friedmann, 2015; Marangolo et al., 2004, 2005). We described in detail two individuals who exhibited previously unreported word-number dissociations: a selective impairment in the visual analysis of numbers, in the process that parses the digit string into triplets; and a selective impairment in the verbal process that generates the number word frame. This set of dissociations leads to the conclusion that word reading and number reading are implemented by completely, or almost completely, separate processes.

Number reading is separate not only from word reading but also from number comprehension, namely, the conversion of multidigit strings to quantity. Multidigit-to-word conversion and multidigit-to-quantity conversion may share common components in the initial parts – the visual parsing of numbers. Their later processing stages, however, are at least partially separate. This separation was directly examined in Chapter 9 via the case of ZN, an individual with aphasia. ZN had a selective impairment in converting multidigit numbers to number words, despite his good visual processing of such numbers, his spared ability to produce them phonologically (e.g., in repetition), and his spared ability to convert single-digit numbers to words. This highly specific pattern can be explained as a selective impairment in a verbal process that handles the number's structure – presumably, the generation of number word frames. Crucially, several tasks showed that ZN could correctly convert multi-digit numbers to quantity. This dissociation indicates that the impaired process handles multidigit-to-verbal conversion but not multidigit-to-quantity conversion, and hence, that multidigit-to-verbal conversion is separate from multidigit-to-quantity conversion.

Multiplication impairments and their remediation

The last section of this dissertation addressed a different kind of digit integration: the association of two digits with their product, when memorizing the multiplication table. Here we reported DL, a 40-year-old woman with a severe lack of knowledge of the multiplication facts. In line with a recent series of studies on multiplication facts knowledge (De Visscher & Noël, 2013, 2014a, 2014b), we showed that DL's enduring failure to learn the multiplication table is accompanied by hypersensitivity to interference – an extreme difficulty in memorizing highly-similar verbal items, such as multiplication facts. To show that hypersensitivity to interference is indeed the reason of her multiplication difficulty, we showed that DL could easily learn the multiplication facts, but only when they were presented to her in low-interference conditions – i.e., when the learning session included only multiplication facts that were sufficiently dissimilar from each other. Thus, this study provided the first causal evidence to hypersensitivity to interference as a source of difficulty in learning the multiplication table.

Importantly, this study too made contribution that is not only theoretical but also clinical. The training program we devised for DL could be used at schools to teach multiplication. Our study suggested that this method would be useful for children who have trouble learning the multiplication table due to hypersensitivity to interference; but another study, which used a similar paradigm (Mark-Zigdon & Katzoff, 2015), strongly suggests that even typically-developing children may benefit from this method.

Conclusion

This dissertation described several specific cognitive mechanisms that specifically handle multi-digit numbers. Many of these mechanisms are similar in the sense that they integrate digits into more complex structures, yet they are still separate from each other, each with its own role.

Studies of symbolic number processing traditionally distinguished between lexical and syntactic processes – the former handling single digit or number words, and the latter handling integration of digits or number words. Under this classification, many of the processes we characterized here can be described as syntactic. This dissertation therefore offers a possible concrete definition of the term “syntax” in the context of number processing: “number syntax” is not a single process, but rather a collection of separate processes that integrate numeric elements. This view fits well with previous studies that showed much specificity in syntactic processing. In particular, many numerical or mathematical syntactic processes are dissociable from language syntax (Brysbaert et al., 1998; Maruyama et al., 2012; Monti et al., 2012; Varley et al., 2005).

It was hypothesized that syntactic representations require certain cognitive abilities that are unique to humans, and that for this reasons human alone can form complex syntactic representations (Dehaene, Meyniel, Wacongne, Wang, & Pallier, 2015; Hauser et al., 2002). Yet even if this is true, and a common cognitive ability underlies all syntactic mechanisms, there are still several distinct syntactic processes. I hope that the research presented here would prove to have contributed to our understanding of such mechanisms, and in turn – to our ability to diagnose and treat individuals who have impairments in these mechanisms.

Appendices

Appendix A. Supplementary material for Chapter 3

A.1. The small-number advantage is not a motor effect

The small-number advantage effect – the faster deviation of the finger towards small numbers than towards large numbers – was taken in Chapter 3 as a numeric effect. An alternative interpretation, however, could attribute this effect to a motor rather than a numeric process. We hereby describe two control experiments (mentioned briefly in Section 3.2.3) that refute this motor interpretation.

A.1.1. Experiment A1: Number-to-position task with left-handed participants

A motor interpretation of the small-number advantage may attribute the effect to the asymmetry resulting from the fact that all participants in the Experiments described in Chapter 3 were right handed. For example, the types of muscle activity required to push the finger left or right, may make leftward movements faster than rightward movements. Such a view predicts a reversed effect (large-number advantage) if the experiment is performed by left-handed participants.

A.1.1.1. Method

Seventeen left-handed adults, aged 26;7 ± 3;9, participated in this experiment. Their mother tongue was Hebrew and they had no reported cognitive disorders. They performed the silent number-to-position task with a 0-40 number line and 4 trials per target.

The horizontal movement onset time was calculated per trial using the method described in Section 3.2.2.4.1. This succeeded for 79% of the trials by the automatic onset-detection algorithm, and for 98% after manual encoding. The factors affecting onset times were analyzed with a 2-way repeated measures ANOVA. The dependent measure was the horizontal movement onset time, the subject was a random factor, and there were 2 within-subject factors: the target side (< 20, left; or > 20, right), and a numeric factor given by the absolute distance between the target number and 20.

A.1.1.2. Results

The failed trial rate was 6.9% ± 6.8%. The endpoint error was 1.71 ± 0.49 numerical units, the endpoint bias was -0.68 ± 0.47 numerical units, and movement time was 1197 ± 166 ms. There was no significant difference between the left-handed and right-handed group in any of these measures ($t(33) < 1.75$, two-tailed $p > 0.09$) except the failed trial rate – the left-handed group had more errors (t(33) = 2.3, two-tailed $p = .03$). Fig. A.1a shows the mean trajectories per target number.

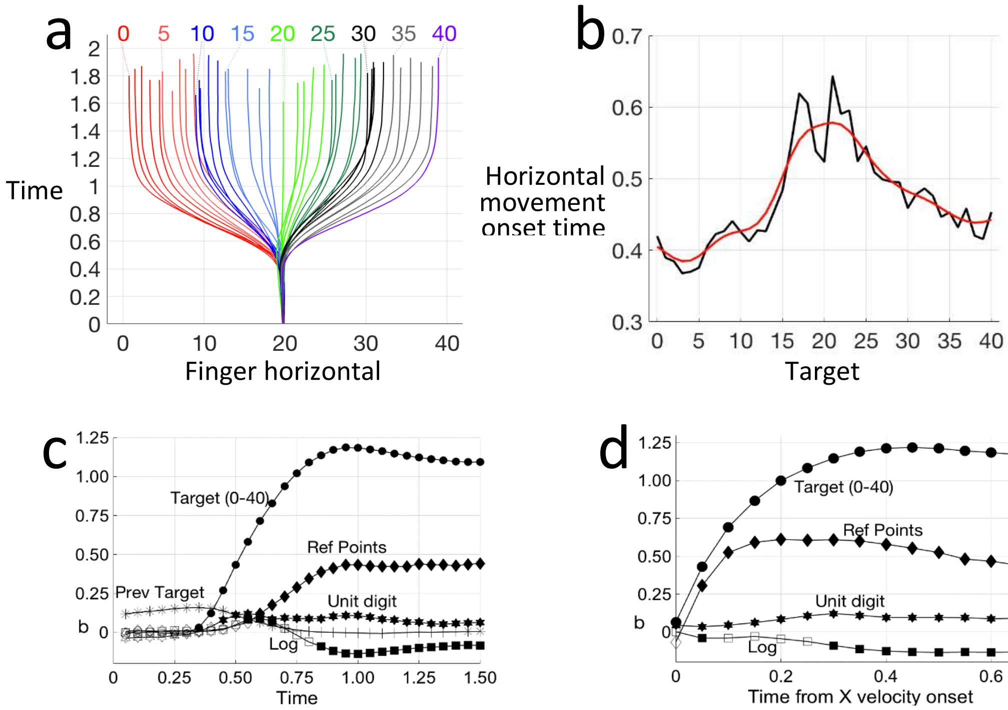

Fig. A.1. Results of Experiment A1. (a) Median trajectories per target. (b) The mean horizontal movement onset time, averaged over all participants, as a function of target number. The red line shows the same data with Gaussian smoothing, σ = 2. (c) Regression b values, with the trajectories aligned by the target onset. (d) Regression b values, with the trajectories aligned by the horizontal movement onset time.

Contrary to the interpretation of the small-number advantage as a motor effect, the left-handed participants showed a small-number advantage just like the right-handed participants in the previous experiments: the horizontal movement onset times (Fig. A.1b) were smaller for target numbers < 20 (mean = 415 ms) than for target numbers > 20 (mean = 478 ms). The Side x Distance repeated measures ANOVA showed that this small-number advantage was significant (a main effect of Side, $F(1,16) = 12.87$, $p = .002$, $\eta_p^2 = .45$, $\eta^2 = .16$). The ANOVA also showed, similarly to Experiment 1 in Chapter 3, that movement onset times were affected by the target distance from the middle of the number line (main effect of Distance, $F(1,16) = 27.97$, $p < .001$, $\eta_p^2 = .64$, $\eta^2 = .10$), and there was no Side x Distance interaction ($F(1,16) = 1.57$, $p = .23$).

We compared the small-number advantage between the left-handed group (Experiment A1) and the right-handed group (silent condition in Experiment 3.1). The data of both experiments was submitted to a mixed-design ANOVA – we repeated the Side x Distance ANOVA, while adding the Experiment as a between-subject factor. The small-number advantage was not significantly different between the two experiments (Side x Condition interaction: $F < 1$).

To further confirm that the left-handed group behaved similarly to the right-handed group, the trajectory data was submitted to the regression analyses presented in Section 3.2.1.3. The dependent variable was the implied endpoint and the predictors were the target number N_{0-40}, log'(N_{0-40}), the unit digit, the spatial-reference-points-based bias function SRP, and the target number of the previous trial. One regression was run per participant and time point in 50 ms intervals. The per-subject regression b values of each time point and predictor were compared versus zero using t-test. The results (Fig. A.1c) were very similar to the pattern observed in the silent condition in Experiment 3.1: dominant linear factor, transient logarithmic factor, SRP contribution in the late trajectory parts, and an effect of the previous trial in early trajectory parts. In a second regression, in which the trials were aligned by their horizontal movement onset time, the log effect was no longer significant (Fig. A.1d), like in Chapter 3.

A.1.2. Experiment A2: Point towards an arrow

If the small-number advantage has a motor origin, it should also appear in a non-numeric mapping-to-position task. Experiment A2 did exactly that: it was almost identical with the number-to-position task, but the stimuli were presented non-numerically, as an arrow pointing to the target location (this is the same experiment as described in Section 2.3.5).

A.1.2.1. Method

Nineteen right-handed adults, aged 32;3 ± 12;9, participated in this experiment. Their mother tongue was Hebrew and they had no reported cognitive disorders.

The method was similar to the number-to-position task, with a single difference: the target stimulus was not a number, but a downward-pointing arrow placed at the target location along the top line. The participants were instructed to move their finger towards the arrow. Each target arrow could appear in one of 41 positions (corresponding with the positions of the numbers 0-40), and each position was presented four times.

The horizontal movement onset time was calculated per trial using the method described in Section 3.2.2.4.1. This succeeded for 81% of the trials by the automatic onset-detection algorithm, and for 95% after manual encoding. The factors affecting onset times were analyzed with a 2-way repeated measures ANOVA: the dependent measure was the horizontal movement onset time, the subject was a random factor, and there were 2 within-subject factors: the target side (left or right), and a numeric factor given by the absolute distance between the target number and the middle location.

A.1.2.2. Results

The failed trial rate was 1.85% ± 1.92%. The endpoint error was 0.40 ± 0.11 numerical units, the endpoint bias was 0.11 ± 0.15 numerical units, and movement time was 730 ± 154 ms. Fig. A.2a shows the mean trajectories per target location.

Contrary to the interpretation of the small-number advantage as a motor effect, the participants did not show a left-side advantage. In fact, they showed the opposite pattern – a right-side advantage: the horizontal movement onset times (Fig. A.2b) were larger for left-side target locations (mean = 227 ms) than for right-side locations (mean = 215 ms). The Side x Distance repeated measures ANOVA showed that this large-number advantage was significant (a main effect of Side, $F(1,16) = 4.77$, $p = .04$, $\eta_p^2 = .21$, $\eta^2 = .06$). Movement onset times were also affected by the distance of the target location from the middle (main effect of Distance, $F(1,16) = 35.74$, $p < .001$, $\eta_p^2 = .67$, $\eta^2 = .10$), and there was no Side x Distance interaction ($F < 1$).

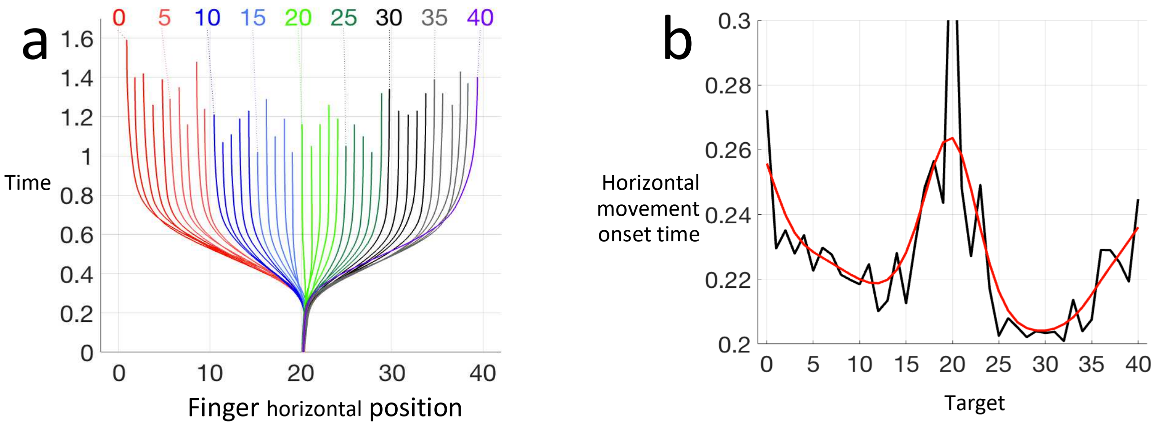

Fig. A.2. Results of Experiment A2 – pointing to arrow. (a) Median trajectories per target. (b) The mean horizontal movement onset time, averaged over all participants, as a function of target number. The red line shows the same data with Gaussian smoothing, σ = 2.

A.1.3. Conclusion from Experiments A1 and A2

Both experiments clearly refute the motor hypothesis as an interpretation of the small-number advantage: the effect, which was observed for right-handed participants, was observed in the number-to-position task also for left-handed participants, whose motor movements are reversed. In contrast, the effect did not exist when right-handed participants pointed to arrows, a task that involved the same motor responses as the number-to-position task.

Taken together, the experiment clearly refute the notion of a motor-driven advantage of the left side (or of the non-dominant side). In fact, Experiment A2 even suggests the opposite: when the number-processing part was eliminated, we observed earlier movement towards locations on the right side than to locations on the left side.

A.2. The horizontal movement onset time detection algorithm: methodological notes

A.2.1. Separating intentional movements from jitter

The onset detection algorithm used the velocities during the time window 0-250 ms as a baseline for random movements. It is hard to know whether a movement is intentional or not, however, we can show that the horizontal velocities in this early time window are categorically different from the velocities in later time windows.

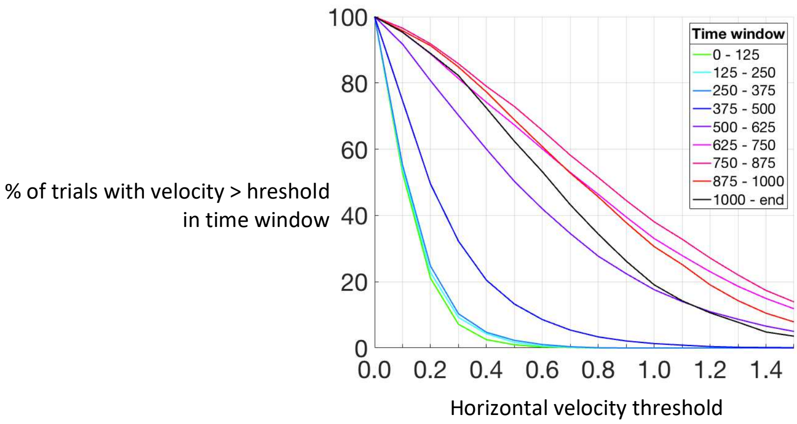

Fig. A.3. Velocities in different time windows in Experiment 3.1 silent condition. Until 375 ms the velocities are low and remain quite unchanged. Then the velocities start increasing quickly, and after 500 ms the velocity distribution remains relatively stable.

Fig. A.3 shows the distribution of horizontal velocities in different 125-ms time windows (the data is from the silent condition in Experiment 3.1). It clearly shows that the velocity distribution hardly changed in the first three time windows, up to 375 ms post stimulus onset. Only in the next time window (375 – 500 ms) velocities start building up. By 500 ms, the velocities are already quite close to their peak value.Unsurprisingly, this figure is in almost perfect match with the regression analysis (Fig. 3.2a). The finger doesn't start deviating sideways before 350-400 ms, and this can be seen not only when inspecting the finger direction (implied endpoint) over all trials in a regression analysis, but even when inspecting the detailed, per-trial velocity information, as done here.

This figure strongly suggests that intentional movements are almost non-existent, or at least extremely small, in the first 375 ms of each trial. Thus our assumption, that the first 250 ms of each trial are random movements or jitter, is even conservative.

A.2.2. Another measure for horizontal movement onset

To validate our conclusions about horizontal movement onsets, we examined an additional measure of horizontal movement onset time. This measure, *acceleration initiation time*, is the time when we first observe strong leftward or rightward acceleration. An acceleration peak was defined as a time window of at least 150 ms during which the acceleration was constantly above a certain threshold (0.175 numerical units / sec^2), and the acceleration initiation time was defined as the beginning of the first acceleration peak. To obtain accelerations, the horizontal velocities (calculated as described in the Chapter 3) were smoothed with Gaussian (σ = 20 ms) and derived.

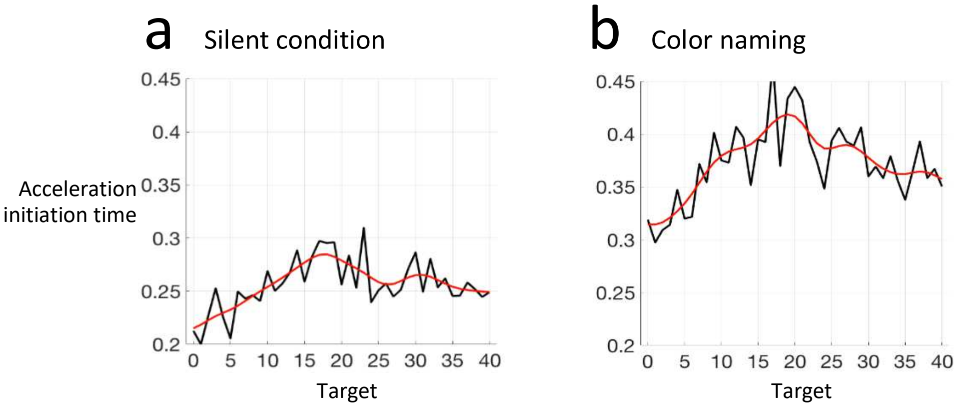

Fig. A.4. Horizontal movement onset times in Experiment 3.1 as measured by the *acceleration initiation time* – the beginning of the earliest horizontal acceleration peak. This measure too shows a small-number advantage, i.e., earlier horizontal movement onset for smaller numbers than for large numbers. The red line shows the same data with Gaussian smoothing, σ = 2.

The pattern of acceleration initiation times was very similar to the pattern of the horizontal movement onset times obtained with our onset-detection algorithm – both in visual inspection (Fig. A.4) and when applying the Condition x Side x Distance repeated measures ANOVA as described in the Chapter 3. Specifically, acceleration initiation times were later in color naming than in the silent condition (main effect of Condition, $F(1,17) = 73.47$, $p < .001$, $\eta_p^2 = .81$, $\eta^2 = .35$). There was a small-number advantage (main effect of Side, $F(1,17) = 7.78$, $p = .12$,

η_p^2 = .31, η^2 = .01). Unlike the onset-detection algorithm described in the Chapter 3, here we did not observe a difference in small-number advantage between the two conditions (no Condition x Side interaction, $F < 1$). Finally, acceleration initiation times were later for target numbers near the middle of the number line (main effect of Distance, $F(1,17) = 60.8$, $p < .001$, $\eta_p^2 = .78$, $\eta^2 = .04$), and this effect was marginally stronger in color naming than in the silent condition (Condition x Distance interaction, $F(1,17) = 3.50$, $p = .08$, $\eta_p^2 = .17$, $\eta^2 < .01$).

Thus, the analysis of acceleration initiation times replicated the main findings obtained with the onset detection algorithm in the Chapter 3. Acceleration initiation times may have the advantage of being a more intuitive measure, and that the way to calculate them is simpler. However, we believe that the onset-detection algorithm is a truer measure of the participants' intention to move. For example, on some trials the acceleration initiation time may reflect an initial bias, prior to processing the target number.

Appendix B. Supplementary material for Chapter 5

B.1. Did the participants comply with the instructions for default direction?

Visual inspection of the average trajectories indicates that the participants followed the initial direction (which was implicit in Experiment 5.1 and specifically instructed in Experiment 5.2). For a more specific analysis of the finger's initial directions, we calculated the initial direction per trial as the mean implied endpoint until t = 50 ms (Fig. B.1). In Experiment 5.1, the initial directions were -4.2º (SD = 4.3º, SD refers to the standard deviation of the per-subject means), -1.3º (SD = 3.9º), and 0.3º (SD = 4.5º) for the small-biased, unbiased, and large-biased conditions, respectively. Namely, the directions were similar across the conditions (repeated measures ANOVA with the Condition as a numeric within-subject factor: $F(1,16) = 1.01$, $p = .33$). In Experiment 5.2, the initial directions were -42º (SD = 10º), -1.3º (SD = 6º), and 43º (SD = 8º) for the left, middle, and right conditions, respectively – i.e., similar values in opposite directions in the left and right conditions (paired $t(23) = .24$, 2-tailed $p = .81$).

These results clearly show that the participants followed the instructions regarding the initial finger aiming.

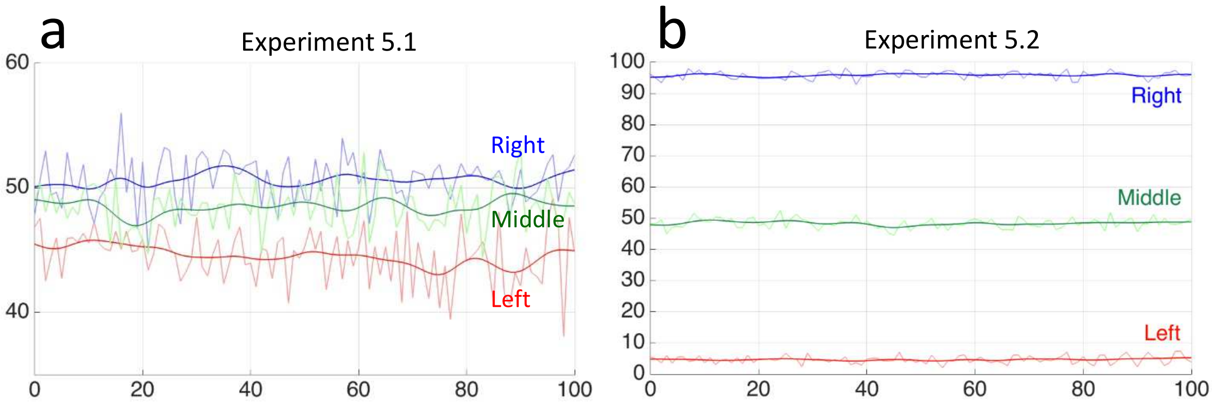

Fig. B.1. The finger's initial direction (average implied endpoint in the first 50 ms of a trial), averaged per target and condition. The thick lines show the same data after smoothing (Gaussian, σ = 3).

B.2. The duration of prior-based pointing

In Chapter 7, we reported the significance of each regression predictor by comparing the b values to 0. Examining the magnitude of these b values may provide additional information, because we used a meaningful scale for all predictors: in all cases, b=1.0 indicates "proper" weighting of the predictor.

Here we specifically examine Experiment 5.2 results. We observed that for most predictors, the b values suggest "proper" weighting of the relevant effect: b[Condition], which reflects the finger's initial direction, was almost 1.0 when the finger started moving, suggesting that subjects moved towards 0, 50, or 100, as instructed. Similarly, b[target number] was almost exactly 1.0 at 650 ms post stimulus onset and thereafter, suggesting that the target number was encoded on a linear scale and that the target location was adjusted to the length of the number line. The situation was different, however, with respect to b[constant], which reflects pointing by the Bayesian prior: its value was not 1.0, as it should have been were the participants consistently pointing towards 50. The b[constant] value was much lower, and reached a peak of about 0.25.

This under-effect of the constant may have two interpretations. One possibility is that the prior-based aiming to 50 was partial – either the prior had only a partial effect on each trial, or it affected only some trials (whereas in other trials the prior-based pointing was skipped and the participant switched directly from the default direction to the target location). The peak of b[constant] – about 25% of its expected value – suggests that the prior effect is reduced to about 25%, or alternatively that it affects about 25% of the trials.

A second interpretation is that the prior affected all trials, but its effect was relatively short and not synchronized between trials. As a result, in each time point only some trials were affected by the prior. Our regressions consider one time point at a time, and in this time point they average over all trials, only some of which are affected by the prior, and this makes the b value lower. Namely, what appears in the regression as a weak and long effect is actually the sum of many stronger and shorter effects. We can even estimate the durations of the "real" (per-trial) effect: the b[constant]'s peak value is 0.25, i.e., about ¼ of the "real" effect size, so the σ of b[constant]'s regression curve should correspondingly about 4 times the σ of the "real" curves. In the regression (Fig. 2c), the b[constant] effect is about 350 ms long, so the duration of the real point-according-to-prior stage is about ¼ than that – about 90 ms.

B.3. Validating the comparability of conditions

Our main analysis method in Section A was a regression analysis that pooled together identical post-stimulus-onset time points from different trials. Pooling together trials in this way has some underlying assumptions, and we hereby examined two main assumptions.

B.3.1. Differential progress of the cognitive process?

When comparing the same time point in different trials, we assume that at this time point the cognitive process reaches a similar stage in different trials – i.e. that on average, the cognitive process develops in the same speed in different trials. This assumption is not trivial: in fact, in Chapter 3 we found that sometimes the cognitive process develops in *different* speeds in different trials – the number-to-position task is performed faster for smaller numbers than for larger numbers ("small number advantage"). Such differences can bias the regressions: e.g., in the case of small-number advantage, the result of the temporal bias was that even a linearly-organized mapping to positions may appear in the regressions as logarithmic.

Table B.1. Trial-level measures per condition (grand mean ± standard deviation of the per-subject means). There were no significant differences between the conditions in either experiment.

	Left	Middle	Right	ANOVA
Experiment 5.1				
Movement time (ms)	1091 ± 146	1089 ± 137	1098 ± 151	$F(2,34) = 0.10$, $p = .90$
Endpoint bias (0-100)	-1.15 ± 1.13	-1.02 ± 1.07	-0.99 ± 1.13	$F(2,34) = 0.25$, $p = .78$
Endpoint error (0-100)	3.77 ± 0.80	3.90 ± 0.85	3.82 ± 0.83	$F(2,34) = 0.58$, $p = .56$
% Failed trials	11.94 ± 2.09	12.94 ± 3.23	12.02 ± 1.97	$F(2,34) = 1.04$, $p = .36$
Experiment 5.2				
Movement time (ms)	917 ± 142	899 ± 130	914 ± 115	$F(2,46) = 0.65$, $p = .53$
Endpoint bias (0-100)	-0.83 ± 1.35	-0.91 ± 1.21	-0.70 ± 1.51	$F(2,46) = 0.45$, $p = .64$
Endpoint error (0-100)	4.29 ± 1.18	4.32 ± 1.18	4.26 ± 1.21	$F(2,46) = 0.12$, $p = .89$
% Failed trials	6.07 ± 6.05	5.36 ± 5.51	3.72 ± 2.84	$F(2,46) = 1.87$, $p = .17$

The "ANOVA" column shows the results of repeated measures ANOVA with the condition as a single within-subject factor.

In the present study, such problems could arise in case of differences between the conditions, e.g., if one condition has faster movement than another condition, or one condition is more difficult than another. To refute these possibilities, we compared the conditions with each other using trial-level measures that reflect, at least to some extent, processing speed and difficulty: the *endpoint bias* is a trial's endpoint (the position where the finger crossed the number line) and the target number. *Endpoint error* is the absolute value of endpoint bias. *Movement time* is the duration from stimulus onset to the moment when the finger reached the number line. Finally, the rate of failed trials also reflects experimental block's difficulty to some extent (Chapter 3). We did not compare here within-trial measures, because for most of these measures we in fact predicted some systematic condition effects, as described in the Chapter 7.

Table B.1 shows that for each of the two experiments, no significant differences between the conditions was fond in each of the above measures (repeated measures ANOVA with the condition as a between-subject factor).

B.3.2. Differential finger speed?

Another kind of artifact may arise from the geometrical aspects of the task. The starting point was always in the bottom-middle of the screen, and the finger moves towards the number line. Importantly, the finger's distance from the number line may affect the participant's movement strategy: for example, changes of direction may be sharper when the finger is closer to the number line, not for any cognitive reason but simply because there isn't enough room to take a relaxed curve. The finger's y coordinate may therefore affect the implied endpoint for non-cognitive reasons.

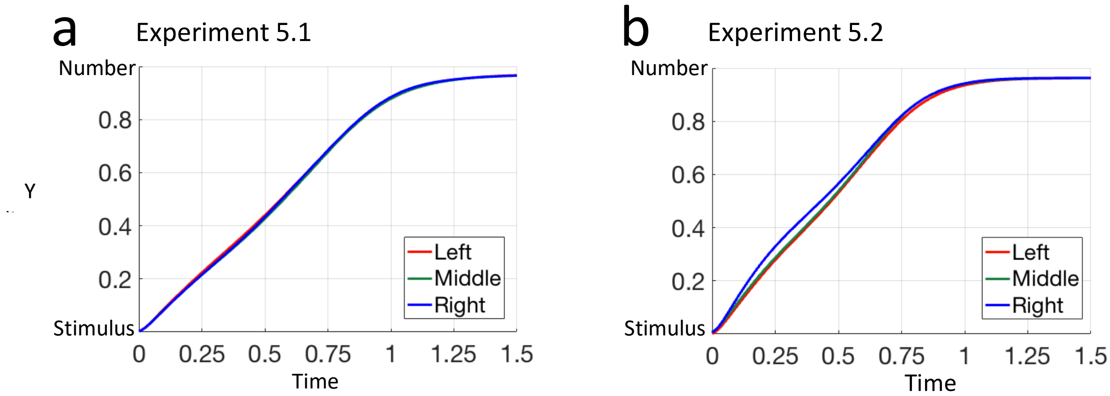

Fig. B.2. In both experiments, the Y coordinates per time point are similar between conditions. This refutes possible artifacts of geometrical movement strategies resulting from the finger vertical position.

Our analyses compared the conditions using regressions with implied endpoints as the dependent variable. If the implied endpoints were biased – for example, because we regressed together trials from y=30% in condition A and trials from y=70% in condition B, this may bias the regressions. For the regressions to be valid, we should assume that for each time point, the different conditions should have similar y coordinates. In Experiment 5.1, this was indeed the case: the y coordinates did not significantly differ between the conditions in any time point (Fig. B.2a; repeated measures ANOVA with the condition as a between-subject factor and the subject as the random factor: $F(2,17) < 0.80$, $p \geq .46$ in all time points). In Experiment 5.2, however, some differences were observed in early time points (until 470 ms, $F(2,23) \geq 3.2$, $p \leq .05$; but in later time points, $F(2,23) \leq 3.0$, $p \geq .06$). Specifically, the finger moved faster in the "right" condition (Fig. B.2b). To refute the possibility that this difference can serve as an alternative interpretation to Experiment 5.2 results, we re-ran the Experiment 5.2 regressions as described in the Chapter 7, with the only difference that now we ran one regression per y coordinate (rather than per time point) – a method that eliminates the potential artifact. Contrary to the prediction of the alternative interpretation, the results were essentially the same as before. Specifically, there was a significant transient effect of Const (from y=20% to y=65%, peak b[const] = 0.18 at y=40%), as well as a concurrent effect of N-1.

B.4. Additional factors affecting finger movement

In previous studies with the trajectory-tracked number-to-position paradigm, we included in the regressions a logarithmic predictor to account for potential logarithmic quantity representation. In Chapter 3 we showed that this log predictor captures a temporal bias rather than a logarithmic representation. It was therefore not used here. Nevertheless, in both experiments, adding the log predictor essentially did not change the effects of the other predictors, and the log effect itself was very small and not significantly higher than 0 in any time point.

Our previous studies also included a bias-function predictor (SRP, defined in Section 2.3.2.6). The SRP predictor was not included in the regressions in the present chapter for the sake of simplicity, as this predictor was not relevant to the theoretical question we examined. In both experiments, adding this predictor did not change the results: the other predictors were essentially unchanged, and the SRP predictor itself behaved similarly to our previous experiments – its effect was considerable (b ~= 0.25 in the endpoints) and started rising together with the target number regression effect (in Experiment 5.2) or slightly after it (in Experiment 5.1).

Appendix C. Supplementary material for Chapter 10

How similarity between multiplication facts should be measured

When memorizing a set of multiplication facts, the different facts in the set interfere with each other, and the degree of interference depends on the degree of similarity between the set items. How should we measure this similarity? The basic idea we used here was that for a given set, a similarity was computed as the sum of pairwise similarities of all pairs in the set. Thus, given a set of 4 facts, we computed the similarity index between each of the 6 possible pairs of facts, and summed these values to get the set's similarity level.

In turn, pairwise similarity can be defined in several ways. Perhaps the most intuitive index of similarity could be the number of digits shared between the two facts. However, following De Visscher and Noël (2014b), we used a different index – the number of shared digit pairs. These two indices are tightly correlated, but the former index increases linearly with the number of shared features (digits) between the facts, whereas De Visscher and Noël's similarity index increases in an over-linear manner, in accord with findings from similarity analyses in other domains (Tversky & Gati, 1982).

A comparative evaluation of possible similarity indices would require more than one participant and more than one training intervention, so such evaluation was not possible in the present study. Nevertheless, we compared several possible similarity indices and examined how each of them would account for our data. Our main finding was that DL's performance in the high-similarity set (set#2) was poorer than in the three other sets. A good similarity index should reflect this distinction, i.e., it should yield higher similarity for set#2 than for the other sets.

We examined the following indices of pairwise similarity:

1. The number of identical digits that appear anywhere in the two facts. For example, the facts 3*4=12 and 3*5=15 have 2 common digits, so their similarity is 2.
2. The measure that was used in the present study: similarity is the number of identical digit *pairs* between the two facts (irrespectively of the digit positions). For example, the facts 8*7=56 and 8*3=24 have similarity=0 because they have no common pair of digits (they only share the digit 8). The facts 3*4=12 and 3*7=21 have similarity=3 due to three common digit pairs (1-2, 2-3, and 1-3).

The two indices above are based on digits. Because multiplication facts are assumed to be stored verbally (Dehaene, 1992; Dehaene & Cohen, 1995), we also evaluated two corresponding measures that are based on number words:

3. The number of number words that appear in both facts. For example, the similarity of 3*4=12 and 3*7=21 is 1, because only the word “three” appears in both facts (the digit “1” also appears in both, but it is expressed as two different words – “twelve”, “one”).
4. The number of number word pairs that appear in both facts.

Finally, we also evaluated the following two indices:

5. The number of number words that appear in both facts in the same role (operand, result). For example, 3*4=12 and 3*7=21 have similarity=1: the word “three” appears in both facts as an operand. However, 3*4=12 and 7*9=63 have similarity=0, because the word “three” is an operand in the first fact and a part of the result in the second fact.
6. A final similarity index considers the operands and ignores the result. The idea is that interference takes an effect before the result was retrieved, and at this time only the operands have an effect. This index was defined as similarity=1 if the two facts have a common operand, and similarity=0 otherwise.

The above six indices were evaluated in two ways: by their compatibility with the training effect (where lower similarity should predict higher effect of the training), and by their compatibility with DL’s pre-training knowledge.

First, we examined which similarity indices best predicts DL’s training results. Specifically, we examined which of the similarity indices assigns high value to set#2, in which DL had the poorest performance, and low values to the three other sets. Table C.1 shows the similarity value of each training set according to each of the six similarity indices. Clearly, De Visscher and Noël’s (2014b) index (no. 2) was the best: it showed the largest gap between the within-set similarity in week #2, in which DL showed poor performance, and the within-set similarities of the other weeks, in which the training succeeded. Notably, it is somewhat surprising that although multiplication facts are stored verbally, the most appropriate similarity index is one that is based on digits rather than on number words.

Table C.1. Within-set similarity per training week, computed using different similarity indices.

Training week no.	1	2	3	4	
DL's performance	Good	Bad	Good	Good	
Facts trained during this week	4*4=16 8*3=24 8*7=56 5*3=15	7*4=28 7*6=42 8*4=32 9*4=36	8*8=64 9*7=63 6*2=12 8*6=48	9*6=54 6*5=30 8*5=40 7*5=35	Pre-training[a] r (1-tail *p*)
Index 1: # of identical digits	12	26	16	20	.25 (.005)
Index 2 (used in the study): No. of identical digit pairs	0	18	6	8	.19 (.02)
Index 3: # of identical number words	7	16	8	14	.25 (.004)
Index 4: # of identical word pairs	1	4	2	7	-.09
Index 5: # of identical words-in-role	6	12	7	12	-.17
Index 6: Common operand exists	4	8	3	8	

[a] Point biserial correlation between DL's pretest successes and failures in a multiplication fact and that fact's similarity with the rest of the table. Positive r values indicate the predicted direction.

Another method to evaluate the similarity indices was based on DL's pre-training knowledge. We reasoned that if between-fact similarity affected DL's knowledge of multiplication facts, she might show lower pre-training knowledge of multiplication facts that have higher similarity with the rest of the multiplication table. We therefore computed, for each multiplication fact, its similarity with the rest of the multiplication table. This was defined as the average of the fact's similarities with all other multiplication facts. To evaluate a similarity index, we computed the point biserial correlation between the fact-table similarity value, computed as described above, and DL's successes and failures on that fact in the pre-training test. This correlation was computed for each similarity index from #1 to #5 (and not for index#6, because its fact-table similarity value was identical for all facts). The correlations for the five similarity indices are shown in the rightmost column in Table C.1. They show that indices #1-#3 were better than #4-#5 in predicting DL's pre-training behavior.

Appendix D. Finger-tracking versus mouse-tracking

Trajectory tracking paradigms are less than a decade old, yet they are already being used in several labs. Some labs track the finger movement in a 3-D space (Finkbeiner et al., 2014, 2008; Friedman et al., 2013; Song & Nakayama, 2008a). Others used mouse tracking (Faulkenberry et al., 2016; Lepora & Pezzulo, 2015; Marghetis et al., 2014; Santens et al., 2011) with software such as Jon Freeman's MouseTracker (Freeman & Ambady, 2010). Our lab, and our collaborators, use finger tracking with a tablet computer.

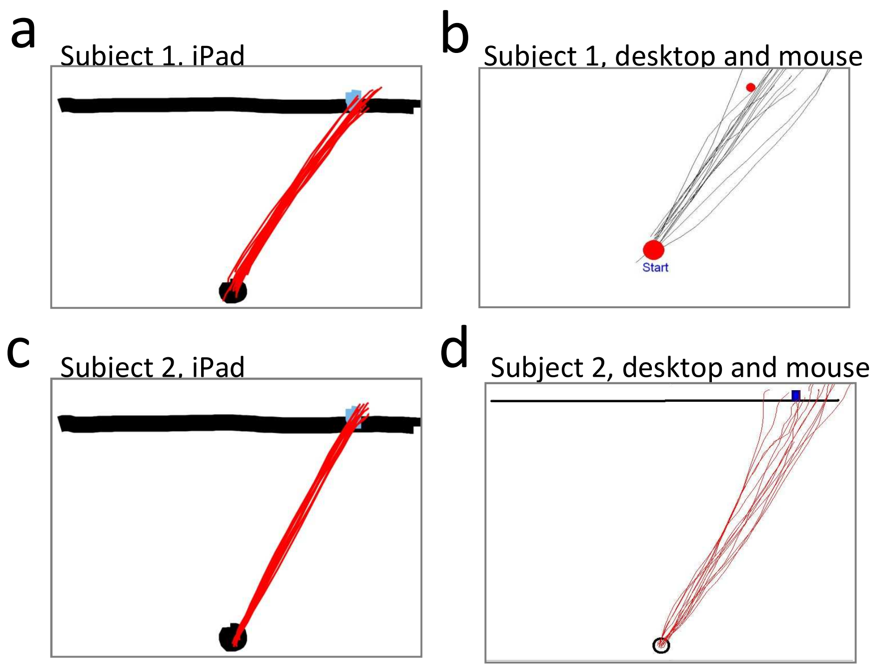

Fig. D.1. Pilot experiment results – raw trajectories. Two participants were asked to drag their finger from an origin position to a target position on an iPad tablet computer (a,c), or on a desktop computer using a mouse (b,d).

We hold that a tablet computer is superior to mouse tracking, because tablets have a more natural motor interface, which allows for better accuracy. To examine this issue, we ran a pilot experiment in which two subjects were asked to drag either their finger (on an iPad) or a mouse cursor (on a desktop computer) several times from an origin position in the bottom of the screen to a target position on the top of the screen. The origin and target positions were marked as dots.

The pilot was run using a simple image drawing software, so the subjects saw the trajectories being drawn. Subject 1 was 40 years old and subject 2 was 27 years old.

Fig. D.1 shows the raw trajectories in this pilot experiment. Although the results were not analyzed quantitatively, and the iPad and desktop conditions were admittedly not well-controlled methodologically, the difference between the conditions seems quite clear: the trajectories on the tablet were better aimed than the mouse trajectories, and the variance in trajectories was lower on tablet.

Appendix E. A battery for assessment of impairments in number processing

As part of the neuropsychological research done during this PhD, Naama Friedmann and I created a battery of neuropsychological tests to assess impairments in number processing (MAYIM, Dotan & Friedmann, 2014). We are using this battery for research purposes. The battery was also included in the curriculum of Tel Aviv University's program for assessment of learning disabilities, and is provided to the students as a clinical tool.

The MAYIM battery includes several sub tests, organized in few topics:

Transcoding:
This section covers all transcoding pathways between the various representations of symbolic numbers: digit strings, oral, and verbal written. In particular, the battery includes tests of number reading, number dictation, number repetition, copying, etc. There are screening tasks, each with 18 numbers, and longer lists of numbers for a more in-depth assessment. The lists of numbers were designed while considering the number length, the presence of 0 and 1, and the existence of double digits in the number (e.g. 252).

The battery includes a pen-and-paper version of all tasks, and a computerized version of some tasks to allow presentation of stimuli with limited exposure duration. The battery further includes the tools needed for error analysis – specific and clear guidelines, and Excel templates.

Tasks to assess the visual analyzer:
To assess the visual analyzer, the battery includes several tasks with digit input and without verbal production: same-different decision, sequence identification, and number matching (these tasks are explained in detail in Chapter 7).

Other tasks:
The MAYIM battery is at present focused mainly on symbolic number processing, however, it also includes tasks for basic assessment of other number processing mechanisms:

- Calculation facts (addition up to 20, multiplication up to 10*10)
- Calculation procedures
- Knowledge of calculation principles (e.g., commutativity)
- Number-to-position mapping and position-to-number mapping

Appendix F. The Number Catcher game°

During my PhD, Stanislas Dehaene and I created a computer game for teaching the basic concepts of number and arithmetic to children. This game – "The Number Catcher" – is freely available online (http://www.thenumbercatcher.com) in three languages – English, French, and Hebrew, and as an app for iOS/Android tablets.

The game was designed primarily for children in the early school grades, or in the last preschool year (age 5-10). It is intended for typically-developing children as well as for children with difficulties in mathematics. Unlike other games, The Number Catcher is primarily focused on training basic cognitive mechanisms of number processing and arithmetic – the three representations of number (digits, words, and quantity) and the basic principles of addition. The game focuses on two-digit numbers, in order to train not only single-digit representations but also multi-digit related processes.

From a cognitive point of view, the game requires the player to collect sets of items (up to 10 items per set) until the total number of items achieves a certain target number (between 2 and 40). The target number and the number of items in each set are indicated in digit format (e.g., "4"), visually as a line of objects (e.g., ♠♠♠♠), verbally by narration ("four"), or as the result a simple addition or subtraction exercise (e.g., 7-3). This setting rehearses fast transcoding among the three number representations (verbal, digital, and quantity), as well as basic addition and subtraction facts and basic arithmetic procedures.

F.1. Game description

The sets of items are visually presented as boxes with objects (fruits, flowers, or fish). The player's goal is to collect these boxes into a vehicle – truck, carriage, or ship. The target number of items to collect is presented verbally by the game narration (e.g., "please load this truck with 15 fruits"), but also visually as the number of slots in the vehicle, and as a number in Arabic format. The size of each box is presented in Arabic format, as the result of a simple

° Game acknowledgements: *Created by* Dror Dotan, Stanislas Dehaene, Manuela Piazza, and Caroline Huron. *Consulting on game design:* Ghislaine Dehaene-Lambertz, Anna Wilson, Shahar Nash, Limor Tabeka, Daled Dotan. *Project management and programming:* Dror Dotan. *User experience:* Eila Shamir. *Graphic design:* Prototype Studio. *Sound:* Daled Dotan. *Narration:* Karine Hyman. *Translation:* Océane le Tarnec, Stanislas Dehaene, Sherry Nabil, and Rick Teplitz. *Funding and financial support:* INSERM, CEA, Collège de France, McDonnell Foundation, Fondation Bettencourt-Schueller, Azrieli Foundation, and The Lieselotte Adler Laboratory for Research in Child Development. We thank Leonid Geldin, Moria Lahis, Naama Friedmann, and Rachel Isyomin for their help.

addition/subtraction exercise, visually – as a concrete number of objects or as the box size (magnitude), or as combinations of some of the above (Fig. F.1a-b). Boxes can also be sawed in order to achieve a specific required size (Fig. F.1c).

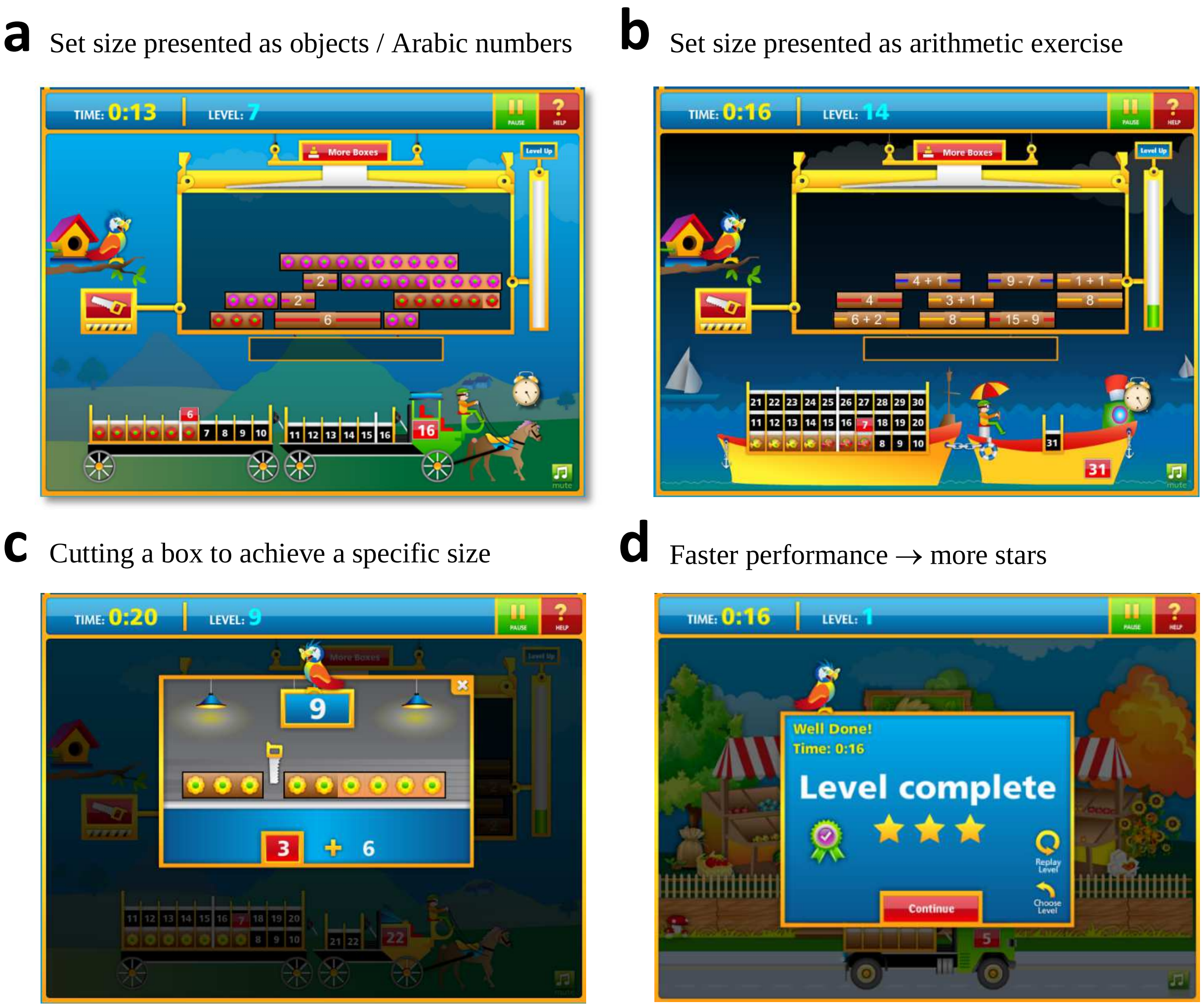

Fig. F.1. The Number Catcher screen layout.

The reaction times in the game are adaptive to the player's level, however, faster reaction times result in a higher score (and more collected stars, Fig. F.1d). Also, the adaptivity is not absolute – the game imposes some minimal reaction time, which becomes faster in higher levels.

The game also includes various functional and graphical elements whose goal is not cognitive but to improve playability and fun.

F.2. Cognitive goals

1. Training the transcoding processes

Numbers can be represented by different cognitive mechanisms – as digits, as number words, and as approximate magnitudes; different operations rely on different representations (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene et al., 2003). The ability to handle the different representations of numbers, and to transform one to another, is a cornerstone of numeric literacy. Being able to transform numbers into the quantity representation is especially important, because we usually see or hear numbers as digits or words, but it is the quantity representation that makes us understand the "meaning" of a number and have a sense of how large it is (Dehaene, 1997). Practicing these transformations may help us process numbers faster and faster, with fewer errors, and with less effort (Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009; Wilson, Dehaene, Dubois, & Fayol, 2009; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006).

Whereas many mathematical games focus just on calculation skills, The Number Catcher is one of only a few games that were specifically designed to teach and practice the more fundamental level – the various representations of numbers and the transformations between them, with a special focus on the quantity representation. This is achieved by presenting number in various formats throughout the game, and encouraging the player to respond with increasing speed.

2. Step-by-step teaching of addition and subtraction

Addition exercises can be solved using various strategies: counting, complement to 10, and memory retrieval. The Number Catcher takes its player, step by step, from basic calculation up to adult strategies:

In the first levels, the game teaches the counting strategy. The objects from the selected boxes are loaded on the vehicle one by one, and as each object is loaded, the game says aloud the total number of objects on the vehicle. This is aimed to strengthen the counting routine and to give it a meaning – the recitation of count words increments the quantity by one at a time.

Subsequent game levels practice the complement to 10 strategy. The game requires filling the vehicles by multiples of 10. For example, if a carriage has 8 flowers on it and you want to get to 13 flowers, first you have to put 2 flowers to fill an exact decade, and only then can you put the remaining 3 flowers on the carriage. This makes the player practice the complement to 10 strategy.

The adult strategy, memory retrieval, is trained in several ways. Addition exercises are presented to the player with their results, both visually and verbally (by the narrator), to encourage rote learning. On top of that, as you progress in the game, the boxes become with fixed size, so the box physical size no longer serves as a magnitude cue to quantity. In the most advanced levels, each box is labeled with addition and subtraction exercises (Fig. F.1b), so the player has to solve several addition and subtraction exercises very quickly, which is possible only by memorization and fast retrieval.

3. Emphasize fluency

The Number Catcher puts a lot of emphasis on automation. We want the player to perform the numeric and arithmetic operations in an automatic manner – not with slow and attention-requiring strategies. The game promotes this in several ways:

- *Adaptive level of difficulty:* The player can move on to the next level only after reaching a certain level of accuracy.
- *Adaptive speed:* when you play faster, boxes fall into the screen faster, which makes the game more challenging. The higher speed allows the player to achieve higher scores, but also leaves him/her less and less time to calculate, and therefore encourages fluency.

  The adaptive speed also serves another purpose: a fast-pace game maximizes the number of exercises encountered per minute and minimizes "cognitive idle time", thereby increasing the learning effect.
- *Multiple solutions:* The game is designed to have more than one correct solution at most times, and yet some solutions are better than others. To achieve high scores, the player must learn to choose the best solution, and this requires quick evaluation and comparison of several solutions.
- *Dual-task technique:* in advanced game levels, more and more factors need to be considered: the spatial organization of the boxes in the container must be considered to prevent them from piling up; the color should be considered if the player wishes to get the color bonus; and there are special game elements such as clocks and bombs. These elements require some of the player's attention, and encourage the advanced player to practice number processing and calculation with less and less attention, i.e., increasingly automatically.

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עיבוד מספרים רב-ספרתיים: מנגנונים קוגניטיביים וליקויים בהם

תקציר

מערכת המספרים היא אחת המערכות הסימבוליות הנפוצות ביותר בתרבות שלנו – כמעט כולנו קוראים, ומבינים מספרים. בתור מערכת פורמלית, היא פשוטה למדי: 10 ספרות, עשרות מעטות של מילות-מספר, ואוסף מצומצם של חוקים – כל אלה מספיקים כדי להגדיר איך לתרגם רצף ספרות לרצף של מילות מספר ובחזרה, ומה הכמות שמיוצגת ע"י רצפים אלה. לעומת זאת, המערכת הקוגניטיבית של עיבוד מספרים אינה כה פשוטה. היא כוללת ייצוגים שונים של מספרים – ספרות, מילות מספר, וכמות – ותהליכים ייעודיים לתרגום מייצוג לייצוג. רמת המורכבות של המערכת הזו באה לידי ביטוי בתופעות שונות – למשל, משך הזמן הארוך שנדרש לילדים כדי להגיע לשליטה במערכת המספרים, וקיומן של לקויות-למידה רבות ושונות שפוגעות בעיבוד מספרים.

אחד האתגרים בעיבוד מספרים הוא תרגומם מייצוג לייצוג. התרגום עשוי להיות קל יחסית במקרה של מספרים חד-ספרתיים – אז ניתן אולי לבצע את התרגום ע"י מיפוי ישיר בין הייצוגים (בין "3", "שלוש" ו-◆◆◆). מספרים רב-ספרתיים, לעומת זאת, מהווים אתגר משמעותי יותר: כדי לתרגם אותם מייצוג אחד למשנהו יש לקחת בחשבון לא רק את הספרות ומילות המספר, אלא גם את היחסים והתלויות ביניהן. למעשה, מספרים רב-ספרתיים הופכים את מערכת המספר ממערכת סימבולית פשוטה למערכת שמייצגת את מבנה המספר – כלומר, מערכת תחבירית.

בעבודת הדוקטורט הזו חקרתי שני תהליכים של המרת מספרים רב-ספרתיים בין ייצוגים: כיצד ממירים רצף ספרות לכמות על מנת להבין את המספר, וכיצד מתרגמים רצף ספרות לרצף מילות מספר על מנת לקרוא את המספר בקול. לגבי כל אחד מתהליכי ההמרה האלה, התמקדתי במנגנונים התחביריים שמעבדים את מבנה המספר.

המרת רצף ספרות לכמות. על מנת לחקור את תהליך ההמרה הזה, פיתחנו שיטת מחקר חדשה: המשתתפים ראו מספרים רב-ספרתיים והצביעו למקום המתאים על פני ציר מספרים, תוך כדי שאנו עוקבים באופן רצוף אחרי תנועת האצבע שלהם. המיקום שהמשתתף מסמן על ציר המספרים משקף את האופן בו הוא מייצג כמות; מסלול האצבע בין נקודת המוצא לבין ציר המספרים משקף את התהליך של בניית הייצוג הכמותי הזה. הפרידגמה הזו שימשה אותנו כדי לחקור מספר שאלות לגבי המרת ספרות לכמות.

שאלה אחת שחקרנו היא האם ייצוג הכמות נסמך על סקלה ליניארית או לוגריתמית. מחקרים קודמים הראו שמבוגרים בעלי השכלה מְמַפִּים מספרים אל ציר מספרים באופן ליניארי. בסדרה של 8 ניסויים שערכנו עם 174 משתתפים, גם הם הראו מיפוי ליניארי שכזה על ציר המספרים; אך במקביל לכך, הם הראו דפוס לוגריתמי כאשר בדקנו את מיקום האצבע בשלבי-הביניים של התנועה. הדפוס הלוגריתמי הזה נבע מזמני עיבוד שונים של כמויות גדולות בהשוואה לקטנות: המשתתפים עיבדו מספרים קטנים מהר יותר ממספרים גדולים, וכתוצאה מכך האצבע

סטתה לכיוון מספר קטן בשלב מוקדם יותר מהשלב בו סטתה לכיוון מספר גדול. האפקט הזה – עיבוד מהיר יותר של מספרים קטנים – נובע ככל הנראה מייצוג לא-ליניארי של כמויות.

שאלה מרכזית נוספת היתה האם אנו מעבדים את הספרות של מספר רב-ספרתי במקביל או באופן סדרתי. כדי לבחון את השאלה הזו, השתמשנו ב-2 ניסויים של מטלת מיפוי מספר למקום. המספרים שהופיעו היו דו-ספרתיים, ודאגנו שספרת העשרות וספרת היחידות יופיעו על המסך עם הפרש-זמן מסוים ביניהן. כאשר ספרת היחידות הופיעה אחרי ספרת העשרות, השפעתה על תנועת האצבע התעכבה גם כן, אך משך העיכוב היה פחות מהצפוי: העיכוב המוטורי (על תנועת האצבע) היה קצר ב-35 מילישניות מהעיכוב הויזואלי (בין ספרת העשרות לספרת היחידות). הדבר מצביע על כך שמסלול העיבוד של ספרת היחידות מכיל פרק זמן של 35 מילישניות במהלכו לא מתבצעת שום פעולה (idle time window). אנו מציעים כי פרק הזמן הזה נובע מכך שהמערכת ממתינה לבנייה של תבנית תחבירית של המספר – ייצוג של המבנה העשרוני של המספר הדו-ספרתי.

בדקנו גם את מנגנוני קבלת ההחלטות המעורבים בתהליך מיפוי המספר למקום. תאוריות בייסיאניות מנבאות שקבלת-החלטות אופטימלית מתחילה בהתפלגות ראשונית (prior) של אפשרויות התגובה, שמתבססת על צעדים קודמים, ומעדכנת אותה בהתאם לצעד הנוכחי. בדקנו את הניבוי הזה באמצעות המטלה שלנו ע"י בקרה על 3 משתנים: הכיוון הראשוני של האצבע, התפלגות מספרי המטרה בצעדים הקודמים, ומספר המטרה בצעד הנוכחי. כפי שניבאה התאוריה הבייסיאנית, תנועת האצבע הושפעה ע"י 3 המשתנים האלה, בסדר זה.

בסיום הפרק העוסק בתרגום ספרות לכמות, אנו מציעים מודל קוגניטיבי מפורט של התהליכים המעורבים במטלת מיפוי מספר למקום. מודל זה כולל 3 שלבים: (1) תרגום רצף הספרות לייצוג הכמות. (2) תהליך איסוף מידע (accumulation of evidence) בייסיאני, שמוביל להחלטה לגבי מיקום היעד – ראשית בהסתמך על צעדים קודמים (prior) ואז לפי המספר שמופיע בצעד הנוכחי. (3) תנועת האצבע. בנוסף, המודל מפרט מספר תתי-שלבים המעורבים בתרגום רצף הספרות לייצוג הכמות: בניית התבנית התחבירית של המספר הרב-ספרתי; שיוך כל ספרה לתפקיד עשרוני בתבנית זו (יחידות, עשרות, וכו'); תרגום הספרה לכמות, תוך כך שלוקחים בחשבון את התפקיד העשרוני שלה; ואינטגרציה של כמויות אלה (של הספרות הבודדות) לכמות כוללת שמתארת את המספר כולו – בייצוג ליניארי ובייצוג לוגריתמי.

המרת רצף ספרות לרצף מילות מספר. על מנת לזהות את השלבים בתהליך ההמרה הזה, חקרנו את תפקודם של 7 משתתפים עם קשיים ספציפיים שונים בקריאת מספרים. לחלק מהמשתתפים היה ליקוי בעיבוד הויזואלי של רצף הספרות – בקידוד סדר הספרות, בקידוד אורך המספר, או בחלוקה של המספר לשלשות. למשתתפים אחרים היה ליקוי במנגנוני הפלט המילולי של מילות מספר – הם הפיקו מספרים במבנה שגוי. בהתבסס על הליקויים שנמצאו בקרב המשתתפים שבדקנו, ובהתבסס על מחקרים קודמים, אנו מציעים מנגנון קוגניטיבי מפורט של התהליכים המעורבים בקריאת מספרים. המודל מניח שבעיבוד הויזואלי של המספר, תהליכים נפרדים מקודדים את זהות הספרות ואת סדרן היחסי, ותהליכים נוספים מקודדים את המבנה העשרוני של המספר: ארכו, החלוקה של הספרות לשלשות, ומיקומי הספרה 0. מנגנוני הפלט המילולי כוללים תהליך אחד שמייצר את המבנה המילולי של המספר, ראשית במבנה

דמוי עץ ואח"כ משטֵחַ אותו לכדי מידע חלקי על רצף מילות המספר; ותהליך נוסף ששולף את הצורה הפונולוגית של כל מילת מספר.

רמת הספציפיות של תהליכי הקריאה האלה נבדקה ב-2 מחקרים נוספים. הראשון ביניהם בדק האם קריאת מספרים (תרגום רצף ספרות לרצף מילים) משתמשת באותם מנגנונים קוגניטיביים כמו קריאת מילים (תרגום רצף אותיות לרצף מילים). לשם כך, סקרנו את תתי-התהליכים השונים המעורבים בקריאת מילים וקריאת מספרים, ולגבי כל אחד מתתי-התהליכים שאלנו האם הוא משרת קריאת מילים, קריאת מספרים, או את שני סוגי הקריאה. סקירה של מחקרים קודמים, בשילוב שתי דיסוציאציות שדיווחנו עליהן במחקר הנוכחי, הביאו למסקנה שמסלולי הקריאה של מילים ומספרים הם נפרדים כמעט לחלוטין. אנו מציעים כי סיבה אפשרית להפרדה הזו היא ההבדלים בין המבנה המורפו-תחבירי של מילים לבין זה של מספרים.

המחקר השני בדק האם קריאת מספרים (תרגום רצף ספרות לרצף מילים) משתמשת באותם מנגנונים קוגניטיביים כמו הבנת מילים (תרגום רצף ספרות לכמות). גם כאן התשובה היתה שלילית: חקרנו את ZN, אדם עם אפזיה שלא היה מסוגל לקרוא בקול מספרים דו-ספרתיים אך היה מסוגל להבין את הכמות שהם מייצגים. הדיסוציאציה הזו מעידה על כך שהתהליכים התחביריים, שמעבדים את מבנה המספר במהלך הקריאה שלו, נפרדים מהתהליכים התחביריים המעבדים את מבנה המספר במהלך התרגום שלו לכמות.

עובדות חישוב. החלק האחרון של עבודת הדוקטורט הזו חקר מקור אפשרי לקושי בלמידת לוח הכפל: רגישות-יתר להפרעה, מצב בו קיים קושי חריף לזכור בע"פ עובדות מילוליות הדומות זו לזו. מחקרים קודמים הראו עדויות למתאם בין רגישות-יתר להפרעה לבין ידע לקוי של לוח הכפל. במחקר הנוכחי, אנו מציגים עדויות סיבתיות לקשר הזה. ערכנו מחקר התערבות בו רמת ההפרעה היתה מבוקרת, והראינו שמצב של הפרעה גבוהה מוביל לקושי משמעותי יותר בשינון לוח הכפל בהשוואה למצב של הפרעה מעטה. אנו מציעים השוואה אפשרית בין רגישות להפרעה בשינון לוח הכפל ובמצבים אחרים, ומעלים את האפשרות שרגישות להפרעה היא תכונה של מערכות תחביריות, שמקודדות את הקשרים בין פריטים. בנוסף, ההצלחה של תכנית ההתערבות שלנו מצביעה על כך שבניגוד לשיטת הלימוד המקובלת בבתי ספר, שינון אפקטיבי של לוח הכפל דורש לקבץ יחדיו תרגילי כפל שונים ככל האפשר זה מזה. מסקנה זו מצביעה על צורך אפשרי לשקול מחדש את אופן הלימוד של לוח הכפל בכיתות היסוד.

לסיכום, עבודת דוקטורט זו הביאה לפיתוח של 2 מודלים קוגניטיביים מפורטים של עיבוד מספרים רב-ספרתיים: מודל אחד מתאר את התהליכים המעורבים בתרגום מספר רב-ספרתי לכמות, ומודל שני מתאר את התהליכים של תרגום מספר רב-ספרתי למילות מספר ולדיבור. במהלך המחקר פיתחנו פרדיגמה מחקרית חדשה (מיפוי מספר למקום עם מעקב אחרי תנועת אצבע), וסוללת מבדקים לאבחון של לקויות למידה בעיבוד מספרים סימבוליים. בנוסף, פיתחנו שיטת התערבות לטיפול בקשיים בלמידת לוח הכפל.

אוניברסיטת תל אביב
בית הספר לחינוך
ע״ש חיים וג׳ואן קונסטנטינר

עיבוד מספרים רב-ספרתיים:
מנגנונים קוגניטיביים וליקויים בהם

חיבור לשם קבלת התואר ״דוקטור לפילוסופיה״

מאת דרור דותן

הוגש לסנאט של אוניברסיטת תל אביב

יוני, 2017